Physics 2D Lecture Slides
Lecture 12: Jan 26th 2005

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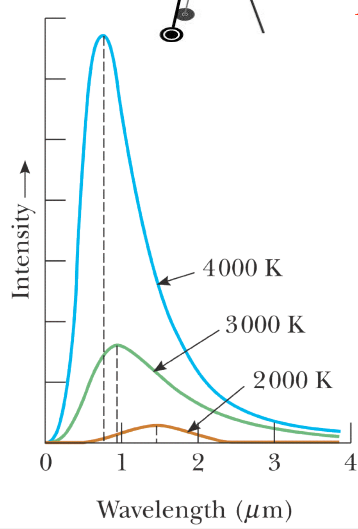
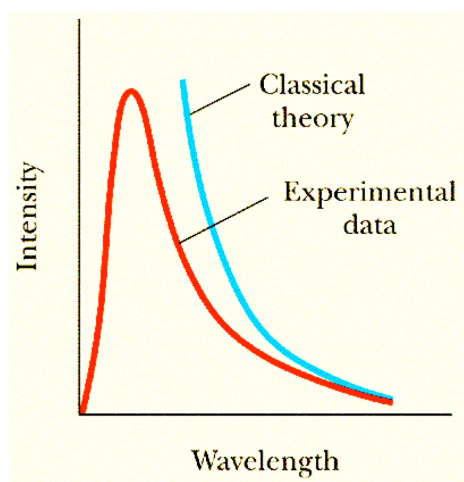
UCSD Physics

Ultra Violet (Frequency) Catastrophe



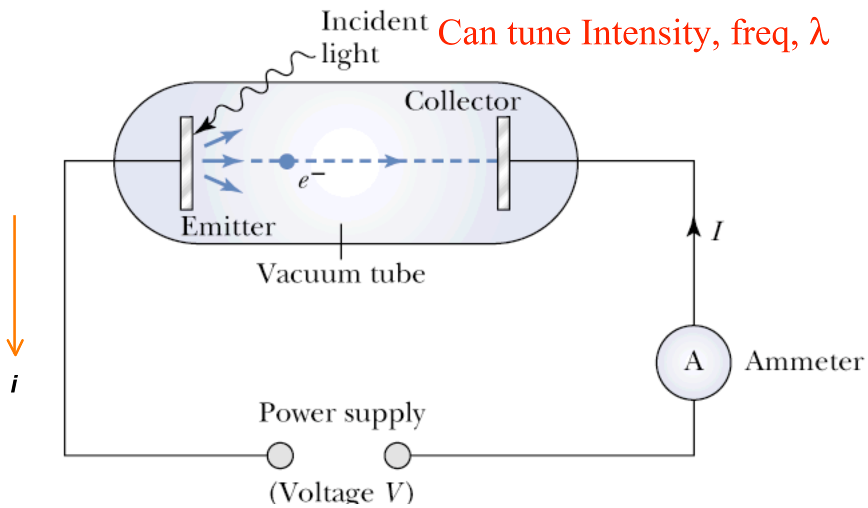
$$\text{Radiancy } R(\lambda) = \frac{c}{4}u(\lambda) = \frac{c}{4}\frac{8\pi}{\lambda^4}kT = \frac{2\pi c}{\lambda^4}kT$$

Radiancy is Radiation intensity per unit λ interval



Disaster # 2 : Photo-Electric Effect

Light of intensity I, wavelength λ and frequency ν incident on a photo-cathode



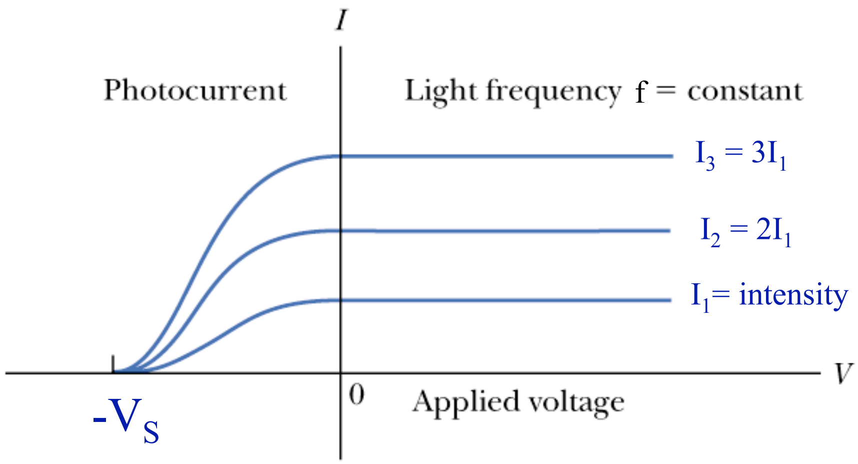
Measure characteristics of current in the circuit as a fn of I, f, λ

Photo Electric Effect: Measurable Properties

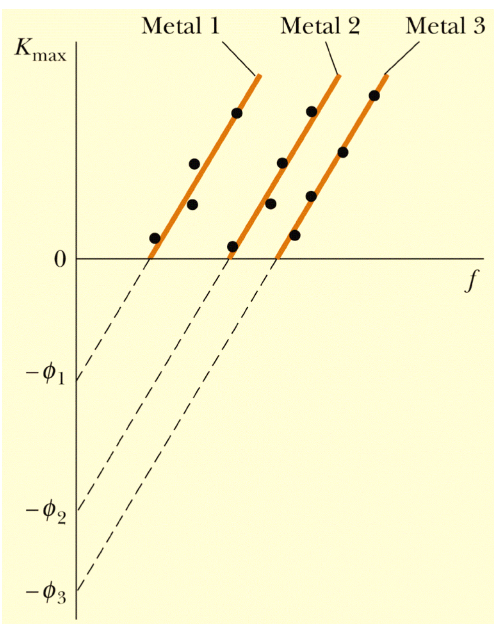
- Rate of electron emission from cathode
  - From current *i* seen in ammeter
- Maximum kinetic energy of emitted electron
  - By applying retarding potential on electron moving towards Collector plate
    - » **$K_{MAX} = eV_S$ (V_S = Stopping voltage)**
    - » **Stopping voltage → no current flows**
- Effect of different types of photo-cathode metal
- Time between shining light and first sign of photo-current in the circuit

