Department of Physics
University of California San Diego

Quantum Universe (4E)
Prof. V. Sharma
Quiz # 1 (April 9 2004)

Problem 1 : Similar But Different ! [10 pts]

In a different version of J.J. Thomson's famous measurement of electron's e/m, non-relativistic electrons are accelerated (from rest) through a measured potential difference V_0. They are then passed into a region of known magnetic field B and the radius R of their circular orbit is measured. (a) Draw a schematic diagram of the setup (b) Write an expression for the kinetic energy K of the electron (in terms of V_0) as they enter the region of B field. (c) Write an expression for the orbit radius R in terms of the charge q, mass m and the speed u of the electron and the external B field. (d) Eliminate speed u from results of (b) and (c) and derive an expression for e/m in terms of the experimentally measurable quantities V_0, B and R.

Problem 2: They Might Be Giants ! [10 pts]

Suppose an electron was bound to a proton, as in the hydrogen atom, by the gravitational force rather than by electric force. Assume, as in the Bohr atom, that the angular momentum is quantized. Write the equations for (a) the total energy of the system and (b) Newton's second law (equality of forces). Now calculate (c) the radius and (d) the energy of the ground state. (e) Compare the size of the atom with that of our observable universe. Would "humans" made of such atoms make a good basketball team ? Would they be sturdy enough to survive a "Shaq attack"?

Facts

Gravitational Constant $G = 6.67 \times 10^{-11} N.m^2/Kg^2$

Planck's Constant $h = 6.626 \times 10^{-34} J.s = 4.136 \times 10^{-15} eV.s$

$1\ eV = 1.60 \times 10^{-19}\ J$

Electron Mass $= 9.1 \times 10^{-31}\ Kg = 0.511\ MeV/c^2$

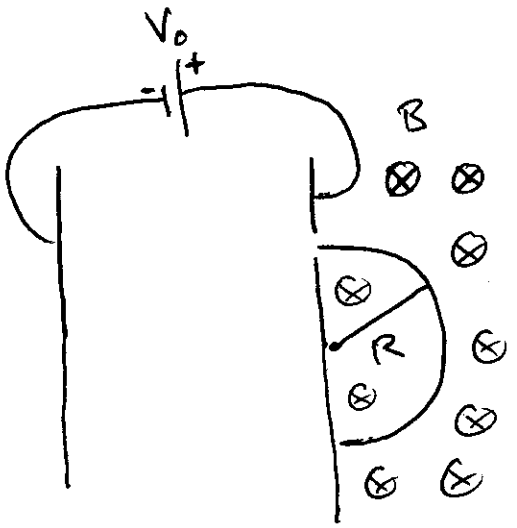
Proton Mass $= 938.3\ MeV/c^2 = 1.673 \times 10^{-27}\ Kg$

Edge of the observable universe $\approx 10^{26}\ m$ away from the Grove Cafe

Phys 4E Quiz 1

1

a



b $\boxed{K = eV_0}$ (KE = PE)

c $euB = \frac{mu^2}{R} \Rightarrow \boxed{R = \frac{mu}{eB}}$

d $eV_0 = \frac{1}{2}mu^2$ and $u = \frac{eBR}{m}$, so

$$eV_0 = \frac{1}{2}m\left(\frac{eBR}{m}\right)^2 \Rightarrow eV_0 = \frac{1}{2}\frac{e^2B^2R^2}{m}$$

So $\boxed{\frac{e}{m} = \frac{2V_0}{B^2R^2}}$

2]

a] $E = \frac{1}{2} m_e v^2 - \frac{G m_e m_p}{r}$

b] $\frac{G m_e m_p}{r^2} = \frac{m_e v^2}{r}$

c] Use $m_e v r = n\hbar$ to get $v = \frac{n\hbar}{m_e r}$

$$\text{So } \frac{G m_e m_p}{r^2} = \frac{m_e}{r}\left(\frac{n^2\hbar^2}{m_e^2 r^2}\right) \Rightarrow r = \left(\frac{\hbar^2}{G m_e^2 m_p}\right) n^2$$

So $\boxed{r_1 = 1.2 \times 10^{29}\ m}$

d] $E = \frac{1}{2} m_e v^2 - \frac{G m_e m_p}{r} = -\frac{1}{2}\left(\frac{G m_e m_p}{r}\right)$

$$\Rightarrow E_n = -\frac{1}{2}(G m_e m_p)\left(\frac{G m_e^2 m_p}{\hbar^2}\right)\frac{1}{n^2} = -\frac{1}{2}\frac{G^2 m_e^3 m_p^2}{\hbar^2}\frac{1}{n^2}$$

$\boxed{E_1 = -4.2 \times 10^{-97}\ J = -2.6 \times 10^{-78}\ eV}$

e) It's bigger than the size of the universe!

Such humans would make a kick-ass basketball team, but be so easily ionized that Shaq would kill them, bad.