Measurement Error: \( x \pm \Delta x \)

- Measurement errors are unavoidable since the measurement procedure is an experimental one.
- True value of a measurable quantity is an abstract concept.
- In a set of repeated measurements with random errors, the distribution of measurements resembles a Gaussian distribution characterized by the parameter \( \sigma \) or \( \Delta \) characterizing the width of the distribution.

\[
G_{X,\sigma}(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-(x-X)^2/2\sigma^2}.
\]
Interpreting Measurements with random Error: $\Delta$

**Figure 5.12.** The shaded area between $X \pm t\sigma$ is the probability of a measurement within $t$ standard deviations of $X$. 

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>1.0</th>
<th>1.25</th>
<th>1.5</th>
<th>1.75</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob (%)</td>
<td>0</td>
<td>20</td>
<td>38</td>
<td>55</td>
<td>68</td>
<td>79</td>
<td>87</td>
<td>92</td>
<td>95.4</td>
<td>98.8</td>
<td>99.7</td>
<td>99.95</td>
<td>99.99</td>
</tr>
</tbody>
</table>
Comparing Measurements With Errors

(dis?) agreement between measurements

Back to Sharma’s weight: Mass measured with poor precision

1000 ± 700 kg is consistent with 70 ± 15 kg
Measurements with Errors

• If your measuring apparatus has an intrinsic error of $\Delta p$
• Then results of measurement of momentum $p$ of an object at rest can easily yield a range of values accommodated by the measurement imprecision:
  – $-\Delta p \leq p \leq \Delta p$
• Similarly for all measurable quantities!
Wave Packets & Uncertainty Principle

in space x: $\Delta k \cdot \Delta x = \pi$ \Rightarrow since $k = \frac{2\pi}{\lambda}$, $p = \frac{h}{\lambda}$

$\Rightarrow \Delta p \cdot \Delta x = \frac{h}{2}$

usually one writes $\Delta p \cdot \Delta x \geq \frac{\hbar}{2}$ approximate relation

In time t: $\Delta w \cdot \Delta t = \pi$ \Rightarrow since $\omega = 2\pi f$, $E = hf$

$\Rightarrow \Delta E \cdot \Delta t = \frac{\hbar}{2}$

usually one writes $\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$ approximate relation

What do these inequalities mean physically?
Act of Watching: A Thought Experiment

Before collision

Incident photon

Electron

After collision

Scattered photon

Recoiling electron

Observed Diffraction pattern

Photons that go through are restricted to this region of lens

Lens

Eye

Scattered photon

\[ p = \frac{h}{\lambda} \]

\( \alpha \)

\( \theta \)

Scattered electron

\( e^- \) initially at rest

\( \Delta x \)

\( p_e \)

Incident photon

\[ p_0 = \frac{h}{\lambda_0} \]
Diffracted image of a point source of light thru a lens (circular aperture of size $d$)

First minimum of diffraction pattern is located by

$$\sin \theta = 1.22 \frac{\lambda}{d}$$

See previous picture for definitions of $\theta$, $\lambda$, $d$.
Image of 2 separate point sources formed by a converging lens of diameter $d$, ability to resolve them depends on $\lambda$ & $d$ because of the inherent diffraction in image formation.

Resolving power $\Delta x \approx \frac{\lambda}{2 \sin \theta}$

$\theta$ Depends on $d$
- Incident light \( (p, \lambda) \) scatters off electron
- To be collected by lens \( \rightarrow \gamma \) must scatter thru angle \( \alpha \)
  - \( -\vartheta \leq \alpha \leq \vartheta \)
- Due to Compton scatter, electron picks up momentum
  - \( P_X, P_Y \)

\[ -\frac{\hbar}{\lambda} \sin \theta \leq P_x \leq \frac{\hbar}{\lambda} \sin \theta \]

Electron momentum uncertainty is
\[ \Delta p \approx \frac{2\hbar}{\lambda} \sin \theta \]

- After passing thru lens, photon “diffracts”, lands somewhere on screen, image (of electron) is fuzzy
- How fuzzy? Optics says shortest distance between two resolvable points is:
\[ \Delta x = \frac{\lambda}{2 \sin \theta} \]
- Larger the lens radius, larger the \( \vartheta \Rightarrow \) better resolution
Putting it all together: act of Observing an electron