Observation: Photo-Current Vs Frequency of Incident Light

Stopping voltage V_S is a measure of the Max kinetic energy of the electron



Stopping Voltage V_s For Different Photocathode Surfaces

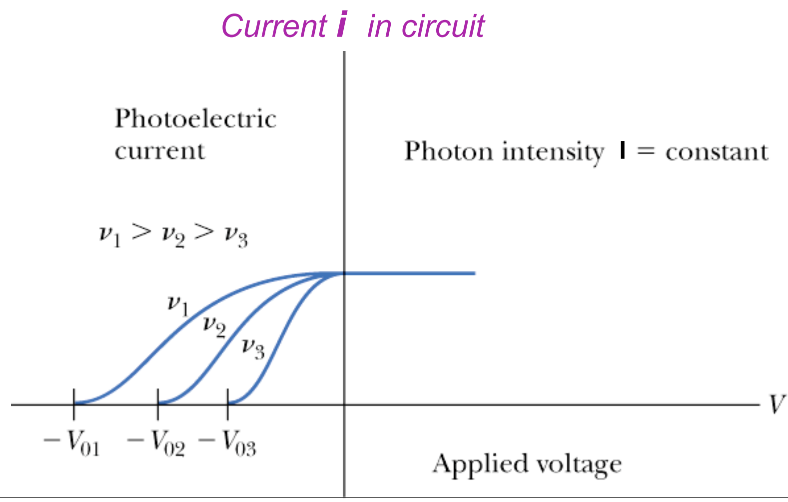


$eV_S = K_{MAX}$ = max KE

Retarding Potential Vs Light Frequency (f)

Shining Light With Constant Intensity But different frequencies

Larger the frequency of light, larger is the stopping voltage (and thus the kinetic energy of the "photoelectrons")



Conclusions from the Experimental Observation

- Max Kinetic energy K_{MAX} independent of Intensity I for light of same frequency
- No photoelectric effect occurs if light frequency f is below a threshold no matter how high the intensity of light
- For a particular metal, light with $f > f_0$ causes photoelectric effect IRRESPECTIVE of light intensity.
  - f_0 is characteristic of that metal
- Photoelectric effect is instantaneous !...not time delay

Can one Explain all this Classically !

Classical Explanation of Photo Electric Effect

- As light Intensity increased $\Rightarrow$ $\vec{E}$ field amplitude larger
  - E field and electrical force seen by the "charged subatomic oscillators" Larger
    - $\vec{F} = e\vec{E}$
    - More force acting on the subatomic charged oscillator
    - $\Rightarrow$ More energy transferred to it
    - $\Rightarrow$ Charged particle "hooked to the atom" should leave the surface with more Kinetic Energy KE !! The intensity of light shining rules !
- As long as light is intense enough , light of ANY frequency f should cause photoelectric effect
- Because the Energy in a Wave is uniformly distributed over the Spherical wavefront incident on cathode, thould be a noticeable time lag ΔT between time is incident & the time a photo-electron is ejected : Energy absorption time
  - How much time ? Lets calculate it classically.

Classical Physics: Time Lag in Photo-Electric Effect

- Electron absorbs energy incident on a surface area where the electron is confined ≅ size of atom in cathode metal
- Electron is "bound" by attractive Coulomb force in the atom, so it must absorb a minimum amount of radiation before its stripped off
- Example : Laser light Intensity I = $120 W/m^2$ on Na metal
  - Binding energy = 2.3 eV= "Work Function"
  - Electron confined in Na atom, size ≅ 0.1nm ..how long before ejection ?
  - Average Power Delivered $P_{AV} = \mathbf{I} . \mathbf{A}$, $A = \pi r^2 \cong 3.1 \times 10^{-20}$ m^2
  - If all energy absorbed then $\Delta E = P_{AV} . \Delta T \Rightarrow \Delta T = \Delta E / P_{AV}$

$$\Delta T = \frac{(2.3eV)(1.6\times10^{-19} J/eV)}{(120W/m^2)(3.1\times10^{-20} m^2)} = 0.10\ S$$

  - Classical Physics predicts Measurable delay even by the primitive clocks of 1900
  - But in experiment, the effect was observed to be instantaneous !!
  - Classical Physics fails in explaining all results

That's Disaster # 2 !

Max Planck & Birth of Quantum Physics

Back to Blackbody Radiation Discrepancy

Planck noted the UltraViolet Catastrophe at high frequency

"Cooked" calculation with new "ideas" so as bring:

$R(\lambda) \rightarrow 0$ as $\lambda \rightarrow 0$

$f \rightarrow \infty$

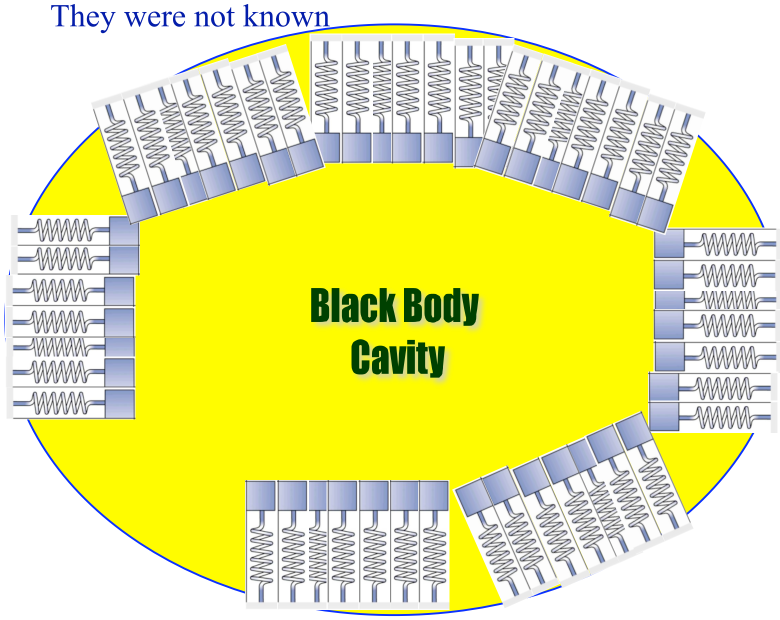
- Cavity radiation as equilibrium exchange of energy between EM radiation & "atomic" oscillators present on walls of cavity
- Oscillators can have any frequency f
- But the Energy exchange between radiation and oscillator NOT continuous and arbitarary…it is discrete …in packets of same amount
- $E = n\,hf$, with n = 1,2 3…. ∞

  h = constant he invented, a very small number he made up

Planck's "Charged Oscillators" in a Black Body Cavity

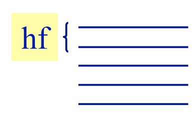
Planck did not know about electrons, Nucleus etc:

They were not known

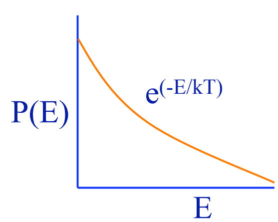


Planck, Quantization of Energy & BB Radiation

- Keep the rule of counting how many waves fit in a BB Volume
- Radiation Energy in cavity is quantized
- EM standing waves of frequency f have energy
  - E = n hf (n = 1,2 ,3 …10 ….1000…)



