Physics 2D Lecture Slides
Lecture 9 : Jan 19th 2005

Vivek Sharma
UCSD Physics

Definition (without proof) of Relativistic Momentum

$$\vec{p} = \frac{m\vec{u}}{\sqrt{1-(u/c)^2}} = \gamma m\vec{u}$$

With the new definition relativistic momentum is conserved in all frames of references : Do the exercise

New Concepts

Rest mass = mass of object measured
In a frame of ref. where object is at rest

$$\gamma = \frac{1}{\sqrt{1-(u/c)^2}}$$

u is velocity of the object
NOT of a reference frame !

Relativistic Force & Acceleration

$$\vec{p} = \frac{m\vec{u}}{\sqrt{1-(u/c)^2}} = \gamma m\vec{u}$$

Relativistic Force And Acceleration

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}\left(\frac{m\vec{u}}{\sqrt{1-(u/c)^2}}\right) \quad use \quad \frac{d}{dt} = \frac{du}{dt}\frac{d}{du}$$

$$F = \left[\frac{m}{\sqrt{1-(u/c)^2}} + \frac{mu}{\left(1-(u/c)^2\right)^{3/2}} \times \left(\frac{-1}{2}\right)\left(\frac{-2u}{c^2}\right)\right]\frac{du}{dt}$$

$$F = \left[\frac{mc^2 - mu^2 + mu^2}{c^2\left(1-(u/c)^2\right)^{3/2}}\right]\frac{du}{dt}$$

$$F = \left[\frac{m}{\left(1-(u/c)^2\right)^{3/2}}\right]\frac{du}{dt} \quad \text{: Relativistic Force}$$

Since Acceleration $\vec{a} = \frac{d\vec{u}}{dt}$, [rate of change of velocity]

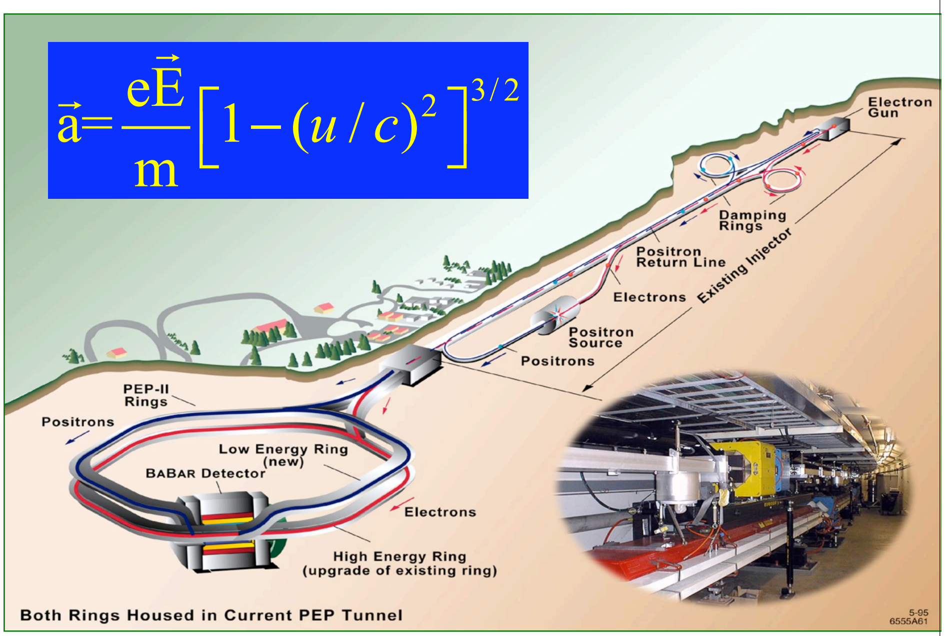
$$\Rightarrow \vec{a} = \frac{\vec{F}}{m}\left[1-(u/c)^2\right]^{3/2}$$

Reason why you cant quite get up to the speed of light no matter how hard you try!

