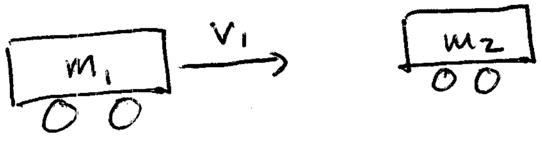
Physics 2D Winter '03
Week 1 HW Solns.

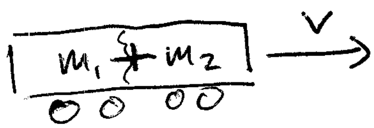
Ch. 1: #3, 4, 5, 6, 7, 8, 11, 13, 15, 17, 18, 19, 21, 23

3] In the frame where the 1500 kg car is at rest, let's write

where $m_1 = 2000$ kg
$v_1 = 20$ m/s
$m_2 = 1500$ kg.



for before the collision and

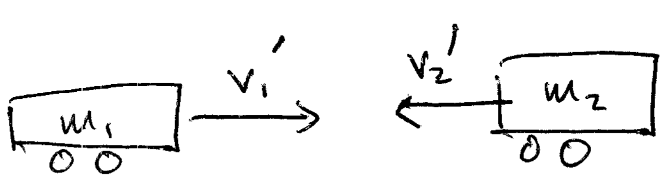


for after they stick together.

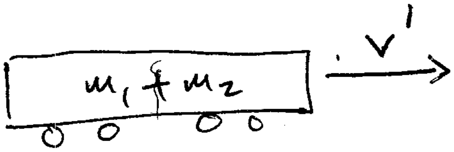
Conservation of momentum gives $\boxed{m_1 v_1 = (m_1 + m_2) v}$ in this frame.

In a frame moving @ 10 m/s in the direction of the moving car, we see

where $v_1' = v_1 - 10$ m/s $= 10$ m/s
$v_2' = 0 - 10$ m/s $= -10$ m/s



before the collision and



where $v' = v - 10\,\text{m/s}$

after.

The problem asks us to show that momentum is conserved here. In other words, check that

$$m_1 v_1' + m_2 v_2' = (m_1 + m_2) v'.$$

Well, $m_1 v_1' = m_1(v_1 - 10) = m_1 v_1 - 10 m_1$

$m_2 v_2' = m_2 (0 - 10) = -10 m_2$

and $(m_1 + m_2) v' = (m_1 + m_2)(v - 10)$

So

$$m_1 v_1' + m_2 v_2' = (m_1 + m_2) v'$$

$$\Downarrow \qquad \qquad \Downarrow$$

$$m_1 v_1 - 10 m_1 - 10 m_2 = (m_1 + m_2) v - 10(m_1 + m_2)$$

$$\Downarrow$$

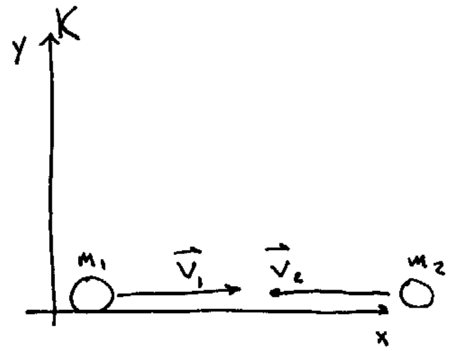
$$m_1 V_1 = (m_1 + m_2) v$$

which is true!

So, working backwards, we can see that momentum is conserved in the moving frame b/c it's conserved in the rest frame. ▥

Physics 2D Homework - Ch.1

#4.) Let frame K be the frame in which $m_1 = 0.3\,kg$, $u_{1_0} = 5\,m/s$, $m_2 = 0.2\,kg$, $u_{2_0} = -3\,m/s$

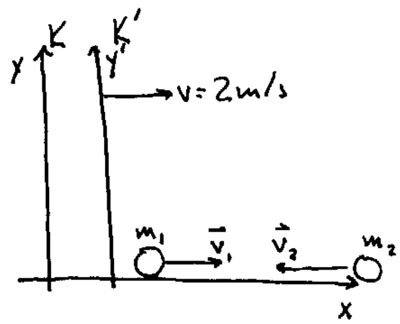


Because momentum is conserved, we know

$$\vec{p}_{1_0} + \vec{p}_{2_0} = \vec{p}_{1f} + \vec{p}_{2f}$$

$$\therefore \boxed{m_1 u_{10} + m_2 u_{20} = m_1 u_{1f} + m_2 u_{2f}} \qquad (1)$$

