Department of Physics
University of California San Diego

Modern Physics (2D)
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Final Exam (Mar 20 2003)

Some Useful Numbers, Equations and Identities

$$\text{Speed of Light, } c = 3.0 \times 10^8 \text{m/s}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$x' = \gamma(x - vt)$$

$$t' = \gamma(t - \frac{xv}{c^2})$$

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$$

$$u'_z = \frac{u_z}{\gamma(1 - \frac{u_z v}{c^2})}$$

$$p = \frac{mu}{\sqrt{1 - u^2/c^2}}$$

$$E = \frac{mc^2}{\sqrt{1 - u^2/c^2}} = K + mc^2$$

$$f_{obs} = \frac{\sqrt{1 + v/c}}{\sqrt{1 - v/c}} f_{source}$$

$$E^2 = p^2c^2 + m^2c^4$$

$$\text{Planck's Constant, } h = 6.626 \times 10^{-34} \text{J} \times \text{S}$$

$$\text{Planck's Constant, } h = 4.136 \times 10^{-15} \text{eV} \times \text{S}$$

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{J}$$

$$1 \text{ MeV/c} = 5.344 \times 10^{-22} \text{Kg.m/s}$$

$$hc = 1240 \text{ eV. nm}$$

$$\text{Coulomb's Constant, } k = 8.99 \times 10^9 \text{N} \times \text{m}^2/\text{C}^2$$

$$\text{Coulomb's law : } F = k\frac{Q_1 Q_2}{r^2}$$

$$\text{Gravitational Constant, } G = 6.67 \times 10^{-11} \text{N} \times \text{m}^2/\text{kg}^2$$

$$\vec{F} = q\vec{v} \times \vec{B}$$

Centripetal Acc. $= v^2/r$ where v and r are velocity and radius of orbit

Intensity of Black Body Radiation $I = \sigma \times T^4$

Stefan – Boltzmann's Constant, $\sigma = 5.670399 \times 10^{-8} W/m^2 \times K^4$

Wien's Wavelength Displacement Constant $= 2.898 \times 10^{-3} m \times K$

Photoelectric Effect $K_{max} = hf - \phi$

Boltzmann's Constant, $k = 1.381 \times 10^{-23} J/K$

Compton Wavelength for scattering off electron = 0.00243 nm

$$\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

Rydberg Constant $R = 1.0973732 \times 10^7 m$

Bohr Radius $a_0 = 0.529 \times 10^{-10} m$

Electron Mass $= 9.11 \times 10^{-31} Kg$

Electron Charge $= 1.602 \times 10^{-19} C$

Atomic Mass Unit $u = 1.6606 \times 10^{-27} Kg$ or $931.5\ MeV/c^2$

Proton Mass $= 1.67 \times 10^{-27} Kg$ or 1.0073 u

Neutron Mass $= 1.6750 \times 10^{-27} Kg$ or 1.0087 u

Electron Rest Energy $= 0.511\ MeV/c^2$

Proton Rest Energy $= 938\ MeV/c^2$

Atomic number of Gold Z = 79

In Rutherford Scattering of alpha particles : $\Delta n = \frac{k^2 Z^2 e^4 N n A}{4R^2 (1/2 m_\alpha v_\alpha^2)^2 \sin^4(\phi/2)}$

$1 kW \times Hour = 3.6 \times 10^6 J$

Construct. interfer. when path diff. between two adjacent rays is $d \sin\phi = n\lambda$

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} + U(x)\,\Psi(x,t) = i\hbar\frac{\partial \Psi(x,t)}{\partial t}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x)}{\partial x^2} + U(x)\psi(x) = E\psi(x)$$

Ground state energy E_1 for particle in a 1D rigid box $= \dfrac{\pi^2\hbar^2}{2mL^2}$

Wavefunction for particle in a 1D rigid box $= \sqrt{\dfrac{2}{L}}\sin(\dfrac{n\pi}{L}x)$

$$[\hat{p}] = \frac{\hbar}{i}\frac{\partial}{\partial x} \; and \; [\hat{p^2}] = -\hbar^2\frac{\partial^2}{\partial x^2}$$

$$\Delta Q = \sqrt{\langle Q^2\rangle - \langle Q\rangle^2}$$

For Stationary States: $\Psi(r,t) = \psi(\vec{r})e^{-i\omega t}$

$$< f(r) >= \int_0^\infty f(r)P(r)dr$$

Transmission Coeff. over a general 1D Barrier : $T(E) \approx e^{-\frac{2}{\hbar}\sqrt{2m}\int\sqrt{U(x)-E}\,dx}$

