Bohr’s Explanation of Hydrogen like atoms

- Bohr’s Semiclassical theory explained some spectroscopic data → Nobel Prize: 1922
- The “hotch-potch” of classical & quantum attributes left many (Einstein) unconvinced
  - “appeared to me to be a miracle – and appears to me to be a miracle today ...... One ought to be ashamed of the successes of the theory”
- Problems with Bohr’s theory:
  - Failed to predict INTENSITY of spectral lines
  - Limited success in predicting spectra of Multi-electron atoms (He)
  - Failed to provide “time evolution” of system from some initial state
  - Overemphasized Particle nature of matter-could not explain the wave-particle duality of light
  - No general scheme applicable to non-periodic motion in subatomic systems
- “Condemned” as a one trick pony! Without fundamental insight …raised the question: Why was Bohr successful?
Atomic Excitation by Electrons: Franck-Hertz Expt

Other ways of Energy exchange are also quantized! Example:
- Transfer energy to atom by colliding electrons on it
- Accelerate electrons, collide with Hg atoms, measure energy transfer in inelastic collision (retarding voltage)
Atomic Excitation by Electrons: Franck-Hertz Expt

Plot # of electrons/time (current) overcoming the retarding potential (V)

Equally spaced Maxima and minima in I-V curve

Atoms accept only discrete amount of Energy, no matter the fashion in which energy is transferred
Prince Louise de Broglie

- Key to Bohr atom was Angular momentum quantization
- Why Quantization $mvr = |L| = nh/2\pi$?
- Invoking symmetry in nature the Prince deBroglie postulated
  - Because photons have wave and particle like nature → particles must have wave like properties
  - Electrons have accompanying “pilot” wave (not EM) which guide particles thru spacetime.

- Matter Wave:
  - “Pilot wave” of Wavelength $\lambda = h/p = h/(\gamma mv)$
  - frequency $f = E/h$

- If matter has wave like properties then there would be interference (destructive & constructive)
- Use analogy of standing waves on a plucked string to explain the quantization condition of Bohr orbits
Matter Waves: How big, how small

1. Wavelength of baseball, \( m = 140 \text{g}, v = 27 \text{m/s} \)

\[
\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34} \text{ J.s}}{(.14 \text{kg})(27 \text{m/s})} = 1.75 \times 10^{-34} \text{m}
\]

\[\Rightarrow \quad \lambda_{\text{baseball}} \ll \text{size of nucleus}\]

\[\Rightarrow \quad \text{Baseball "looks" like a particle}\]

2. Wavelength of electron \( K = 120 \text{eV} \) (assume NR)

\[K = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mK}\]

\[= \sqrt{2(9.11 \times 10^{-31}) (120 \text{eV})(1.6 \times 10^{-19})} = 5.91 \times 10^{-24} \text{ Kg.m/s}\]

\[\lambda_e = \frac{h}{p} = \frac{6.63 \times 10^{-34} \text{ J.s}}{5.91 \times 10^{-24} \text{ kg.m/s}} = 1.12 \times 10^{-10} \text{m}\]

\[\Rightarrow \quad \lambda_e \approx \text{Size of atom}!!\]
Models of Vibrations on a Loop: Model of e in atom

- Modes of vibration when a integral # of \( \lambda \) fit into loop (Standing waves) vibrations continue indefinitely
- Fractional # of waves in a loop can not persist due to destructive interference

Circumference = 2 wavelengths
Circumference = 4 wavelengths
Circumference = 8 wavelengths
Standing waves in H atom:
Constructive interference when
\[ n\lambda = 2\pi r \]

since
\[ \lambda = \frac{h}{p} = \frac{h}{mv} \]

\[ \Rightarrow \frac{nh}{mv} = 2\pi r \]
\[ \Rightarrow n\hbar = mvr \]

Angular momentum
Quantization condition!

This is too intense! Must verify such “loony tunes” with experiment
Reminder: Light as a Wave: Bragg Scattering Expt

Range of X-ray wavelengths scatter off a crystal sample.
X-rays constructively interfere from certain planes producing bright spots.

Interference → Path diff = 2d sin θ = nλ

(a) X rays
Crystal
Photographic plate with Laue spots

Range of X-ray wavelengths scatter off a crystal sample.
X-rays constructively interfere from certain planes producing bright spots.