- Probability Distribution: At an equilibrium temp T, possible Energy of wave is distributed over a spectrum of states: $P(E) = e^{(-E/kT)}$
- Modes of Oscillation with :
  - Less energy E=hf = favored
  - More energy E=hf = disfavored



By this statistics, large energy, high f modes of EM disfavored

Planck's Calculation

$$R(\lambda) = \left(\frac{c}{4}\right)\left(\frac{8\pi}{\lambda^4}\right)\left[\frac{hc}{\lambda}\left(\frac{1}{e^{\frac{hc}{\lambda kT}} - 1}\right)\right]$$

Odd looking form

When $\lambda \to \text{large} \Rightarrow \dfrac{hc}{\lambda kT} \to \text{small}$

Recall $\boxed{e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...}$

$$\Rightarrow e^{\frac{hc}{\lambda kT}} - 1 = (1 + \frac{hc}{\lambda kT} + \frac{1}{2}\left(\frac{hc}{\lambda kT}\right)^2 + ...] - 1$$

$$= \frac{hc}{\lambda kT}$$ plugging this in $R(\lambda)$ eq:

$$\boxed{R(\lambda) = \left(\frac{c}{4}\right)\left(\frac{8\pi}{\lambda^4}\right)\frac{hc}{\lambda kT}}$$

Graph & Compare
With BBQ data

Planck's Formula and Small λ

When λ is small (large f)

$$\frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \cong \frac{1}{e^{\frac{hc}{\lambda kT}}} = e^{-\frac{hc}{\lambda kT}}$$

Substituting in $R(\lambda)$ eqn:

$$R(\lambda) = \left(\frac{c}{4}\right)\left(\frac{8\pi}{\lambda^4}\right)e^{-\frac{hc}{\lambda kT}}$$

As $\lambda \to 0$, $e^{-\frac{hc}{\lambda kT}} \to 0$

$\Rightarrow R(\lambda) \to 0$

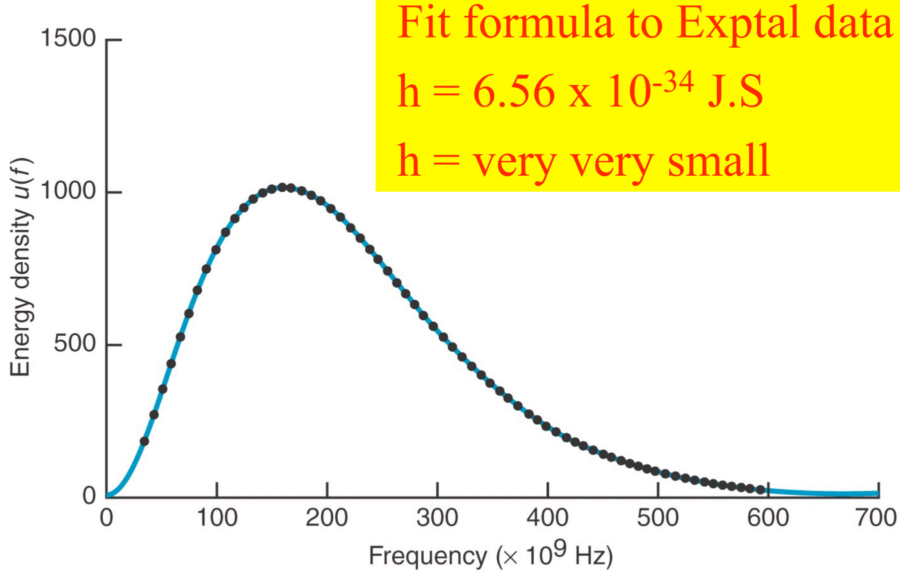
Just as seen in the experimental data

Planck's Explanation of BB Radiation

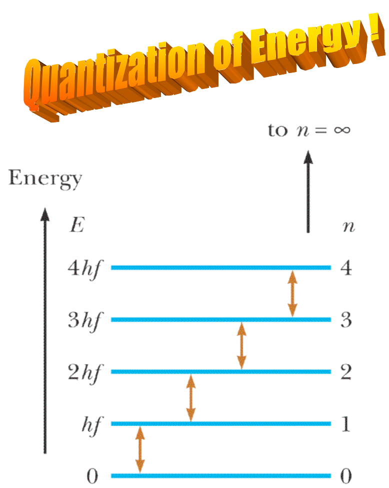
Fit formula to Exptal data

h = 6.56 x 10^{-34} J.S

h = very very small



Major Consequence of Planck's Formula



Resolving Disaster #2: Who You Gonna Call ?

Amongst his lesser known talents was his ability to communicate.

here he is greeting old friend: Conrad Habich

What are you up to? you frozen whale, you smoked, dried, canned piece of soul

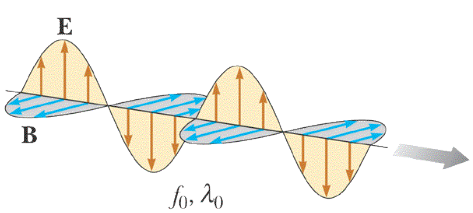
Clearly , like the electron, the phrase

" Whaddup Dog !"

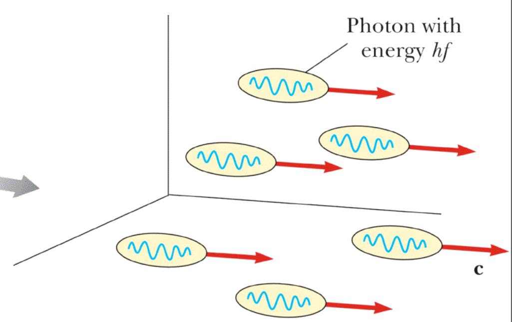
had not been discovered by then !

Einstein's Explanation of PhotoElectric Effect

What Maxwell Saw of EM Waves



What Einstein Saw of EM Waves



Light as bullets of "photons"
Energy concentrated in photons
Energy exchanged instantly
Energy of EM Wave E= hf

Einstein's Explanation of Photoelectric Effect

- Energy associated with EM waves in not uniformly distributed over wave-front, rather is contained in packets of "stuff" ⇒ PHOTON
- $E = hf = hc/\lambda$ [but is it the same h as in Planck's th.?]
- Light shining on metal emitter/cathode is a stream of photons of energy which depends on frequency f
- Photons knock off electron from metal instantaneously
  - Transfer all energy to electron
  - Energy gets used up to pay for Work Function Φ (Binding Energy)
    - Rest of the energy shows up as KE of electron $KE = hf - \Phi$
- Cutoff Frequency $hf_0 = \Phi$ (pops an electron, KE = 0)
- Larger intensity I → more photons incident
- Low frequency light f → not energetic enough to overcome work function of electron in atom

Photo Electric & Einstein (Nobel Prize 1915)

