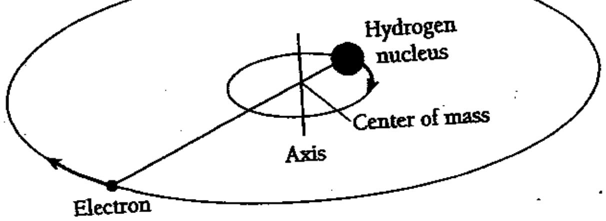
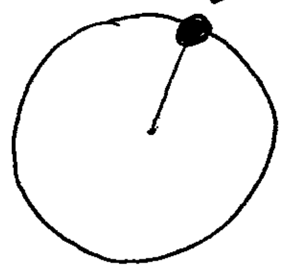
Reduced Mass

When you're dealing with two objects moving around, it is often convenient to reduce the problem to an equivalent <u>one</u>-body situation. The idea of reduced mass allows you to do this, and it's useful in (for example) the hydrogen atom.



$\Longrightarrow$

one object w/ mass = reduced mass.



To figure out what the reduced mass is, use this simple argument. Say you have 2 objects of masses m_1 and m_2 interacting with each other by means of a central force (a force that depends only on the distance between them).

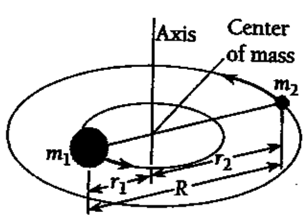


Figure 8.16 A diatomic molecule can rotate about its center of mass.

The rotational inertia of this system is

$$I = m_1 r_1^2 + m_2 r_2^2$$

We know that, if we say the center of mass is at the origin (as in the picture)

$$m_1 r_1 = m_2 r_2 .$$

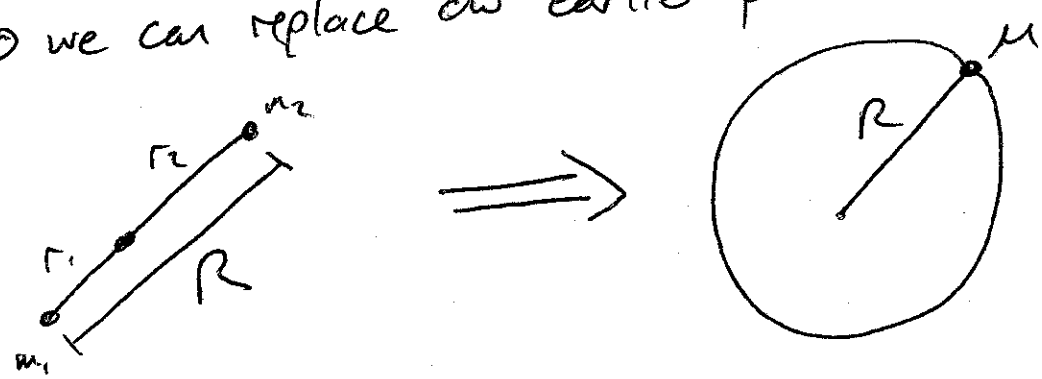
So then

$$I = \frac{m_1 m_2}{m_1 + m_2}(r_1 + r_2)^2 = \mu R^2 \quad \text{(after some algebra).}$$

with $\boxed{\mu = \frac{m_1 m_2}{m_1 + m_2}}$

This looks like an object of mass μ rotating a distance R from the origin.

So we can replace our earlier picture



And that's where the reduced mass comes from! Since all forces only depend on R, just go through the problem as before, but replace

$\vec{F} = m_1\vec{a}_1 + m_2\vec{a}_2$ with $\boxed{\vec{F} = \mu\vec{a},}$ where $\vec{a} = \frac{d^2\vec{R}}{dt^2}$.