Note: As $u/c \to 1$, $\vec{a} \to 0$!!!!
Its harder to accelerate when you get closer to speed of light

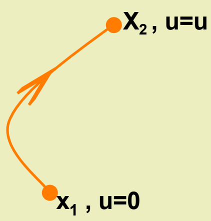
Linear Particle Accelerator : 50 GigaVolts Accelating Potential

$$\vec{a} = \frac{e\vec{E}}{m}\left[1-(u/c)^2\right]^{3/2}$$



PEP-II accelerator schematic and tunnel view

Relativistic Work Done & Change in Energy



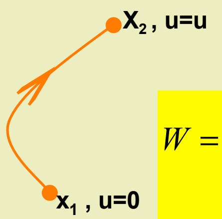
$$W = \int_{x_1}^{x_2} \vec{F}.d\vec{x} = \int_{x_1}^{x_2} \frac{d\vec{p}}{dt}.d\vec{x}$$

$$p = \frac{mu}{\sqrt{1-\frac{u^2}{c^2}}} \quad \therefore \quad \frac{d\vec{p}}{dt} = \frac{m\frac{du}{dt}}{\left[1-\frac{u^2}{c^2}\right]^{3/2}},$$

substitute in W,

$$\therefore \quad W = \int_0^u \frac{m\frac{du}{dt}udt}{\left[1-\frac{u^2}{c^2}\right]^{3/2}} \quad (\text{change in var x} \rightarrow \text{u})$$

Relativistic Work Done & Change in Energy



$$W = \int_0^u \frac{mudu}{\left[1-\frac{u^2}{c^2}\right]^{3/2}} = \frac{mc^2}{\left[1-\frac{u^2}{c^2}\right]^{1/2}} - mc^2$$

$$= \gamma mc^2 - mc^2$$

Work done is change in energy (KE in this case)

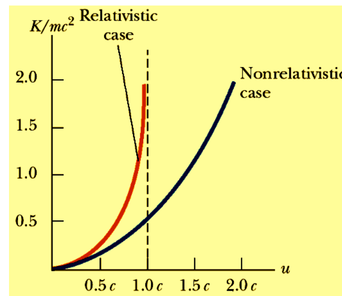
$K = \gamma mc^2 - mc^2$ or Total Energy $E = \gamma mc^2 = K + mc^2$

But Professor… Why Can's ANYTHING go faster than light ?

$$K = \frac{mc^2}{\left[1-\frac{u^2}{c^2}\right]^{1/2}} - mc^2 \Rightarrow \left(K+mc^2\right)^2 = \left(\frac{mc^2}{\left[1-\frac{u^2}{c^2}\right]^{1/2}}\right)^2$$

$$\Rightarrow \left[1-\frac{u^2}{c^2}\right] = m^2c^4\left[K+mc^2\right]^{-2}$$

$$\Rightarrow \boxed{u = c\sqrt{1-(\frac{K}{mc^2}+1)^{-2}}} \quad \text{(Parabolic in u Vs } \frac{K}{mc^2}\text{)}$$

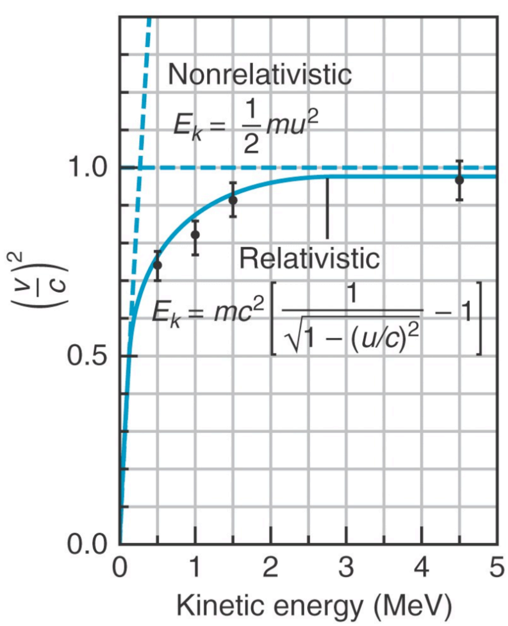


As u $\rightarrow$ c , Kinetic Energy K $\rightarrow \infty$

$\Rightarrow$ Need to do infinite amount of work on the particle to rev it up to the speed of light!

