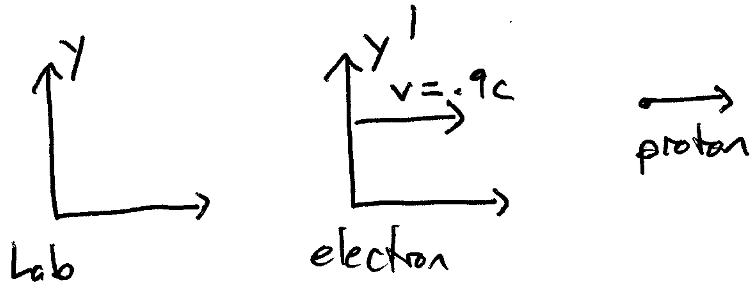
21]



Using $u = \dfrac{u' + v}{1 + uv/c^2}$, where u = speed of proton in lab frame

u' = speed of proton in e^- frame,

we find $u = \dfrac{.7c + .9c}{1 + (.7)(.9)} = \boxed{0.98c}$ as the speed of the proton in the lab frame.

23] The speed of light in the water's (moving) frame is c/n. And the water is moving at a speed v.

So in the lab frame, we will measure

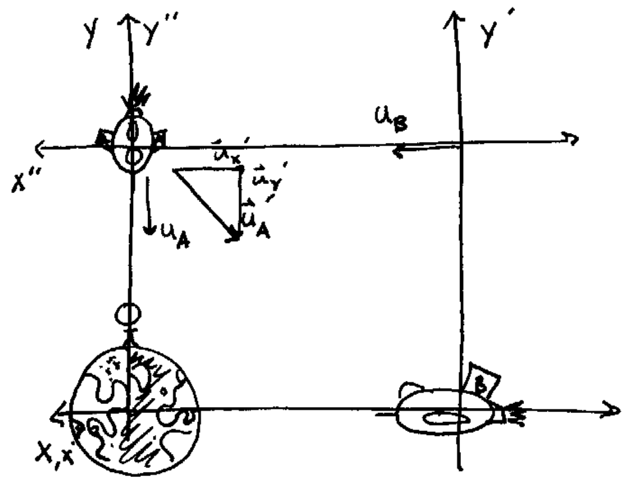
$$u = \frac{c/n + v}{1 + \frac{(c/n)v}{c^2}} = \boxed{\frac{c}{n}\left[\frac{1 + nv/c}{1 + v/nc}\right]}$$

b] For $v/c \ll 1$, $\dfrac{1}{1 + \frac{v}{nc}} \approx 1 - \dfrac{v}{nc}$

So $\dfrac{c}{n}\left[\dfrac{1 + nv/c}{1 + v/nc}\right] \approx \dfrac{c}{n}\left(1 + \dfrac{nv}{c}\right)\left(1 - \dfrac{v}{nc}\right) \approx \boxed{\dfrac{c}{n} + v - \dfrac{v}{n^2}}$, where I've dropped the $\left(\frac{v}{c}\right)^2$ term b/c it's small.

#25.) As seen from Earth, two spaceships A and B are approaching along perpendicular directions. If A is observed by an Earth observer to have velocity $u_y = -0.9c$ and B to have velocity $u_x = 0.90c$, find the speed of ship A as measured by the pilot of B.

This is a little more complicated. The picture is:



We need to Lorentz transform both x and y components of ship A's velocity using eqs. 1.32 and 1.33

$$u_i' = \frac{u_i - v}{1 - \frac{u_i v}{c^2}}$$

$$\Longrightarrow u_{Ax}' = \frac{u_{Ax} - u_{Bx}}{1 - \frac{u_{Ax}u_{Bx}}{c^2}} = -u_{Bx} = -0.90\,c$$

$$u_{Ay}' = \frac{u_{Ay} - u_{By}}{\gamma\left(1 - \frac{u_{Ay}u_{By}}{c^2}\right)} = \frac{u_{Ay}}{\gamma} = -0.392\,c$$

The speed of A seen by B is

$$\boxed{V_A' = \sqrt{u_{Ax}'^2 + u_{Ay}'^2} = 0.982\,c}$$

29

a) So $F = qE = \dfrac{dp}{dt}$.

Relativistically, though, $p = \dfrac{mv}{\sqrt{1-v^2/c^2}}$.