Putting them together

\[ \Delta p \cdot \Delta x = \left( \frac{2h \sin \theta}{\lambda} \right) \left( \frac{\lambda}{2 \sin \theta} \right) = \hbar \]

\[ \Rightarrow \Delta p \Delta x \geq \hbar / 2 \]

- Can not EXACTLY measure Location and momentum of particle at the same time
- Can measure both \( P_x \) and \( Y \) component exactly but not \( P_x \) and \( X \)
Pseudo-Philosophical Aftermath of Uncertainty Principle

• Newtonian Physics & Deterministic physics topples over
  – Newton’s laws told you all you needed to know about trajectory of a particle
    • Apply a force, watch the particle go!
      – Know everything! X, v, p, F, a
      – Can predict exact trajectory of particle if you had perfect device

• No so in the subatomic world!
  – Of small momenta, forces, energies
  – Can’t predict anything exactly
    • Can only predict probabilities
      – There is so much chance that the particle landed here or there
      – Can’t be sure!...cognizant of the errors of thy observations

Philosophers went nuts!...what has happened to nature
Philosophers just talk, don’t do real life experiments!
Matter Diffraction & Uncertainty Principle

Incident Electron beam In Y direction

Momentum measurement beyond Slit show particle not moving exactly in Y direction, develops a X component Of motion $\Delta P_X = h/(2\pi a)$

Diffraction pattern seen on screen

$\Delta P_X$
Particle at Rest Between Two Walls

- Object of mass $M$ at rest between two walls originally at infinity
- What happens to our perception of George as the walls are brought in?

On average, measure $\langle p \rangle = 0$ but there are quite large fluctuations!

Width of Distribution $= \Delta P$

$$\Delta P = \sqrt{(P^2)_{ave} - (P_{ave})^2}; \quad \Delta P \sim \frac{\hbar}{L}$$
Quantum Behavior: Richard Feynman

See Chapters 1 & 2 of Feynman Lectures in Physics Vol III

Or Six Easy Pieces by Richard Feynman: Addison Wesley Publishers
An Experiment with Indestructible Bullets

Erratic Machine gun sprays in many directions

Made of Armor plate

Probability $P_{12}$ when Both holes open

$P_{12} = P_1 + P_2$
An Experiment With Water Waves

Intensity $I_{12}$ when both holes open

(by measuring amplitude of displacement)

\[ I_{12} = h_1 + h_2^2 = I_1 + I_2 + 2 \sqrt{I_1 I_2} \cos \delta \]
Interference Phenomenon in Waves

\[ n\lambda = d \sin \theta \]
An Experiment With Electrons

Probability $P_{12}$ when both holes open

$P_{12} \neq P_1 + P_2$
Interference in Electrons Thru 2 slits

Growth of 2-slit Interference pattern thru different exposure periods

Photographic plate (screen) struck by:

- 28 electrons
- 1000 electrons
- 10,000 electrons
- $10^6$ electrons

White dots simulate presence of electron
No white dots at the place of destructive Interference (minima)
Probability $P'_{12}$ when both holes open and I see which hole the electron came thru.

$$P'_{12} = P'_{1} + P'_{2}$$
Watching The Electrons With Dim Light

Probability $P_{12}$ when both holes open and I see which hole the electron came thru.
Probability $P_{12}$ when both holes open and I
Don’t see which hole the electron came thru

\[ P_{12} = |\phi_1 + \phi_2|^2 \]
Compton Scattering: Shining light to observe electron

\[ \lambda = \frac{h}{p} = \frac{hc}{E} = \frac{c}{f} \]

The act of Observation DISTURBS the object being watched, here the electron moves away from where it was originally.
Watching Electrons With Light of $\lambda >>$ slitsize but High Intensity

Probability $P_{12}$ when both holes open but can't tell from flash which hole the electron came thru
Why Fuzy Flash? → Resolving Power of Light

Image of 2 separate point sources formed by a converging lens of diameter $d$, ability to resolve them depends on $\lambda$ & $d$ because of the inherent diffraction in image formation.

\[
\Delta x \approx \frac{\lambda}{2\sin \theta}
\]
1. Probability of an event is given by the square of amplitude of a complex # $\Psi$: Probability Amplitude

2. When an event occurs in several alternate ways, probability amplitude for the event is sum of probability amplitudes for each way considered separately. There is interference:

$$\Psi = \Psi_1 + \Psi_2$$

$$P_{12} = |\Psi_1 + \Psi_2|^2$$

3. If an experiment is done which is capable of determining whether one or other alternative is actually taken, probability for event is just sum of each alternative

- Interference pattern is LOST!
Is There No Way to Beat Uncertainty Principle?