Let frame K' be the frame with velocity $V = 2\,m/s$ relative to K



Because $u_{10}, u_{20} \ll c$, transform coordinates using Galilean transformation (eq. 1.1):

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

$$\Rightarrow \quad x = x' + vt$$
$$y = y'$$
$$z = z'$$
$$t = t'$$

$$\therefore \quad u_x = u_x' + v$$
$$u_y = u_y'$$
$$u_z = u_z'$$

Plug these transformations into equation (1):

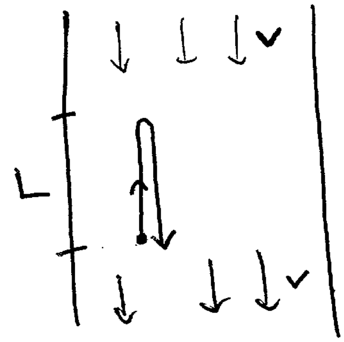
$$m_1(u_{10}' + v) + m_2(u_{20}' + v) = m_1(u_{1f}' + v) + m_2(u_{2f}' + v)$$

$$(m_1 u_{10}' + m_2 u_{20}') + (m_1 + m_2)v = (m_1 u_{1f}' + m_2 u_{2f}') + (m_1 + m_2)v$$

$$\therefore \quad m_1 u_{10}' + m_2 u_{20}' = m_1 u_{1f}' + m_2 u_{2f}'$$

$$\Rightarrow \boxed{\vec{p}_{10}' + \vec{p}_{20}' = \vec{p}_{1f}' + \vec{p}_{2f}'} \quad \blacksquare$$

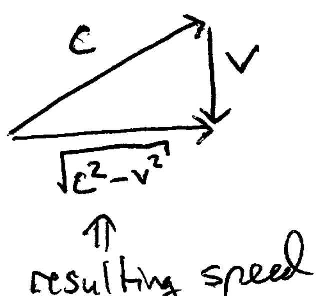
5] A picture:



for flying upwind & downwind

$$\text{Time} = \frac{\text{Dist}}{\text{Speed}} = \underset{\Uparrow \atop \text{upwind}}{\frac{L}{c-v}} + \underset{\Uparrow \atop \text{downwind}}{\frac{L}{c+v}} = \boxed{\frac{2Lc}{c^2-v^2} \equiv t_1}$$

For flying crosswind, the plane has to aim up and to the right, since the wind will push it down. On the return trip, it must aim up and to the left. The resulting speed in either case ~~is~~ is $\sqrt{c^2-v^2}$, so $\boxed{t_2 = \frac{2L}{\sqrt{c^2-v^2}}}$ (for ~~both~~ total trip).



It's easy to compare these: Just write $t_1 = \frac{2L}{c(1-v^2/c^2)}$ and $t_2 = \frac{2L}{c\sqrt{1-v^2/c^2}}$. If $v<c$, then $1-v^2/c^2 < \sqrt{1-v^2/c^2}$

So $t_1 > t_2$.

b] Plugging in $L=100$ mi, $c=500$ mi/h, $v=100$ mi/h gives $t_1 - t_2 = \frac{2L}{c}\left(\frac{1}{1-v^2/c^2} - \frac{1}{\sqrt{1-v^2/c^2}}\right) = \boxed{.008 \text{ hrs} = 0.5 \text{ min.}}$

#6.) With what speed will a clock have to be moving in order to run at a rate that is one half the rate of a clock at rest?