$$\int \sin x\, dx = -cos x$$

$$\int \sin^2 x\, dx = \frac{x}{2} - \frac{\sin 2x}{4}$$

$$\int x\sin^2 x\, dx = \frac{x^2}{4} - \frac{x\sin 2x}{4} - \frac{\cos 2x}{8}$$

$$\int \cos^2 x\, dx = \frac{x}{2} + \frac{\sin 2x}{4}$$

$$\int x\cos^2 x\, dx = \frac{x^2}{2} + \frac{x\sin 2x}{4} + \frac{\cos 2x}{8}$$

$$\int_{-1}^{+1} e^{-ax}\, dx = \frac{1}{a}[e^a - e^{-a}]$$

$$\int_{-1}^{+1} xe^{-ax}\, dx = \frac{1}{a^2}[e^a - e^{-a} - a(e^a + e^{-a})]$$

$$\int_0^{+\infty} e^{-ax^2}\, dx = \frac{1}{2}\sqrt{\frac{\pi}{a}},\; a > 0$$

$$\int_0^{+\infty} x^2 e^{-ax^2}\, dx = \frac{1}{4a}\sqrt{\frac{\pi}{a}},\; a > 0$$

$$\int_0^{+\infty} x^{(2n)} e^{-ax^2}\, dx = \frac{1.3..(2n-1)}{2^{(n+1)}}\sqrt{\frac{\pi}{a^{2n+1}}},\ a > 0$$

$$\int_0^{+\infty} x^{(2n+1)} e^{-ax^2}\, dx = \frac{n!}{2a^{(n+1)}},\ a > 0$$

$$\int_0^{+\infty} x^n e^{-x}\, dx = n!$$

$$\int_1^{+\infty} e^{-ax}\, dx = \frac{e^{-a}}{a}$$

$$\int_1^{+\infty} x e^{-ax}\, dx = \frac{e^{-a}}{a^2}(1+a)$$

$$\int_0^{b} x^2 e^{-x}\, dx = 2 - (2 + 2b + b^2)e^{-b}$$

$$\int_0^{b} x^3 e^{-x}\, dx = 6 - (6 + 6b + 3b^2 + b^3)e^{-b}$$

$$\int_0^{b} x^4 e^{-x}\, dx = 24 - (24 + 24b + 12b^2 + 4b^3 = b^4)e^{-b}$$

$$\int_0^{b} x^2 e^{-x}\, dx = 2 - (2 + 2b + b^2)e^{-b}$$

The ground state wavefunction for 1D oscillator under $U(x) = \frac{1}{2}m\omega^2 x^2$ has the form:

$$\psi_0(x) \propto e^{-\frac{m\omega x^2}{2\hbar}}$$

The wavefunction for first excited state:

$$\psi_1(x) \propto \sqrt{\frac{m\omega}{\hbar}}\, x e^{-\frac{m\omega x^2}{2\hbar}}$$

Next excited state:

$$\psi_2(x) \propto (1 - \frac{2m\omega x^2}{\hbar})e^{-\frac{m\omega x^2}{2\hbar}}$$

The energy of the nth state $E_n = (n + \frac{1}{2})\hbar\omega$.

Volume element in Sph. polar coordinates is $dV = r^2 dr sin\theta d\theta d\phi$

Information about Hydrogenic atom with Z protons in the nucleus

$$\text{Reduced Mass } \mu = \frac{M_1 . M_2}{M_1 + M_2}$$

$$R_{1,0}(r) = \left(\frac{Z}{a_o}\right)^{3/2} 2e^{-\frac{Zr}{a_o}}$$

$$R_{2,0}(r) = \left(\frac{Z}{2a_o}\right)^{3/2} \left(2 - \frac{Zr}{a_0}\right)e^{-\frac{Zr}{a_o}}$$

$$R_{2,1}(r) = \left(\frac{Z}{2a_o}\right)^{3/2} \frac{Zr}{\sqrt{3}a_0}e^{-\frac{Zr}{a_o}}$$

$$Y_o^o(\theta, \phi) = \Theta(\theta)\Phi(\phi) = \frac{1}{2\sqrt{\pi}}$$

$$Y_1^o(\theta, \phi) = \frac{1}{2}\sqrt{\frac{3}{\pi}} \cos\theta \text{ and } Y_1^{\pm 1}(\theta, \phi) = \mp\frac{1}{2}\sqrt{\frac{3}{2\pi}} \sin\theta e^{\pm i\phi}$$

$$\text{Orbit radius } r_n = \frac{n^2 \hbar^2}{Z m_e k e^2}$$

$$\text{bohr radius } a_0 = \frac{\hbar^2}{m_e k e^2} = 0.0529 \text{ nm}$$

$$\text{Energy } E_n = \frac{-kZ^2e^2}{2a_o}\frac{1}{n^2} \text{ for } n = 1, 2, 3, 4, ...$$

$$|L| = \sqrt{\ell(\ell+1)}\hbar\,,\ L_z = m_l\hbar; \quad |J| = \sqrt{j(j+1)}\hbar\,,\ J_z = m_j\hbar$$