Interference → Path diff = 2d sin θ = nλ
If electrons have associated wave-like properties, expect interference pattern when incident on a layer of atoms (reflection diffraction grating) with inter-atomic separation $d$ such that

$$\text{path diff AB} = \text{disint} = n\lambda$$

Atomic lattice as diffraction grating

If electrons have associated wave-like properties, expect interference pattern when incident on a layer of atoms (reflection diffraction grating) with inter-atomic separation $d$ such that

$$\text{path diff AB} = \text{disint} = n\lambda$$

Veritication of Matter Waves: Davisson & Germer Expt
Electrons Diffract in Crystal, just like X-rays

Diffraction pattern produced by 600eV electrons incident on an Al foil target. Notice the waxing and waning of scattered electron intensity. What to expect if electron had no wave-like attribute?
Davisson-Germer Experiment: 54 eV electron Beam

Polar graphs of DG expt with different electron accelerating potential when incident on same crystal (d = const)

Peak at $\Phi=50^\circ$ when $V_{\text{acc}} = 54$ V
Analyzing Davisson-Germer Expt with de Broglie idea

de Broglie $\lambda$ for electron accelerated thru $V_{\text{acc}} = 54 \text{V}$

- $\frac{1}{2}mv^2 = K = \frac{p^2}{2m} = eV \Rightarrow v = \sqrt{\frac{2eV}{m}}$; $p = mv = m\sqrt{\frac{2eV}{m}}$

If you believe de Broglie

$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{m\sqrt{\frac{2eV}{m}}} = \frac{h}{\sqrt{2meV}} = \lambda^{\text{predict}}$

For $V_{\text{acc}} = 54 \text{ Volts} \Rightarrow \lambda = 1.67 \times 10^{-10} \text{ m}$ (de Broglie)

Exptal data from Davisson-Germer Observation:

d$_{\text{nickel}} = 2.15 \text{ Å} = 2.15 \times 10^{-10} \text{ m}$ (from Bragg Scattering)

$\theta_{\text{diff}}^{\text{max}} = 50^\circ$ (observation from scattering intensity plot)

Diffraction Rule: $d \sin \phi = n\lambda$

$\Rightarrow$ For Principal Maxima ($n=1$); $\lambda^{\text{meas}} = (2.15 \text{ Å})(\sin 50^\circ)$

$\lambda^{\text{predict}} = 1.67 \text{ Å}$  $\lambda^{\text{observ}} = 1.65 \text{ Å}$  Excellent agreement
Davisson Germer Experiment: Matter Waves!

\[ \frac{h}{\sqrt{2\text{meV}}} = \lambda_{\text{predict}} \]

Excellent Agreement

\[ V_0^{-1/2} \]

\[ \gamma \]

\[ \chi \]
Practical Application: Electron Microscope
Electron Micrograph: Excellent Resolving Power

Showing Bacteriophage Viruses in E. Coli bacterium

The bacterium is $\approx 1 \mu$ size
Just **WHAT** is Waving in Matter Waves?

- For waves in an ocean, it’s the water that “waves”
- For sound waves, it’s the molecules in medium
- For light it’s the \( E \) & \( B \) vectors
- What’s waving for matter waves?
  - It’s the PROBABILITY OF FINDING THE PARTICLE that waves!
  - Particle can be represented by a wave packet in
    - Space
    - Time
    - Made by superposition of many sinusoidal waves of different \( \lambda \)
    - It’s a “pulse” of probability

Imagine Wave pulse moving along a string: its localized in time and space (unlike a pure harmonic wave)

Wave packet represents particle prob

localized
Making Wave packets with Sinusoidal Waves: Model

Ex: Phenomenon of "Beating" in Sound:

Add two waves of slightly different $\lambda$, $f$

$\Rightarrow$ Wave with : $f = \left(\frac{f_1 + f_2}{2}\right)$, Amplitude $A \propto \left(\frac{f_1 - f_2}{2}\right)$

Start with two waves

$y_1 = ACos(k_1x - w_1t), \quad y_2 = ACos(k_2x - w_2t): \quad k = \frac{2\pi}{\lambda}, \quad w = 2\pi f$