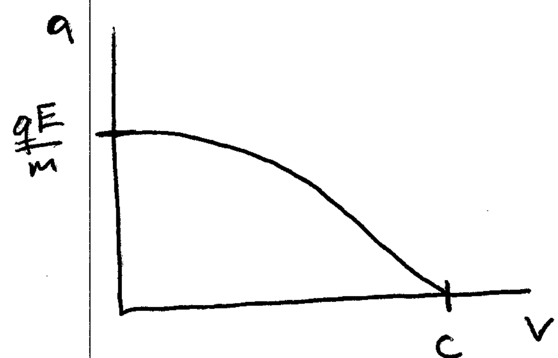
$$\text{So } qE = m\left[\frac{dv}{dt}\frac{1}{\sqrt{1-v^2/c^2}} + \frac{\cancel{m}v}{(1-v^2/c^2)^{3/2}}\cdot\frac{-1}{2}\overbrace{\left(-\frac{2v}{c^2}\right)\frac{dv}{dt}}^{\text{from } \frac{d}{dt}\left(\frac{v^2}{c^2}\right) = \frac{2v}{c^2}\frac{dv}{dt}}\right]$$

$$\Rightarrow qE = m\left[\frac{dv}{dt}\frac{1}{\sqrt{1-v^2/c^2}} + \frac{v^2/c^2}{(1-v^2/c^2)^{3/2}}\frac{dv}{dt}\right]$$

$$\Rightarrow qE = m\frac{dv}{dt}\left(1-v^2/c^2\right)^{-3/2}$$

$$\text{So } \boxed{a = \frac{dv}{dt} = \frac{qE}{m}\left(1-v^2/c^2\right)^{3/2}}$$

b) Let's graph this:



So as $v \to c$, $a \to 0$. Can't accelerate past $v = c$!

c] $$\frac{dv}{dt} = \frac{qE}{m}\left(1 - v^2/c^2\right)^{3/2}$$

So $$\frac{dv}{(1-v^2/c^2)^{3/2}} = \frac{qE}{m}\,dt$$

Integrate both sides: $$\int_0^v \frac{dv'}{(1-v'^2/c^2)^{3/2}} = \int_0^t \frac{qE}{m}\,dt'$$

$$\int \frac{d\cancel{x}}{(1-v^2/c^2)^{3/2}} = \frac{v}{(1-v^2/c^2)^{1/2}}$$ (look in table or do it)

So $$\frac{v}{(1-v^2/c^2)^{1/2}} = \frac{qE}{m}t$$

$$\Rightarrow \frac{v^2}{1-v^2/c^2} = \left(\frac{qE}{m}\right)^2 t^2 \Rightarrow \boxed{v = \frac{qE}{m}\left[\frac{t}{\left(1+\left(\frac{qEt}{mc}\right)^2\right)^{1/2}}\right]}$$

Note: As $t \to \infty$, $v \to c$, which is easy to see from rewriting $v = \dfrac{qEct}{\sqrt{(mc)^2+(qEt)^2}}$. Nice!

Using $v = \frac{dx}{dt}$, integrate again:

$$\int_0^x dx' = \int_0^t \frac{qE}{m}\left(\frac{t'}{\sqrt{1+\left(\frac{qEt'}{mc}\right)^2}}\right) dt'$$

Writing (for the RHS) $u = 1 + \left(\frac{qEt'}{mc}\right)^2$, we see

$$du = 2t'\left(\frac{qE}{mc}\right)^2 dt'$$

So $$\int \frac{qE}{m}\left(\frac{t'\,dt'}{\sqrt{1+\left(\frac{qEt'}{mc}\right)^2}}\right) = \int \frac{qE}{m}\, du \cdot \frac{1}{2}\left(\frac{mc}{qE}\right)^2 u^{-1/2}$$

$$= \frac{1}{2}c^2\left(\frac{m}{qE}\right)\cdot 2u^{1/2}\Bigg|_{u=1}^{u=1+\left(\frac{qEt}{mc}\right)^2}$$

$$= c^2\left(\frac{m}{qE}\right)\left[\sqrt{1+\left(\frac{qEt}{mc}\right)^2} - 1\right]$$

So $$\boxed{x = \frac{c}{qE}\left[\sqrt{(mc)^2 + (qEt)^2} - mc\right]}$$

As $t \to \infty$, $x \to ct$. As expected for something moving with speed c!

#37.) An electron has a speed of 0.75c. Find the speed of a proton that has

a.) The same kinetic energy as the electron

The kinetic energy of the $\bar{e}$ is

$$K_e = E_e - m_e c^2$$
$$= (\gamma_e - 1) m_e c^2$$
$$= \left[\frac{1}{\sqrt{1-\left(\frac{0.75c}{c}\right)^2}} - 1\right](0.511 \text{ MeV})$$
$$= 261.6 \text{ keV}$$

