Some Useful Numbers, Equations and Identities

$$\text{Speed of Light, } c = 2.998\times10^{8}\text{m/s};\ \gamma = \frac{1}{\sqrt{1-v^2/c^2}}$$

$$x' = \gamma(x - vt);\quad t' = \gamma(t - \frac{xv}{c^2})$$

$$u'_x = \frac{u_x - v}{1-\frac{u_x v}{c^2}};\ u'_z = \frac{u_z}{\gamma(1-\frac{u_z v}{c^2})}$$

$$p = \frac{mu}{\sqrt{1-u^2/c^2}};\ E = \frac{mc^2}{\sqrt{1-u^2/c^2}} = K + mc^2$$

$$E^2 = p^2c^2 + m^2c^4$$

$$f_{obs} = \frac{\sqrt{1+v/c}}{\sqrt{1-v/c}}\, f_{source}$$

$$For\ x < 1; (1\pm x)^n = 1 \pm nx + \frac{n(n-1)}{2}x^2 \pm \frac{n(n-1)(n-2)}{3}x^3 + ..$$

$$\text{Planck's Constant, } h = 6.626\times10^{-34}\text{J}\times\text{ S} = 4.136\times10^{-15}\text{eV}\times\text{ S}$$

$$1\text{ eV} = 1.60\times10^{-19}\text{J};\quad 1\text{ MeV/c} = 5.344\times10^{-22}\text{Kg.m/s}$$

$$\text{Coulomb's Constant, } k = 8.99\times10^{9}\text{N}\times\text{m}^2/\text{C}^2$$

$$\text{Gravitational Constant, } G = 6.67\times10^{-11}\text{N}\times\text{ m}^2/\text{kg}^2$$

$$\text{Stefan} - \text{Boltzmann's Constant, } \sigma = 5.670399\times10^{-8}\text{W/m}^2\times\text{K}^4$$

$$\text{Wien's Wavelength Displacement Constant} = 2.898\times10^{-3}\text{m}\times\text{K}$$

$$\text{Boltzmann's Constant, } k = 1.381\times10^{-23}\text{J/K}$$

$$\text{Electron Mass} = 9.11\times10^{-31}\text{Kg};\quad \text{Electron Charge} = 1.602\times10^{-19}\text{C}$$

$$\text{Atomic Mass Unit } u = 1.6606\times10^{-27}\text{Kg or } 931.5\text{ MeV/c}^2$$

$$\text{Proton Mass} = 1.673\times10^{-27}\text{Kg or } 1.0073\text{u}$$

$$\text{Neutron Mass} = 1.675\times10^{-27}\text{Kg or } 1.0087\text{u}$$

$$\text{Electron Rest Energy} = 0.511\text{ MeV/c}^2;\quad \text{Proton Rest Energy} = 938\text{ MeV/c}^2$$

$$1\text{kW}\times\text{Hour} = 3.6\times10^{6}\text{J}$$

Intensity of Black Body Radiation $\mathrm{I} = \sigma \times \mathrm{T}^4$

$$\overrightarrow{F} = q\overrightarrow{v} \times \overrightarrow{B}$$

Centripetal Acc. $= v^2/r$ where v and r are velocity and radius of orbit

Photoelectric Effect $K_{max} = hf - \phi$

Compton Wavelength λ_c for scattering off electron $= 0.00243$ nm

In Compton Scattering $\Delta\lambda = \dfrac{h}{m_e c}(1 - \cos\theta)$

Construct. interfer. when path diff. between two adjacent rays is $d\sin\phi = n\lambda$

$$\text{TDSE}: \ -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} + U(x)\,\Psi(x,t) = i\hbar\frac{\partial \Psi(x,t)}{\partial t}$$

$$\text{TISE}: -\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x)}{\partial x^2} + U(x)\psi(x) = E\psi(x)$$

Ground state energy E_1 for particle in a 1D rigid box $= \dfrac{\pi^2\hbar^2}{2mL^2}$

Wavefunction for particle in a 1D rigid box $= \sqrt{\dfrac{2}{L}}\sin(\dfrac{n\pi}{L}x)$

$$[\hat{p}] = \frac{\hbar}{i}\frac{\partial}{\partial x}, \quad [\hat{p^2}] = -\hbar^2\frac{\partial^2}{\partial x^2}, \ [\hat{E}] = i\hbar\frac{\partial}{\partial t}$$

$$\Delta Q = \sqrt{\langle Q^2\rangle - \langle Q\rangle^2}$$

Time dependent form of Wave Function: $\Psi(\vec{r},t) = \psi(\vec{r})e^{-i\omega t}; \omega = E/\hbar$

$$< Q > = \int\int \Psi(x,t)^*[Q]\Psi(x,t)dxdt$$

$$< f(r) > = \int_0^\infty f(r)P(r)dr$$

$$\int u \cdot dv = uv - \int v \cdot du$$

$$\int \sin x\, dx = -cosx$$

$$\int \sin^2 x\,dx = \frac{x}{2} - \frac{\sin 2x}{4}\,; \quad \int \cos^2 x\,dx = \frac{x}{2} + \frac{\sin 2x}{4}$$

$$\int x\sin^2 x\,dx = \frac{x^2}{4} - \frac{x\sin 2x}{4} - \frac{\cos 2x}{8}$$

$$\int x\cos^2 x\,dx = \frac{x^2}{2} + \frac{x\sin 2x}{4} + \frac{\cos 2x}{8}$$

$$\sin(\theta_1)\sin(\theta_2) \;=\; (1/2)[\cos(\theta_1-\theta_2) - \cos(\theta_1+\theta_2)]$$