Light shining on metal cathode is made of photons

Energy E, depends on frequency f , $E = hf = h(c/\lambda)$

This QUANTUM of energy is used to knock off electron

$$E = hf = \varphi + KE_{electron}$$

$$eV_s = KE = hf - \varphi$$

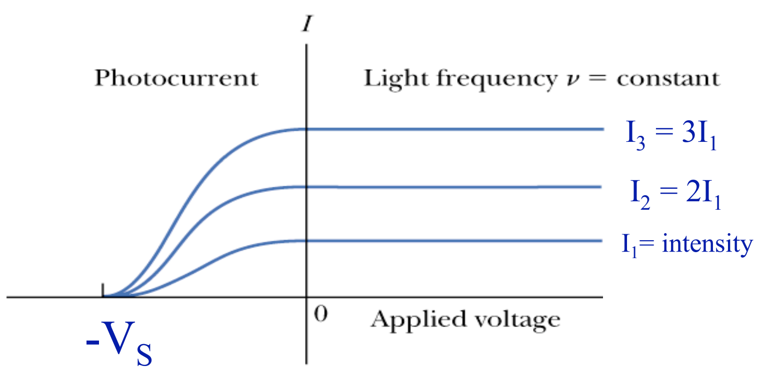
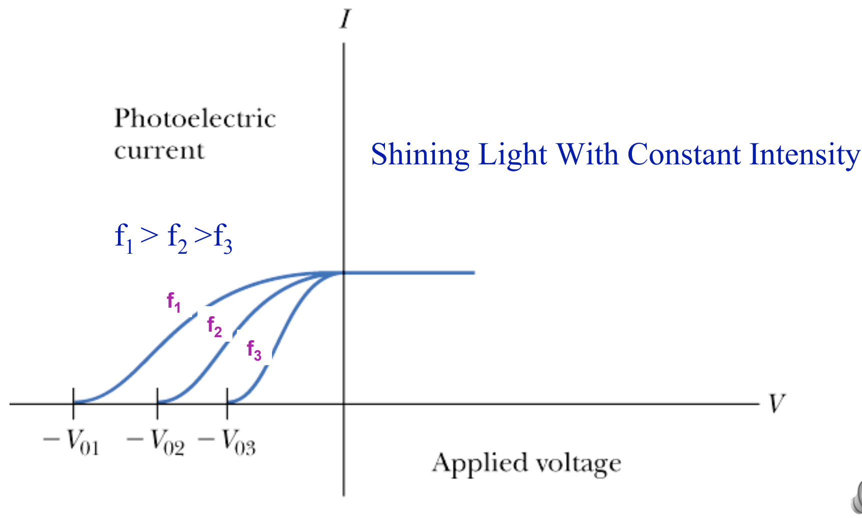


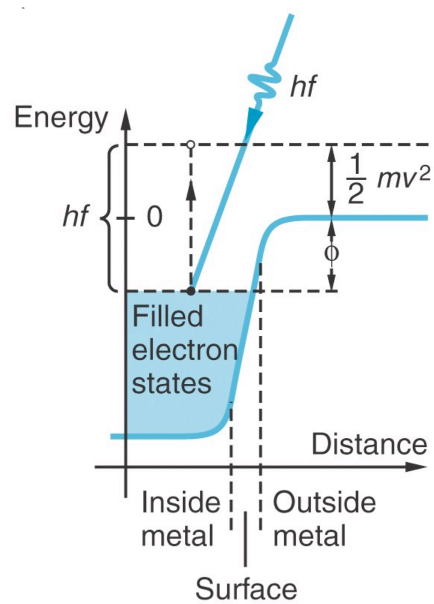
Photo Electric & Einstein (Nobel Prize 1915)

Light shining on metal cathode is made of photons

Quantum of Energy $E = hf = KE + \varphi \Rightarrow KE = hf - \varphi$



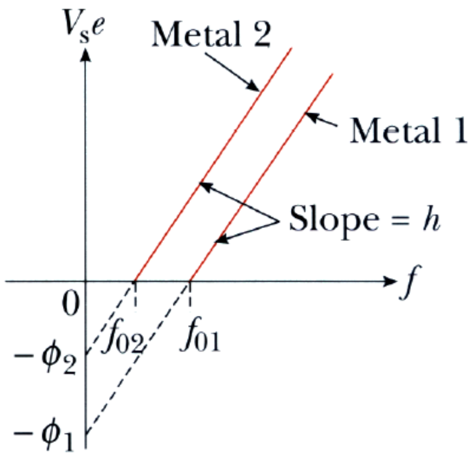
Modern View of Photoelectric Effect



Is “h” same in Photoelectric Effect as BB Radiation?

Slope h = 6.626 x 10^{-34} JS

Einstein → Nobel Prize!



No matter where you travel
in the galaxy and beyond…
..no matter what experiment
You do

h : Planck’s constant is same

NOBEL PRIZE FOR PLANCK

Work Function (Binding Energy) In Metals

TABLE 3-1 Photoelectric work functions

Element	ϕ (eV)
Na	2.28
C	4.81
Cd	4.07
Al	4.08
Ag	4.73
Pt	6.35
Mg	3.68
Ni	5.01
Se	5.11
Pb	4.14

Photoelectric Effect on An Iron Surface:

Light of Intensity I = 1.0 μW/cm^2 incident on 1.0cm^2 surface of Fe

Assume Fe reflects 96% of light

further only 3% of incident light is Violet region (λ= 250nm)

barely above threshold frequency for Ph. El effect

(a) Intensity available for Ph. El effect I =3% ×4%×(1.0 μW/cm^2)

(b) how many photo-electrons emitted per second ?

$$\# \text{ of photoelectrons} = \frac{\text{Power}}{\text{h f}} = \frac{3\% \times 4\% \times (1.0\ \mu\text{W/cm}^2)\ \lambda}{\text{hc}}$$

$$= \frac{(250\times10^{-9}m)(1.2\times10^{-9}J/s)}{(6.6\times10^{-34}J\bullet s)(3.0\times10^{8}m/s)}$$

$$= 1.5\times10^{9}$$

(c) Current in Ammeter : $i = (1.6\times10^{-19}C)(1.5\times10^{9}) = 2.4\times10^{-10}A$

(d) Work Function $\Phi = hf_0 = (4.14\times10^{-15}eV\bullet s)(1.1\times10^{15}s^{-1})$