Use Lorentz time transformation (eq. 1.28 or 1.30)

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

If we measure a length of time Δt with the clock at rest, we require

$$\Delta t' = 2\,\Delta t$$

$$t_f' - t_o' = \gamma\left(t_f - \frac{vx}{c^2}\right) - \gamma\left(t_o - \frac{vx}{c^2}\right)$$ (2)

$\boxed{\Delta t' = \gamma \Delta t}$ Time Dilation formula

$$2\Delta t = \gamma \Delta t$$

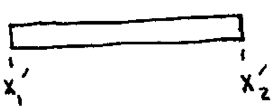
$$\Rightarrow \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 2$$

$$1 - \frac{v^2}{c^2} = \frac{1}{4}$$

$$\therefore \boxed{v = \sqrt{\frac{3}{4}}\,c = 0.866\,c}$$

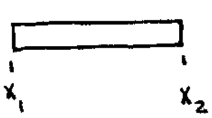
#9) A meter stick moving in a direction parallel to its length appears to be only 75 cm long to an observer. What is the speed of the meter stick relative to the observer?

At rest, the meter stick is observed to look like



$$\Delta x' = x_2' - x_1' = 1\,m$$

When moving with relative velocity v, it appears to look like



$$\Delta x = x_2 - x_1 = 0.75\,m = 0.75\,\Delta x'$$

7] $L = L'\sqrt{1 - v^2/c^2}$

$L/L' = 1/2$, so $\frac{1}{2} = \sqrt{1 - v^2/c^2}$

$$\Rightarrow \boxed{v = \frac{\sqrt{3}}{2}c}$$

8] 1 day = 86,400 sec.

So the clock ticks off 86,399 sec in what we measure to be 86,400 sec.

In $\Delta t = \gamma \Delta t'$, $\Delta t = 86,400$ sec

$\Delta t' = 86,399$ sec

$\Rightarrow \gamma = \Delta t/\Delta t' = 1.000011574$

Using $\gamma \approx 1 + v^2/2c^2$, we see $\frac{1}{2}\frac{v^2}{c^2} = 1.15 \times 10^{-5}$

$$\Rightarrow \boxed{v = 0.0048c = 1.44 \times 10^6 \text{ m/s}}$$

11] $\gamma = \dfrac{1}{\sqrt{1-v^2/c^2}} = \dfrac{1}{\sqrt{1-(.95)^2}} = 3.2$

So mean lifetime is $3.2(2.6\times10^{-8}\,s) = \boxed{8.32\times10^{-8}\,s}$

b] $\text{Dist} = \text{vel}\times\text{time} = (.95c)(8.32\times10^{-8}\,s) = \boxed{23.7\,m}$

#13.) An astronaut at rest on Earth has a heartbeat rate of 70 beats/min. What will this rate be when the astronaut is traveling in a spaceship at 0.90c as measured

a.) by an observer also in the ship?

The observer is at rest relative to the astronaut.

$\therefore$ $\boxed{\text{rate is still 70 beats/min}}$

b.) by an observer at rest on the Earth?

if heart rate is 70 beats/min, interval between beats is

$$\Delta t = \frac{1}{70} \text{ min/beat}$$

Use time dilation equation (2):

$$\Delta t' = \gamma \Delta t$$

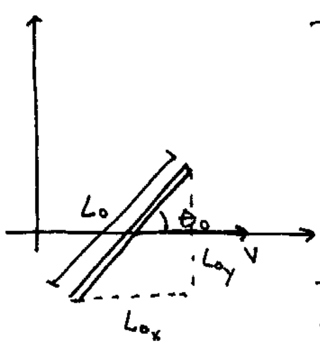
$$\Rightarrow \Delta t' = \frac{\Delta t}{\sqrt{1-\frac{v^2}{c^2}}} = 0.0328 \text{ min/beat}$$

$\therefore$ $\boxed{\text{rate is 30.5 beats/min}}$

#15.) A rod of length L_o moves with speed v along the horizontal direction. The rod makes an angle of θ_o with respect to the x' axis.

a.) Show that the length of the rod as measured by a stationary observer is given by

$$L = L_o\sqrt{1-\frac{v^2}{c^2}\cos^2\theta_o}$$



The length of the rod can be written as

$$L_o^2 = L_{ox}^2 + L_{oy}^2$$
$$= L_o^2\cos^2\theta_o + L_o^2\sin^2\theta_o$$

If the rod moves with velocity v in x direction, L_{ox} will Lorentz contract (use eq. (3)). L_{oy} will remain unaffected.