The kinetic energy of the proton will be

$$K_p = (\gamma_p - 1) m_p c^2 = K_e$$
$$\gamma_p = \frac{K_e}{m_p c^2} + 1 = \frac{1}{\sqrt{1-\frac{v_p^2}{c^2}}}$$
$$\Rightarrow 1 - \frac{v_p^2}{c^2} = \left[\frac{K_e}{m_p c^2} + 1\right]^{-2}$$
$$v_p = c\sqrt{1 - \left[\frac{K_e}{m_p c^2} + 1\right]^{-2}}$$
$$\boxed{v_p = 0.0236\, c}$$

b.) The same momentum as the electron

Relativistic momentum is given by eq. 1.35 as

$$p = \gamma m u$$

We require

$$p_e = p_p$$
$$\gamma_e m_e u_e = \gamma_p m_p u_p$$

#37.) continued

$$\gamma_e m_e u_e = \frac{m_p u_p}{\sqrt{1-\frac{u_p^2}{c^2}}}$$

$$\left(1-\frac{u_p^2}{c^2}\right) p_e^2 = m_p^2 u_p^2$$

$$p_e^2 - \frac{u_p^2 p_e^2}{c^2} = m_p^2 u_p^2$$

$$u_p^2\left[m_p^2 + \frac{p_e^2}{c^2}\right] = p_e^2$$

$$u_p = \frac{p_e}{\sqrt{m_p^2 + \frac{p_e^2}{c^2}}} = \frac{\gamma_e m_e u_e}{\sqrt{m_p^2 + \left(\frac{\gamma_e m_e u_e}{c}\right)^2}}$$

$$\gamma_e = \frac{1}{\sqrt{1-0.75^2}} = 1.51$$

$$\therefore \boxed{u_p = 6.17 \times 10^{-4} c}$$

#38.) Protons in an accelerator at the Fermi National Laboratory near Chicago are accelerated to an energy of 400 times their rest energy.

a.) What is the speed of these protons?

The problem tells us

$$E = 400 mc^2 = \gamma mc^2$$

$$\Rightarrow \gamma = 400$$

From problem 34, eq. (1)

$$v = c\sqrt{1-\frac{1}{\gamma^2}}$$

$\boxed{v = 0.999997c}$ Holy crap!

#38.) continued

b.) What is their kinetic energy in MeV?

$$K = E - mc^2$$

$$= 399\ mc^2$$

$$\boxed{K = 3.74 \times 10^5\ \text{MeV}}$$

#39.) How long will the Sun shine, Nellie? The Sun radiates about 4.0×10^{26} J of energy into space each second. (Who is Nellie? Am I Nellie?)

a.) How much mass is released as radiation each second?

Just use

$$E = mc^2$$

$$\Rightarrow \boxed{m = \frac{E}{c^2} = 4.4 \times 10^9\ \text{kg}}$$

b.) If the mass of the sun is 2.0×10^{30} kg, how long can the sun survive if the energy release continues at the present rate?

From part a.), the sun loses mass at the rate

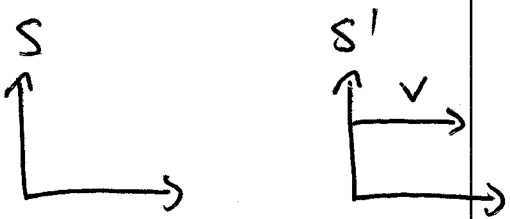
$$\frac{\Delta m}{\Delta t} = \frac{4.4 \times 10^9\ \text{kg}}{1\ \text{sec}}$$

$$\Rightarrow (2.0 \times 10^{30}\ \text{kg})\left(\frac{1\ \text{sec}}{4.4 \times 10^9\ \text{kg}}\right) = 454 \times 10^{18}\ \text{sec}$$

$$\therefore \boxed{\Delta t = 1.4 \times 10^{13}\ \text{yrs.}}$$

Smoke 'em if you got 'em

41]



So $p' = \dfrac{mu'}{\sqrt{1-u'^2/c^2}}$, $E' = \dfrac{mc^2}{\sqrt{1-u'^2/c^2}}$

$$u' = \frac{u-v}{1-\frac{uv}{c^2}}.$$

Thus $1-u'^2/c^2 = 1 - \dfrac{1}{c^2}\left(\dfrac{u-v}{1-\frac{uv}{c^2}}\right)^2 = 1 - \dfrac{(u-v)^2}{c^2\left(1-\frac{uv}{c^2}\right)^2}$

$$\Rightarrow \boxed{p' = m\left[\frac{u-v}{1-\frac{uv}{c^2}}\right]\left[1-\frac{(u-v)^2}{c^2\left(1-\frac{uv}{c^2}\right)^2}\right]^{-1/2}}$$

Nastay!

and $\boxed{E' = mc^2\left[1-\dfrac{(u-v)^2}{c^2\left(1-\frac{uv}{c^2}\right)^2}\right]^{-1/2}}$

(So $p' \neq p$, $E' \neq E$).

b] Lots of algebra, but it works. I'm not gonna do it here.