$= 4.5$ eV

Facts

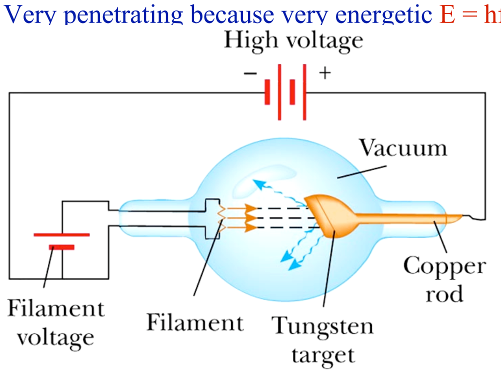
- The human eye is a sensitive photon detector at visible wavelenghts: Need >5 photons of ≅ 550nm to register on your optical sensor
- The Photographic process :
  - Energy to Dissociate an AgBr molecule = 0.6eV
- Photosynthesis Process : 9 sunlight photon per reaction cycle of converting CO_2 and water to carbohydrate & O_2
  - chlorophyll absorbs best at $\lambda \cong$ 650-700 nm
- Designing Space Shuttle “skin” : Why Platinum is a good thing
- designing Solar cells : picking your metal cathode

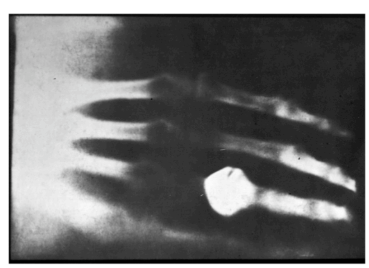
Photon & Relativity: Wave or a Particle ?

- Photon associated with EM waves, travel with speed =c
- For light (m =0) : Relativity says $E^2 = (pc)^2 + (mc^2)^2$
- $\Rightarrow E = pc$
- But Planck tells us : $E = hf = h\,(c/\lambda)$
- Put them together : $hc/\lambda = pc$
  - $\Rightarrow$ $p = h/\lambda$
  - Momentum of the photon (light) is inversely proportional to λ
- But we associate λ with waves & p with particles ….what is going on??
  - A new paradigm of conversation with the subatomic particles : Quantum Physics

X Rays "Bremsstrahlung": The Braking Radiation

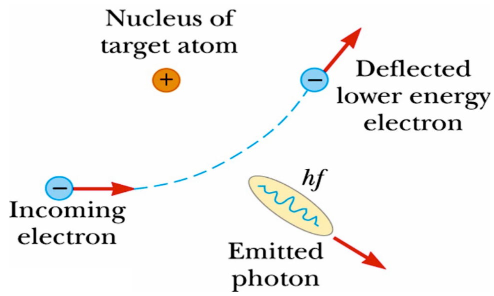
- EM radiation, produced by bombarding a metal target with energetic electrons.
- Produced in general by ALL decelerating charged particles
- X rays : very short $\lambda \cong$ 60-100 pm (10^{-12}m), large frequency f
- Very penetrating because very energetic $E = hf$!!





Useful for probing structure of sub-atomic Particles
(and your teeth)

X Ray Production Mechanism

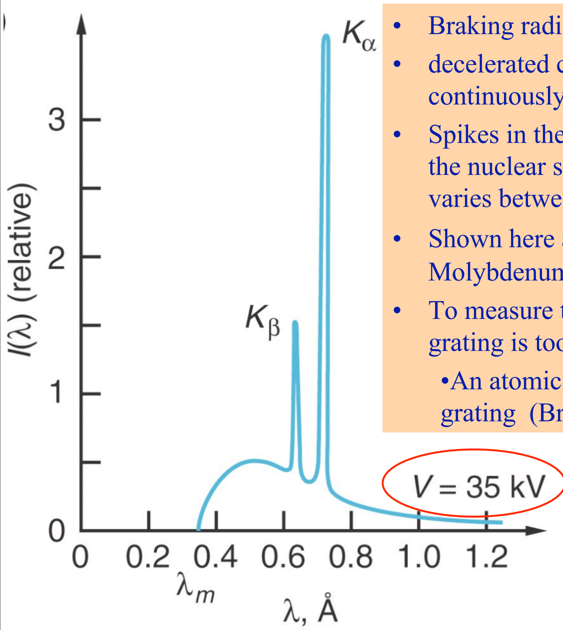


when electron passes near a positively charged target nucleus contained in target material, its deflected from its path because of its electrical attraction , experiences acceleration.

Rules of E&M say that any charged particle will emit radiation when accelerated. This EM radiation "appears" as photons. Since photo carries energy and momentum, the electron must lose same amount. If all of electron's energy is lost in just one single collision then

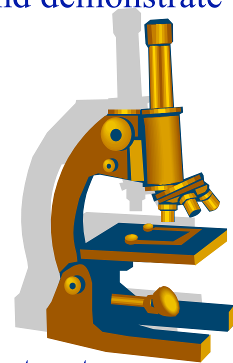
$$e\,\Delta V = \mathrm{hf}_{max} = \frac{hc}{\lambda_{min}} \text{ or } \boxed{\lambda_{min} = \frac{hc}{e\,\Delta V}}$$

X Ray Spectrum in Molybdenum (Mo)



- Braking radiation predicted by Maxwell's eqn
- decelerated charged particle will radiate continuously
- Spikes in the spectrum are characteristic of the nuclear structure of target material and varies between materials
- Shown here are the α and β lines for Molybdenum (Mo)
- To measure the wavelength, diffraction grating is too wide, need smaller slits
  - An atomic crystal lattice as diffraction grating (Bragg)

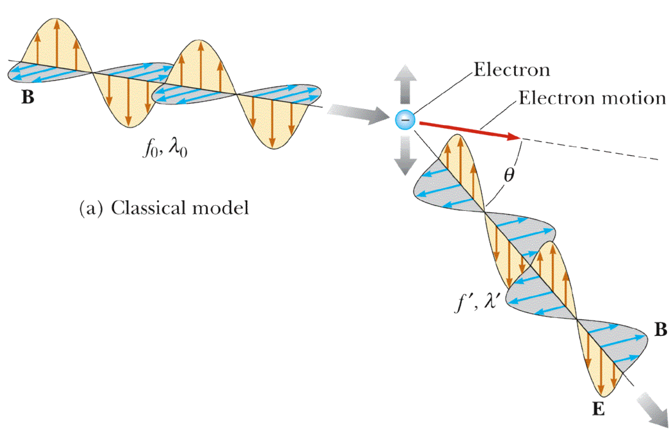
- X rays are EM waves of low wavelength, high frequency (and energy) and demonstrate characteristic features of a wave
  – Interference
  – Diffraction



- To probe into a structure you need a light source with wavelength much smaller than the features of the object being probed
  – Good Resolution → $\lambda << \Delta$
- X rays allows one probe at atomic size (10^{-10})m

Compton Scattering : Quantum Pool !