The Total Magnetic Moment

$$\vec{\mu} = \vec{\mu}_o + \vec{\mu}_s = \frac{-e}{2m_e}[\vec{L} + g\vec{S}]$$

Magnetic Potential Energy Under a B Field

$$U = -\vec{\mu} \cdot \vec{B}$$

The Hitchhiker's guide to the galaxy says on its very first page :
" DON'T PANIC !"

Problem 1: **Star Wars!** [30 pts]

Stardate 44246.3: An armada (collection) of spaceships that is 1.0 ly long in its rest frame moves collectively with a speed 0.80c relative to a ground station S. The armada is headed towards a secret Romulan base. A messenger travels (with critical news) from the very rear of the armada to the front with a speed of 0.95c relative to S. How long does the trip take as measured (a) in the armada's rest frame (b) in the messenger's rest frame and (c) by an observer in frame S?

1 ly (light year) = 9.46×10^{15} m.

Problem 2: Gone in 10^{-10} Seconds [20 pts]

An observer in a laboratory measures a high speed Kaon particle (K^0) to be moving at a speed of 0.9c. This K^0 particle decays into two (equal mass, oppositely charged) pions $K^0 \rightarrow \pi^+\pi^-$. What are (a) the greatest (b) least speeds of the pions as measured by the observer in the lab frame.
You need to know that M_K=498 MeV/c^2 & M_π=140 MeV/c^2 .

Problem 3: Photoelectrons From Aluminum: [20 pts]

Light of wavelength 200 nm falls on an aluminum surface. In metallic Aluminum 4.2 eV are required to remove an electron. (a) What is the kinetic energy of the fastest photoelectron? (b) what is the stopping potential for this wavelength? (c) What is the cutoff wavelength for Al ? (d) If the intensity of the incident light is 2.0W/m^2, what is the average number of photons per second per unit area which strike the aluminum surface?

Problem 4: A Matter-Antimatter Atom: [20 pts]

A "Positronium" atom is a system that consists of a positron (anti-matter form of electron, with the same mass but opposite charge) and an electron that orbit each other. What are the (a) radius (b) energy of the ground state of this system in terms of the Hydrogen atom radius & energy. (c) Compare the wavelengths of the spectral lines of the Positronium with those of the ordinary Hydrogen atom, which one is larger?

Problem 5: You Can Run But You Can't Hide ! [30pts]

Consider a potential given by

$U(x) = 0$ for $x < 0$ and $U(x) = V_0$ for $x > 0$, where $V_0 > 0$.

A beam of particles with energy $E < V_0$ is incident on this potential from the left (the negative x-direction). (a) What is the wavefunction in the region x<0? (b) What is the wavefunction in the region x>0? (c) Write down the conditions that force the wavefunction to be smooth and continuous at x=0. (d) Draw the total wavefunction (for all x). (e) At what point x is the particle's probability $P(x) = \frac{1}{e} P(x=0)$?

Problem 6: Rapping About a 3-D Harmonic Oscillator [30 pts]

Consider a 3D harmonic oscillator of mass m under a potential

$$U(x,y,z) = \frac{1}{2} m(\omega_1^2 x^2 + \omega_2^2 y^2 + \omega_3^2 z^2) \text{ with } \omega_1 < \omega_2 < \omega_3$$

(a) Write the appropriate time-independent Schrodinger equation for this oscillator. (b) Write the wavefunction for the first excited state including the normalization constant (let's call it A) (c) Normalize the wavefunction and calculate the value of the normalization constant A (d) What is the energy of this state? (e) Is this state degenerate? Why (not)? (f) What is average potential energy of this state?