- How about NOT watching the electrons!
- Let's be a bit crafty
- Since this is a Thought experiment → ideal conditions
  - Mount the wall on rollers, put a lot of grease → frictionless
  - Wall will move when electron hits it
  - Watch recoil of the wall containing the slits when the electron hits it
  - By watching whether wall moved up or down I can tell
    - Electron went thru hole # 1
    - Electron went thru hole #2

- Will my ingenious plot succeed?
Measuring The Recoil of The Wall: Not Watching Electron!
Losing Out To Uncertainty Principle

• To measure the RECOIL of the wall ⇒
  – must know the initial momentum of the wall before electron hit it
  – Final momentum after electron hits the wall
  – Calculate vector sum → recoil

• Uncertainty principle:
  – To do this ⇒ $\Delta P = 0 \rightarrow \Delta X = \infty$ [can not know the position of wall exactly]
  – If don’t know the wall location, then don’t know where the holes are
  – Holes will be in different place for every electron that goes thru
  – The center of interference pattern will have different (random) location for each electron
  – Such random shift is just enough to Smear out the pattern so that no interference is observed!

• Uncertainty Principle Protects Quantum Mechanics!
The Bullet Vs The Electron: Each Behaves the Same Way

\[ P_{12} \] (smoothed)

\[ P_{12} \]
Quantum Mechanics of Subatomic Particles

• Act of Observation destroys the system (No watching!)
• If can’t watch then All conversations can only be in terms of Probability P
• Every particle under the influence of a force is described by a Complex wave function $\Psi(x,y,z,t)$
• $\Psi$ is the ultimate DNA of particle: contains all info about the particle under the force (in a potential e.g Hydrogen)
• Probability of per unit volume of finding the particle at some point $(x,y,z)$ and time $t$ is given by
  – $P(x,y,z,t) = |\Psi(x,y,z,t)|^2$
• When there are more than one path to reach a final location then the probability of the event is
  – $\Psi = \Psi_1 + \Psi_2$
  – $P = |\Psi^* \Psi| = |\Psi_1|^2 + |\Psi_2|^2 + 2|\Psi_1| |\Psi_2| \cos \phi$
Although not possible to specify with certainty the location of particle, its possible to assign probability $P(x)dx$ of finding particle between $x$ and $x+dx$

- $P(x) \, dx = |\Psi(x,t)|^2 \, dx$
- E.g intensity distribution in light diffraction pattern is a measure of the probability that a photon will strike a given point within the pattern
Ψ: The Wave function Of A Particle

- The particle must be somewhere
  \[ \int_{-\infty}^{+\infty} |\psi(x,t)|^2 \, dx = 1 \]
- Any \( \psi \) satisfying this condition is NORMALIZED
- Prob of finding particle in finite interval
  \[ P(a \leq x \leq b) = \int_{a}^{b} \psi^*(x,t) \psi(x,t) \, dx \]
- Fundamental aim of Quantum Mechanics
  - Given the wavefunction at some instant (say \( t=0 \)) find \( \psi \) at some subsequent time \( t \)
  - \( \psi(x,t=0) \rightarrow \psi(x,t) \) …evolution
  - Think of a probabilistic view of particle’s “newtonian trajectory”
    - We are replacing Newton’s 2\textsuperscript{nd} law for subatomic systems

The Wave Function is a mathematical function that describes a physical object \( \rightarrow \) Wave function must have some rigorous properties:

- \( \psi \) must be finite
- \( \psi \) must be continuous fn of \( x,t \)
- \( \psi \) must be single-valued
- \( \psi \) must be smooth fn \( \frac{d\psi}{dx} \) must be continuous

WHY?