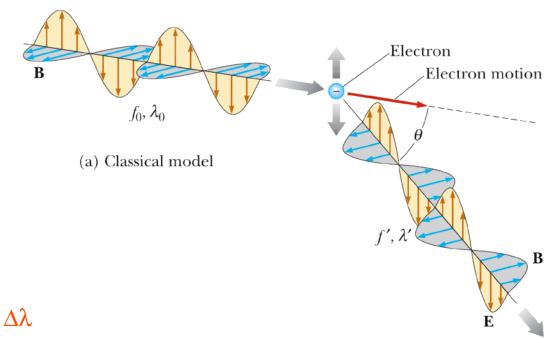
- 1922: Arthur Compton (USA) proves that X-rays (EM Waves) have particle like properties (acts like photons)
  – Showed that classical theory failed to explain the scattering effect of
    - X rays on to free (not bound, barely bound electrons)
- Experiment : shine X ray EM waves on to a surface with "almost" free electrons
  – Watch the scattering of light off electron : measure time + wavelength of scattered X-ray



(a) Classical model

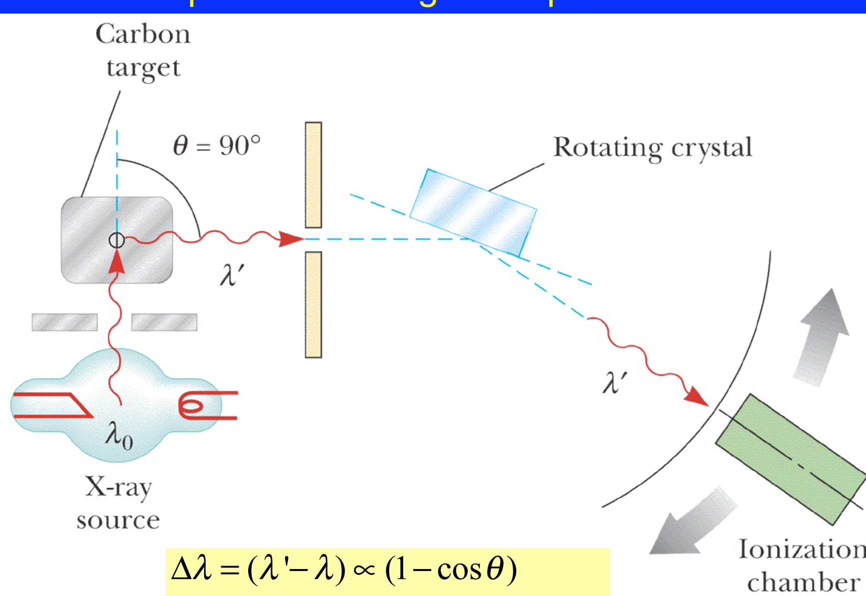
Compton Effect: what should Happen Classically?

- Plane wave [f,λ] incident on a surface with loosely bound electrons → interaction of E field of EM wave with electron: **F** = e**E**
- Electron oscillates with $f = f_{incident}$
- Eventually radiates spherical waves with $f_{radiated} = f_{incident}$
  - At all scattering angles, Δf & Δλ must be zero
- Time delay while the electron gets a "tan" : soaks in radiation



(a) Classical model

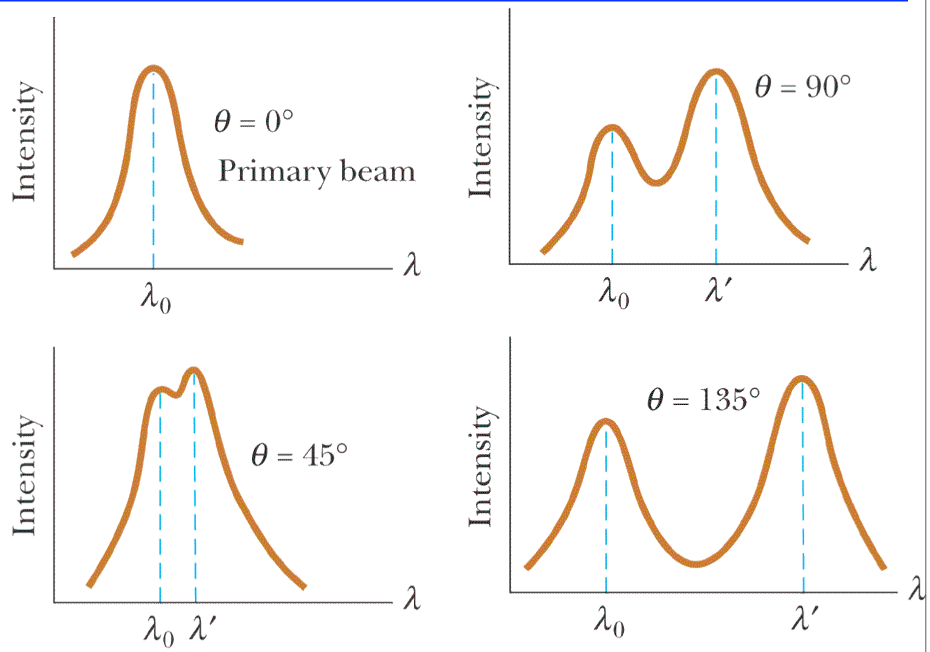
Compton Scattering : Setup & Results



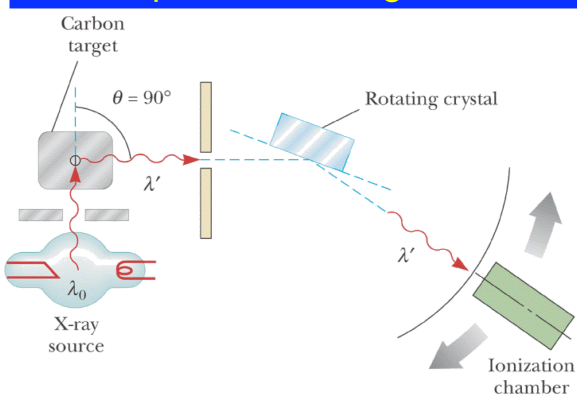
$$\Delta\lambda = (\lambda' - \lambda) \propto (1 - \cos\theta)$$

Scattered λ' larger than incident

Compton Scattering Observations



Compton Scattering : Summary of Observations

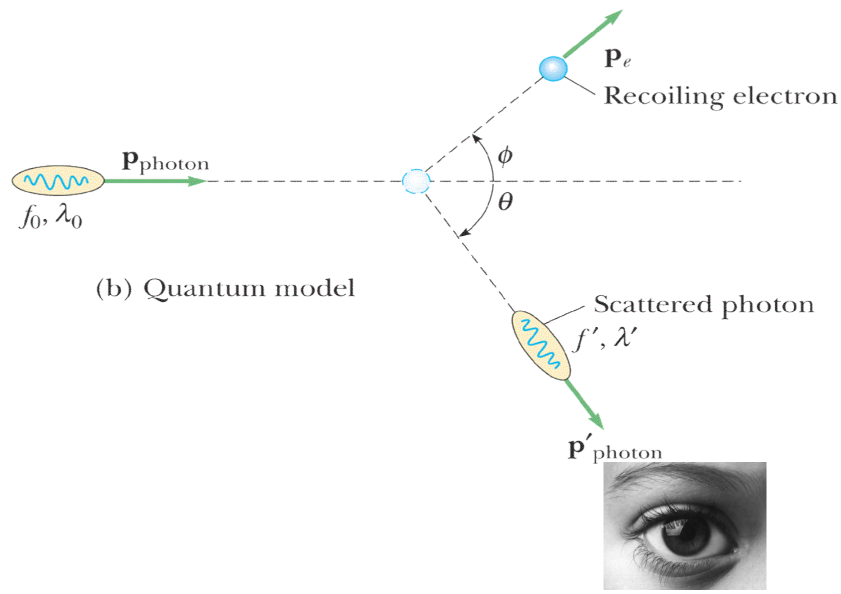


$\Delta\lambda = (\lambda' - \lambda) \propto (1 - \cos\theta)$!