#15.) continued

Lorentz contraction

$\Delta x' = \gamma \Delta x$ (again, don't forget $\Delta x'$ is proper length! ($\Delta x' = L_{0x}$))

$L_0 \cos\theta_0 = \gamma \Delta x$

$$\Rightarrow \Delta x = \frac{L_0 \cos\theta_0}{\gamma} = L_0 \cos\theta_0 \sqrt{1 - \frac{v^2}{c^2}}$$

To observer at rest, the length of the rod will appear to be

$$L^2 = \Delta x^2 + L_{0y}^2$$

$$= L_0^2 \cos^2\theta_0 \left(1 - \frac{v^2}{c^2}\right) + L_0^2 \sin^2\theta$$

$$= L_0^2 (\sin^2\theta_0 + \cos^2\theta_0) - L_0^2 \frac{v^2}{c^2} \cos^2\theta_0$$

$$= L_0^2 \left(1 - \frac{v^2}{c^2} \cos^2\theta_0\right) \qquad (\text{recall } \sin^2\theta + \cos^2\theta = 1)$$

$$\therefore \boxed{L = L_0 \sqrt{1 - \frac{v^2}{c^2} \cos^2\theta_0}} \blacksquare$$

b.) Show that the angle that the rod makes with the x axis is given by the expression

$$\tan\theta = \gamma \tan\theta_0$$

In the rod frame

$$\tan\theta_0 = \frac{L_{0y}}{L_{0x}}$$

To the stationary observer,

$$\tan\theta = \frac{L_{0y}}{\Delta x}$$

from the last part,

$$\Delta x = \frac{L_{0x}}{\gamma}$$

$$\therefore \boxed{\tan\theta = \frac{L_{0y}}{L_{0x}/\gamma} = \gamma \frac{L_{0y}}{L_{0x}} = \gamma \tan\theta_0} \blacksquare$$

17] $f_{obs} = \frac{\sqrt{1+v/c}}{\sqrt{1-v/c}} f_{source}$

$f_{obs} = c/\lambda_{obs}$, $f_{source} = c/\lambda_{source}$

So $\frac{1}{\lambda_{obs}} = \frac{\sqrt{1+v/c}}{\sqrt{1-v/c}} \frac{1}{\lambda_{source}}$

$\Rightarrow \frac{\lambda_{source}}{\lambda_{obs}} = \frac{\sqrt{1+v/c}}{\sqrt{1-v/c}}$. $\lambda_{source} = 650\text{nm}$

$\lambda_{obs} = 550\text{nm}$.

So $\frac{\lambda_{source}}{\lambda_{obs}} = \frac{650}{550} = \frac{13}{11}$.

$\Rightarrow \left(\frac{13}{11}\right)^2 = \frac{1+v/c}{1-v/c} \Rightarrow \left(\frac{13}{11}\right)^2 - \frac{v}{c}\left(\frac{13}{11}\right)^2 = 1+v/c$

$\Rightarrow \frac{v}{c}\left(1+\left(\frac{13}{11}\right)^2\right) = \left(\frac{13}{11}\right)^2 - 1$

$\Rightarrow \boxed{v = 0.16c}$

18] a] Wavelength goes down → freq. goes up.
Galaxy must be approaching us!
Thus the blue shift.

As in ~~prob~~ problem 17, we see $\dfrac{\lambda_{source}}{\lambda_{obs}} = \sqrt{\dfrac{1+v/c}{1-v/c}}$

So using $\lambda_{source} = 550\,nm$, $\lambda_{obs} = 450\,nm$, and solving as in #17, one finds $\boxed{v = 0.198c}$

b] Wavelength ↑ → freq ↓ → galaxy is receding.

$\lambda_{source} = 550\,nm$, $\lambda_{obs} = 700\,nm$, so solving as above we find $\boxed{v = 0.237c}$

#19.) Doppler radar. An important practical application of the Doppler effect is the use of radar to determine the speed of a moving object. In this case the Doppler shift of the electromagnetic radar signal reflected from the moving object is directly proportional to the radial speed of the moving object with respect to the radar transmitter. If a police radar transmitter radiates at 10.0 GHz, calculate the frequency shift observed by the police for a car traveling at

a.) 60.0 mi/h

Use the Doppler shift equation for approaching source (eq. 1.13)

Initially, consider the radar gun as source, the car as the observer.