$$\cos(\theta_1)\cos(\theta_2) \;=\; (1/2)[\cos(\theta_1-\theta_2) - \cos(\theta_1+\theta_2)]$$

$$\sin(\theta_1)\cos(\theta_2) \;=\; (1/2)[\sin(\theta_1-\theta_2) - \sin(\theta_1+\theta_2)]$$

$$\int_{-1}^{+1} e^{-ax}\,dx = \frac{1}{a}[e^a - e^{-a}]$$

$$\int_{-1}^{+1} xe^{-ax}\,dx = \frac{1}{a^2}[e^a - e^{-a} - a(e^a + e^{-a})]$$

$$\int_{0}^{+\infty} e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}},\; a > 0$$

$$\int_{0}^{+\infty} x^2 e^{-ax^2}\,dx = \frac{1}{4a}\sqrt{\frac{\pi}{a}},\; a > 0$$

$$\int_{0}^{+\infty} x^{(2n)} e^{-ax^2}\,dx = \frac{1.3..(2n-1)}{2^{(n+1)}}\sqrt{\frac{\pi}{a^{2n+1}}},\; a > 0$$

$$\int_{0}^{+\infty} x^{(2n+1)} e^{-ax^2}\,dx = \frac{n!}{2a^{(n+1)}},\; a > 0$$

$$\int_{0}^{+\infty} x^n e^{-x}\,dx = n!$$

$$\int_{0}^{+\infty} x^n e^{-x/\alpha}\,dx = n!\alpha^{n+1}$$

$$\int_{1}^{+\infty} e^{-ax}\,dx = \frac{e^{-a}}{a}$$

$$\int_{1}^{+\infty} xe^{-ax}\,dx = \frac{e^{-a}}{a^2}(1+a)$$

$$\int_0^b x^2 e^{-x}\, dx = 2 - (2 + 2b + b^2)e^{-b}$$

$$\int_0^b x^3 e^{-x}\, dx = 6 - (6 + 6b + 3b^2 + b^3)e^{-b}$$

$$\int_0^b x^4 e^{-x}\, dx = 24 - (24 + 24b + 12b^2 + 4b^3 + b^4)e^{-b}$$

$$\int_0^b x^2 e^{-x}\, dx = 2 - (2 + 2b + b^2)e^{-b}$$

The ground state wavefunction for 1D oscillator under $U(x) = \frac{1}{2}m\omega^2 x^2$ has the form:

$$\psi_0(x) \propto e^{-\frac{m\omega x^2}{2\hbar}}$$

The wavefunction for Oscillator's first excited state:

$$\psi_1(x) \propto \sqrt{\frac{m\omega}{\hbar}}\, x e^{-\frac{m\omega x^2}{2\hbar}}$$

Next excited state:

$$\psi_2(x) \propto (1 - \frac{2m\omega x^2}{\hbar})e^{-\frac{m\omega x^2}{2\hbar}}$$

The energy of the nth Oscillator state $E_n = (n + \frac{1}{2})\hbar\omega$.

Volume element in Sph. polar coordinates is $dV = r^2 dr sin\theta d\theta d\phi$

<u>Information about Hydrogenic atom with Z protons in the nucleus:</u>

$$\text{Reduced Mass } \mu = \frac{M_1.M_2}{M_1 + M_2}$$

$$\mathrm{R}_{1,0}(\mathrm{r}) = (\frac{\mathrm{Z}}{\mathrm{a_o}})^{3/2} 2\mathrm{e}^{-\frac{\mathrm{Zr}}{\mathrm{a_o}}}$$

$$\mathrm{R}_{2,0}(\mathrm{r}) = (\frac{\mathrm{Z}}{2\mathrm{a_o}})^{3/2} (2 - \frac{\mathrm{Zr}}{\mathrm{a_0}})\mathrm{e}^{-\frac{\mathrm{Zr}}{2\mathrm{a_o}}}$$

$$R_{2,1}(r) = (\frac{Z}{2a_o})^{3/2} \frac{Zr}{\sqrt{3}a_0} e^{-\frac{Zr}{2a_o}}$$

$$\text{Radial Prob. Density } P(r) \ = \ r^2|R(r)|^2$$

$$Y_o^o(\theta,\phi) = \Theta(\theta)\Phi(\phi) = \frac{1}{2\sqrt{\pi}}$$

$$Y_1^o(\theta,\phi) = \frac{1}{2}\sqrt{\frac{3}{\pi}}\cos\theta \text{ and } Y_1^{\pm 1}(\theta,\phi) = \mp\frac{1}{2}\sqrt{\frac{3}{2\pi}}\sin\theta e^{\pm i\phi}$$

$$\text{Orbit radius } r_n \ = \ \frac{n^2\hbar^2}{Zm_eke^2}$$

$$\text{bohr radius } a_0 = \frac{\hbar^2}{m_eke^2} = 0.0529 \text{ nm}$$

$$\text{Energy } E_n = \frac{-kZ^2e^2}{2a_o}\frac{1}{n^2} \text{ for } n = 1, 2, 3, 4, \ldots$$

$$\text{Rydberg Constant } R = 1.0973732 \times 10^7 m^{-1}$$