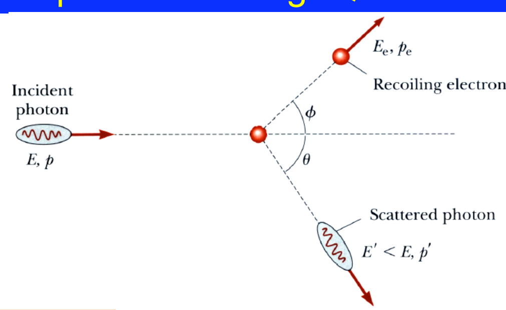
Not isotropy in distribution of scattered radiation

How does one explain this startling anisotropy?

Compton Effect : Quantum (Relativistic) Pool



Compton Scattering: Quantum Picture



Energy Conservation:

$E + m_e c^2 = E' + E_e$

Momentum Conserv:

$p = p'\cos\theta + p_e \cos\phi$

$0 = p'\sin\theta - p_e \sin\phi$

Use these to eliminate electron deflection angle (not measured)

$p_e \cos\phi = p - p'\cos\theta$

$p_e \sin\phi = p'\sin\theta$

Square and add $\Rightarrow$

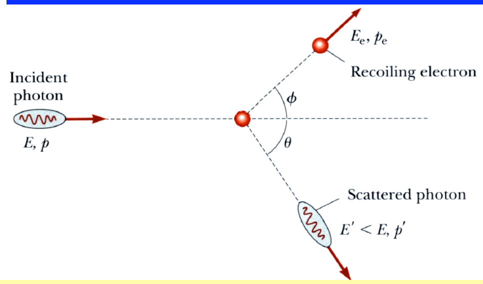
$p_e^2 = p^2 - 2pp'\cos\theta + p'^2$

Eliminate p_e & E_e using

$E_e^2 = p_e^2 c^2 + m_e^2 c^4$ &

$E_e = (E - E') + m_e c^2$

Compton Scattering: The Quantum Picture



Energy Conservation:

$E + m_e c^2 = E' + E_e$

Momentum Conserv:

$p = p'\cos\theta + p_e \cos\phi$

$0 = p'\sin\theta - p_e \sin\phi$

Use these to eliminate electron deflection angle (not measured)

$$\left((E-E')+m_e c^2\right)^2 = \left[p^2 - 2pp'\cos\theta + p'^2\right] + (m_e c^2)^2$$

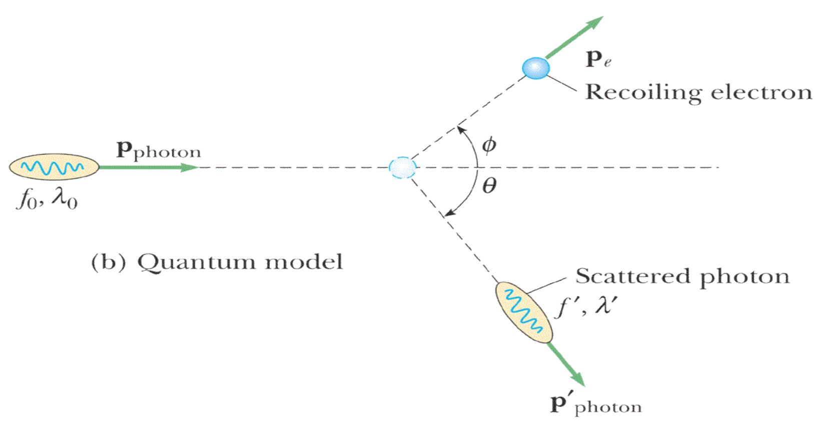
For light $p = \frac{E}{c} \Rightarrow$

$$E^2 + E'^2 - 2EE' + 2(E-E')mc^2 = \left[\frac{E^2}{c^2} - 2\frac{EE'}{c^2}\cos\theta + \frac{E'^2}{c^2}\right]c^2$$

$$\Rightarrow -EE' + (E-E')mc^2 = -EE'\cos\theta$$

$$\Rightarrow \frac{E-E'}{EE'} = -\frac{1}{m_e c^2}(1-\cos\theta) \Rightarrow \boxed{(\lambda' - \lambda) = (\frac{h}{m_e c})(1-\cos\theta)}$$

Rules of Quantum Pool between Photon and Electron

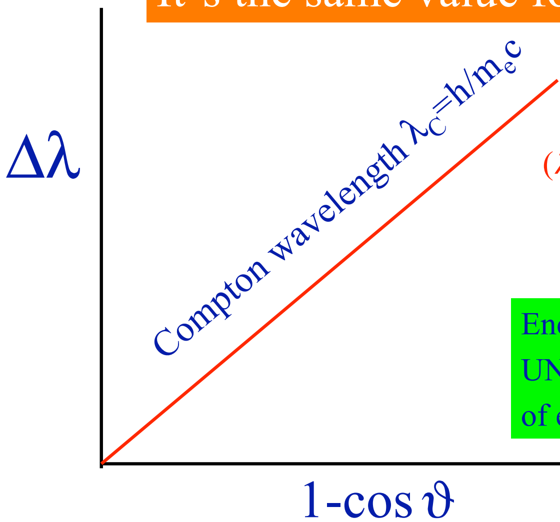


$$(\lambda' - \lambda) = (\frac{\boxed{h}}{m_e c})(1-\cos\theta)$$

Checking for h in Compton Scattering

Plot scattered photon data, calculate slope and measure "h"

It's the same value for h again !!