45] $n \to p + e^- + \bar{\nu}$

Total initial energy $E_i = m_n c^2 = 939.6$ MeV.

Total final energy $E_f = m_p c^2 + m_e c^2 + m_{\bar{\nu}} c^2 + K$

$m_p c^2 = 938.3$ MeV

$m_e c^2 = 0.511$ MeV

$m_{\bar{\nu}} c^2 = 0$

So $E_f = (938.3 + 0.511 + (0.781 \pm 0.005))$ MeV

$= 939.592 \pm 0.005$ MeV

939.6 MeV is in this range, so it's consistent.

<u>DAMN!</u>

#47.) The K° meson is an uncharged member of the particle "zoo" that decays into two charged pions according to

$$K^\circ \rightarrow \pi^+ + \pi^-$$

The pions have opposite charges as indicated and have the same mass, $m_\pi = 140\ \mathrm{MeV/c^2}$. Suppose that a K° at rest decays into two pions in a bubble chamber in which a magnetic field of 2.0T is present. If the radius of curvature of the pions is 34.4cm find

a.) the momenta and speeds of the pions

We can use #31.) from the last homework set

$$p = 300\ BR$$

Plug in values to get

$$\boxed{p = 206\ \mathrm{MeV/c}}$$

Since the K°'s momentum is zero, the pions' momenta must be equal and opposite in direction

$$p_{\pi^+} = -p_{\pi^-}$$

Note that

$$p = \gamma m u$$
$$E = \gamma m c^2$$

$$\Rightarrow \frac{p}{E} = \frac{u}{c^2}$$

$$\therefore \boxed{u = \frac{pc^2}{E} = \frac{(206\ \mathrm{MeV/c})\ c^2}{pc/\sqrt{p^2c^2 + m^2c^4}} = 0.827c}$$

#47.) continued

b.) the mass of the K^0 meson

Conservation of mass-Energy tells us

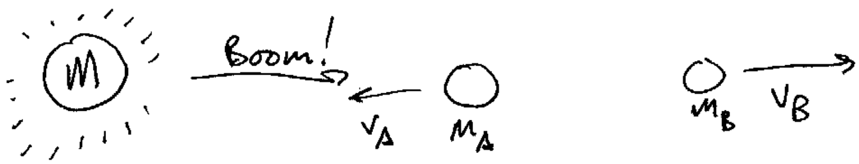
$$E_{K_0} = E_{\pi^+} + E_{\pi^-} = 2E$$

$$E_{K_0} = m_{K^0} c^2$$

$$2E = 2\sqrt{p_\pi^2 c^2 + m_\pi^2 c^4} = 498\,\text{MeV}$$

$$\therefore \quad \boxed{m_{K_0} = 498\ \text{MeV}/c^2}$$

#48.) An unstable particle having a mass of 3.34×10^{-27} kg is initially at rest. The particle decays into two fragments that fly off with velocities of $0.987c$ and $-0.868c$. Find the rest masses of the fragments



Conserve mass energy

$$Mc^2 = \gamma_A m_A c^2 + \gamma_B m_B c^2 \qquad (3)$$

$$\gamma_A = 6.22$$
$$\gamma_B = 2.01$$

Conserve momentum

$$\gamma_A m_A u_A = \gamma_B m_B u_B$$

$$\Rightarrow m_A = \frac{\gamma_B}{\gamma_A} m_B u_B = 0.284\, m_B$$

48.) continued

Substitute into (3) + plug in values

$$3.34\times10^{-27}\,\text{kg} = (6.22)(.284)\,m_A + 2.01\,m_A$$

$$\therefore\ m_A = 8.84\times10^{-28}\,\text{kg}$$

$$m_B = 0.284\,m_A = 2.51\times10^{-28}\,\text{kg}$$

#52.) If astronauts could travel at $V=0.95c$, we on Earth would say it takes $(4.2/0.95) = 4.4$ years to reach Alpha Centauri, 4.2 lightyears away. The astronauts disagree.

a.) How much time passes on the astronaut's clocks?

A time dilation problem. Recall

$$\Delta t' = \gamma \Delta t \quad (\Delta t \text{ proper time})$$

$$\Rightarrow \boxed{\Delta t = \frac{\Delta t'}{\gamma} = 1.37 \text{ years}}$$

b.) What distance to Alpha Centauri do the astronauts measure?

A Lorentz contraction problem. Recall

$$\Delta x' = \gamma \Delta x$$

$$\Rightarrow \boxed{\Delta x = \frac{\Delta x'}{\gamma} = 1.31 \text{ lightyears}}$$