The car will receive the radar signal with frequency

$$f_{car} = \frac{\sqrt{1+\frac{v}{c}}}{\sqrt{1-\frac{v}{c}}} f_{rad}$$

The car will reflect the signals at this frequency. However, since the car is moving, the radar receiver will see them Doppler shifted again

$$f_{rad}' = \frac{\sqrt{1+\frac{v}{c}}}{\sqrt{1-\frac{v}{c}}} f_{car}$$

$$= \frac{1+\frac{v}{c}}{1-\frac{v}{c}} f_{rad} \qquad (1+\tfrac{v}{c})(1-\tfrac{v}{c})^{-1} f_{rad}$$

Just like the last problem, $v \ll c$, so we can Taylor expand and simplify

$$(1-\tfrac{v}{c})^{-1} = (1-x)^{-1} = 1+x+x^2+\ldots \qquad x = \frac{v}{c}$$

$$\simeq 1+x$$

$$\therefore f_{rad}' \simeq (1+x)^2 f_{rad} \simeq (1+2x+\cancel{x^2}^{\to 0}) f_{rad}$$

$$\simeq (1+2x) f_{rad}$$

#19.) continued

$$\Delta f = f_{rad}' - f_{rad} = 2x\, f_{rad}$$

$$\therefore \frac{\Delta f}{f_{rad}} = 2x = \frac{2v}{c}$$

$$\Delta f = \frac{2v}{c} f_{rad} \qquad (4)$$

Make sure your units jive!

$$\boxed{\Delta f_1 = 1790 \text{ Hz}}$$

b.) 70.0 mi/h

Use eq. (4) to get

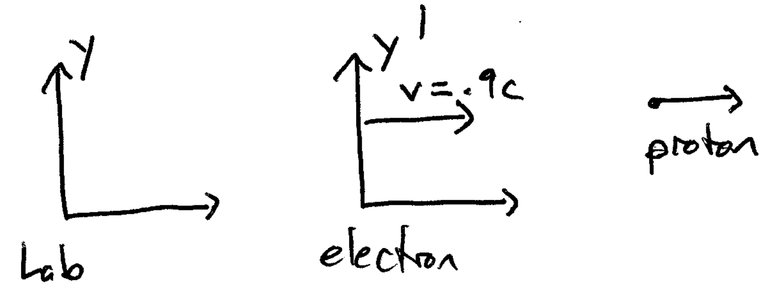
$$\Delta f_2 = 2090 \text{ Hz}$$

c.) The radar must be able to distinguish between the two cars, and therefore it must be sensitive enough to distinguish between the two frequency shifts Δf_1 + Δf_2

$$\Delta f = \Delta f_2 - \Delta f_1 = 300 \text{ Hz}$$

$$\therefore \boxed{\frac{\Delta f}{f} = \frac{300}{1 \times 10^{10}} = 3 \text{ parts in } 10^8}$$

21]



Using $u = \dfrac{u'+v}{1+uv/c^2}$, where u = speed of proton in lab frame

u' = speed of proton in e^- frame,

we find $u = \dfrac{.7c + .9c}{1+(.7)(.9)} = \boxed{0.98c}$ as the speed of

the proton in the lab frame.

23] The speed of light in the water's (moving) frame is c/n. And the water is moving at a ~~at~~ speed v.

So in the lab frame, we will measure

$$u = \frac{c/n + v}{1 + \frac{(c/n)v}{c^2}} = \boxed{\frac{c}{n}\left[\frac{1+nv/c}{1+v/nc}\right]}$$

b] For $v/c \ll 1$, $\dfrac{1}{1+\frac{v}{nc}} \approx 1 - \dfrac{v}{nc}$

So $\dfrac{c}{n}\left[\dfrac{1+nv/c}{1+v/nc}\right] \approx \dfrac{c}{n}\left(1+\dfrac{nv}{c}\right)\left(1-\dfrac{v}{nc}\right) \approx \boxed{\dfrac{c}{n} + v - \dfrac{v}{n^2}}$, where I've

dropped the $\left(\frac{v}{c}\right)^2$ term b/c it's small.