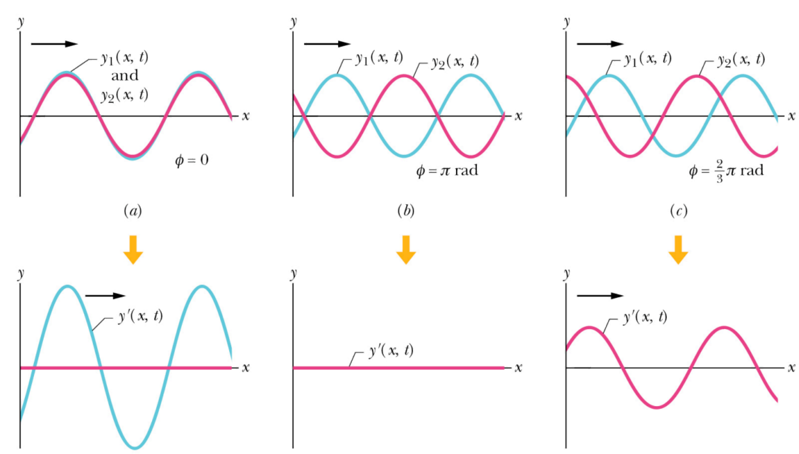
$$(\lambda' - \lambda) = \left(\frac{h}{m_e c}\right)(1 - \cos\theta)$$

Energy Quantization is a UNIVERSAL characteristic of energy transactions !

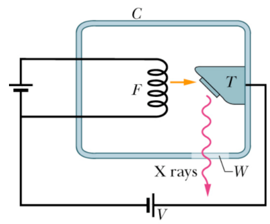
Interference of Waves: A Reminder

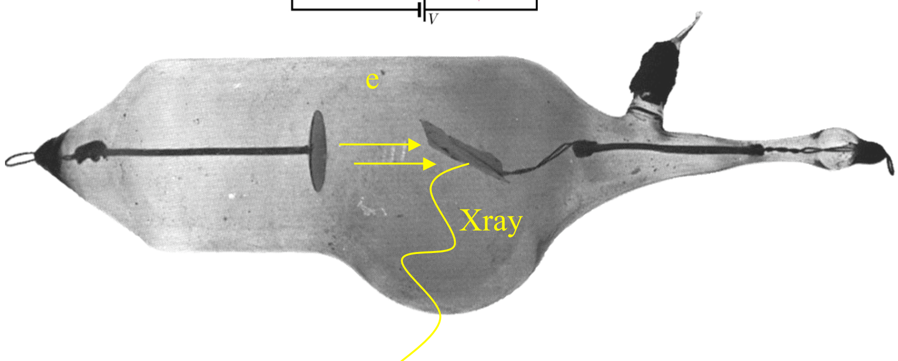
Two Identical waves $y_i(x,t) = y_{\max}\sin(k_i x - \omega_i t + \phi_i)$ travel along +x and interefere to give a resulting wave $y'(x,t)$. The resulting wave form depends on relative phase differenc between 2 waves. Shown for $\Delta\phi = 0,\ \pi,\ \frac{2}{3}\pi$

Read Ch17-8 from Resnick etal held in Ereserve

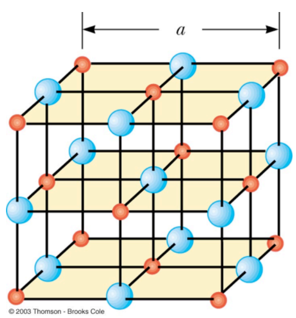


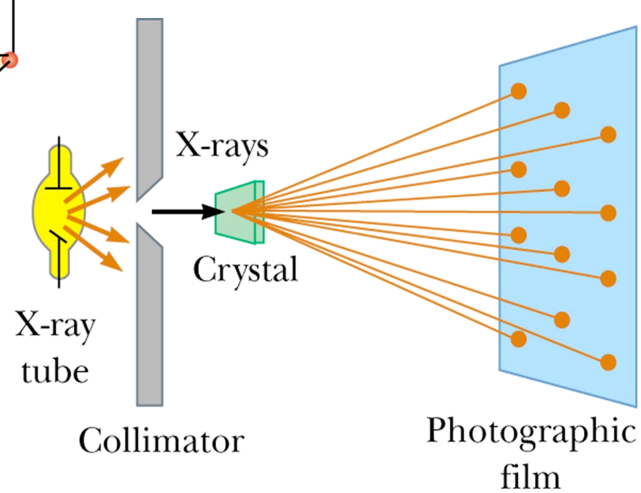
An X-ray Tube from 20th Century



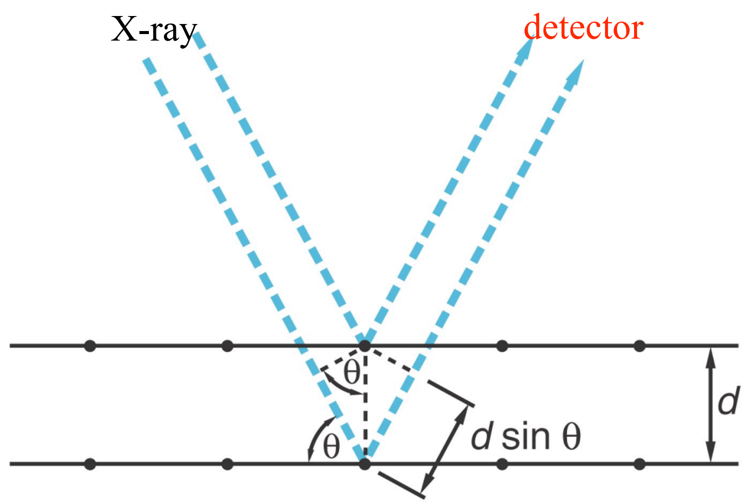


The “High Energy Accelerator” of 1900s: produced energetic light : X Ray , gave new optic to subatomic phenomena



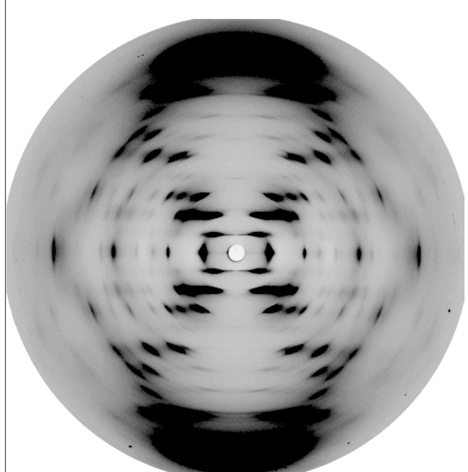


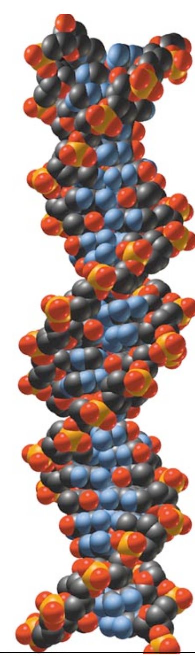
Bragg Scattering: Probing Atoms With X-Rays



Constructive Interference when net phase difference is 0, 2π etc
This implied path difference traveled by two waves must be integral multiple of wavelength : **$n\lambda=2d\sin\vartheta$**

X-Ray Picture of a DNA Crystal





Proteins inside Rhinovirus reconstructed by x-ray diffraction