Problem 7: The Lithium Ion (Li^{++}): Check it out, Yo! [30 pts]

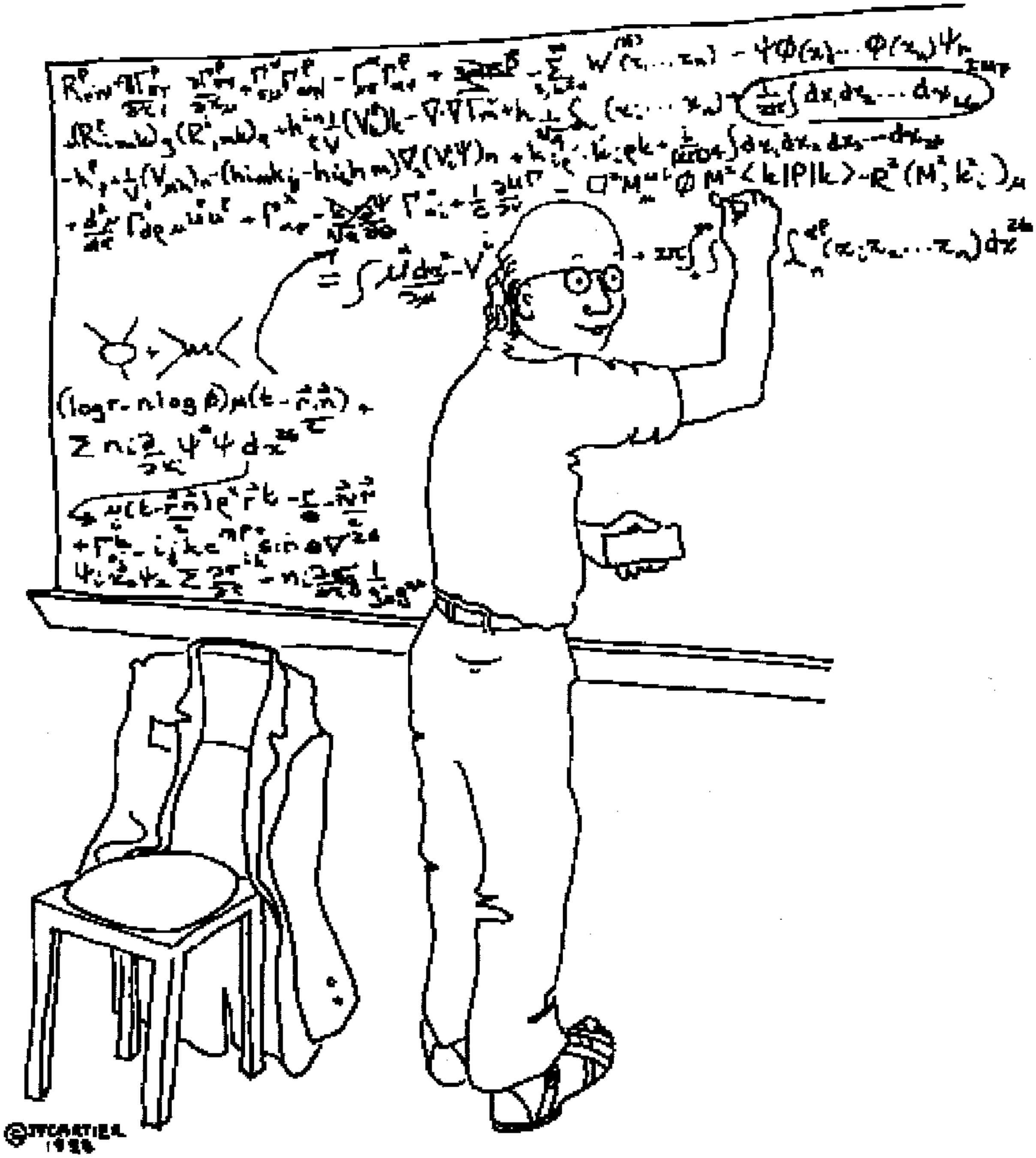
(a) Calculate the average potential and kinetic energy for the electron in the ground state of the Li^{++} ion. (b) Calculate the Uncertainty product $\Delta r.\Delta p$ for this system.

Hint: Use symmetry of the system to avoid needless computation.

Problem 8: A "Spinning" Electron Is Such A Bogus Idea [20 pts]

When the Idea of electron spin was introduced in 1920's, the electron was thought to be a tiny charged sphere (today it is considered a point object with (almost) NO extension in space!!). Find the equatorial speed under the assumption that the electron is a uniform sphere of radius $R=3x10^{-6}$ nm, as early theorists believed. (b) Compare your result with the speed of light in vacuum. We just have to accept that "spin" is a mysterious but necessary degree of freedom for an electron. (c) What would happen to your answer from part (b) if we make the radius of the electron 100 times smaller? (d) How big do we need to make the radius of the electron in order have the orbital speed be less than the speed of light? Hint: Moment of Inertia of a sphere of radius R, I = (2/5) MR^2

Brian, The String Theorist, Five Years Later !



"At this point we notice that this equation is beautifully simplified if we assume that space–time has 92 dimensions."