Non-relativistic case: $\boxed{K = \frac{1}{2}mu^2 \Rightarrow u = \sqrt{\frac{2K}{m}}}$

Relativistic Kinetic Energy Vs Velocity



A Digression: How to Handle Large/Small Numbers

- Example: consider very energetic particle with very large Energy E

$$\gamma = \frac{E}{mc^2} = \frac{mc^2 + K}{mc^2} = 1 + \frac{K}{mc^2}$$

- Lets Say $\gamma = 3 \times 10^{11}$, Now calculate u from → $\frac{u}{c} = \left[1 - \frac{1}{\gamma^2}\right]^{1/2}$
- Try this on your el-cheapo calculator, you will get u/c =1, u=c due to limited precision.
- In fact u≅c but not exactly!, try to get this analytically

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{(1-\beta)(1+\beta)}}$$

Since $\beta = \frac{u}{c} \cong 1, \quad 1+\beta = 2$

$$\gamma \approx \frac{1}{\sqrt{2}\sqrt{1-\beta}}$$

$$\Rightarrow 1-\beta = \frac{1}{2\gamma^2} = 5 \times 10^{-24}, \quad u = \beta c$$

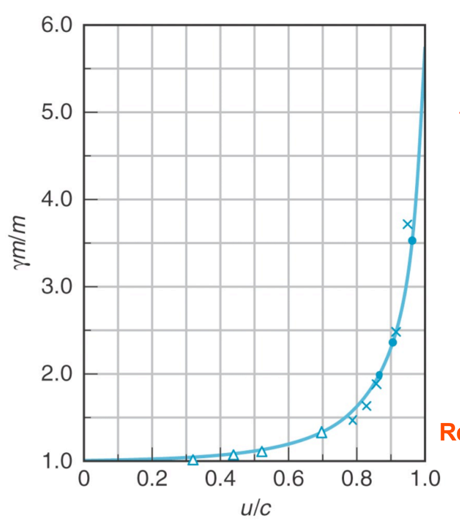
$$\Rightarrow u = 0.999\ 999\ 999\ 999\ 999\ 999\ 999\ 995c \ !!$$

Such particles are routinely produced in violent cosmic collisions

In Quizzes, you are Expected to perform Such simple approximations

When Electron Goes Fast it Gets "Fat"

Total Energy $E = \gamma mc^2 = K + mc^2$



$$E = \gamma mc^2$$

As $\frac{v}{c} \to 1, \quad \gamma \to \infty$

Apparent Mass approaches ∞

New Concept
Rest Mass = particle mass when its at rest

Relativistic Kinetic Energy & Newtonian Physics

Relativistic KE $K = \gamma mc^2 - mc^2$

$$\left[\begin{array}{l}\text{Remember Binomial Theorem} \\ \text{for } x \ll 1;\ (1+x)^n = (1 + \frac{nx}{1!} + \frac{n(n-1)}{2!}x^2 + \text{smaller terms})\end{array}\right]$$

$$\therefore \text{ When } u \ll c, \left[1 - \frac{u^2}{c^2}\right]^{-\frac{1}{2}} \cong 1 + \frac{1}{2}\frac{u^2}{c^2} + \ldots\text{smaller terms}$$

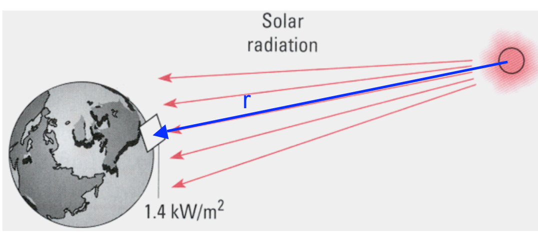
$$\text{so } K \cong mc^2[1 + \frac{1}{2}\frac{u^2}{c^2}] - mc^2 = \frac{1}{2}mu^2 \quad \text{(classical form recovered)}$$

Total Energy of a Particle $E = \gamma mc^2 = KE + mc^2$

For a particle at rest, $u = 0 \Rightarrow$ Total Energy $E = mc^2$

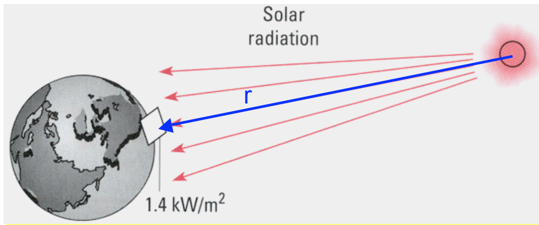
$E=mc^2 \Rightarrow$ Sunshine Won't Be Forever !

Q: Solar Energy reaches earth at rate of 1.4kW per square meter of surface perpendicular to the direction of the sun. by how much does the mass of sun decrease per second owing to energy loss? The mean radius of the Earth's orbit is 1.5×10^{11}m.



- Surface area of a sphere of radius r is $A = 4\pi r^2$
- Total Power radiated by Sun = power received by a sphere whose radius is equal to earth's orbit radius

E= mc² ⇒ Sunshine Won't Be Forever !



Total Power radiated by Sun = power received by a sphere with radius equal to earth-sun orbit radius(r in figure)

$$P_{lost}^{sun} = \frac{P_{incident}^{Earth}}{A} A_{earth-sun} = \frac{P_{incident}^{Earth}}{A} 4\pi r_{earth-sun}^2 = (1.4\times10^3 W/m^2)(4\pi)(1.5\times10^{11})^2$$

$$P_{lost}^{sun} = 4.0\times10^{26} W$$

So Sun loses E = 4.0×10^{26} J of rest energy per second

Its mass decreases by $m = \frac{E}{c^2} = \frac{4.0\times10^{26}\,J}{(3.0\times10^8)^2} = 4.4\times10^9\,kg$ per sec!!

If the Sun's Mass = $2.0\times10^{30} kg$ So how long with the Sun last ?

One day the sun will be gone and the solar system will not be a hospitable place for life

Relationship between P and E

$$\boxed{E = \gamma mc^2} \Rightarrow E^2 = \gamma^2 m^2 c^4$$

$$\boxed{p = \gamma mu} \Rightarrow \boxed{p^2c^2 = \gamma^2 m^2 u^2 c^2}$$

$$\Rightarrow E^2 - p^2c^2 = \gamma^2 m^2 c^4 - \gamma^2 m^2 u^2 c^2 = \gamma^2 m^2 c^2 (c^2 - u^2)$$

$$= \frac{m^2c^2}{1-\frac{u^2}{c^2}}(c^2 - u^2) = \frac{m^2c^4}{c^2-u^2}(c^2-u^2) = m^2c^4$$

$$\boxed{E^2 = p^2c^2 + (mc^2)^2}$$ ....important relation

For particles with zero rest mass like photon (EM waves)

$$\boxed{E = pc \text{ or } p = \frac{E}{c}}$$ (light has momentum!)

Relativistic Invariance : $\boxed{E^2 - p^2c^2 = m^2c^4}$: In all Ref Frames

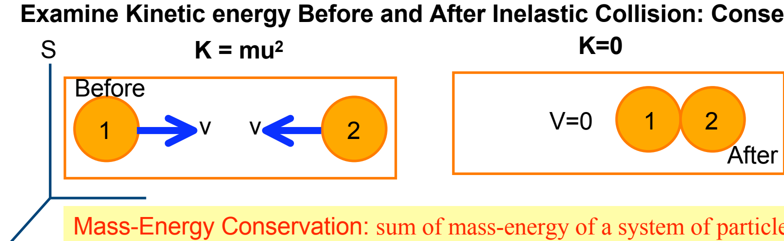
Rest Mass is a "finger print" of the particle

Mass Can "Morph" into Energy & Vice Verca

- In Newtonian mechanics: mass and energy separate concepts
- In relativistic physics : Mass and Energy are the same thing !
- New word/concept : MassEnergy , just like SpaceTime
- It is the mass-energy that is always conserved in every reaction : Before & After a reaction has happened
- Like squeezing a balloon : Squeeze here, it grows elsewhere
  - **If you "squeeze" mass, it becomes (kinetic) energy & vice verca !**
    - **CONVERSION FACTOR = C^2**
    - **This exchange rate never changes !**

Mass is Energy, Energy is Mass : Mass-Energy Conservation

Examine Kinetic energy Before and After Inelastic Collision: Conserved?



Mass-Energy Conservation: sum of mass-energy of a system of particles before interaction must equal sum of mass-energy after interaction

$$E_{before} = E_{after}$$

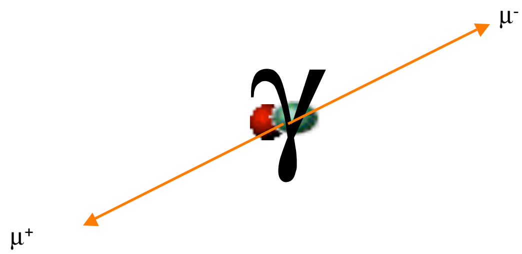
$$\frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}} + \frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}} = Mc^2 \Rightarrow M = \frac{2m}{\sqrt{1-\frac{u^2}{c^2}}} > 2m$$

Kinetic energy has been transformed into mass increase

$$\Delta M = M - 2m = \frac{2K}{c^2} = \frac{2}{c^2}\left(\frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}} - mc^2\right)$$

Kinetic energy is not lost, its transformed into more mass in final state

Creation and Annihilation of Particles

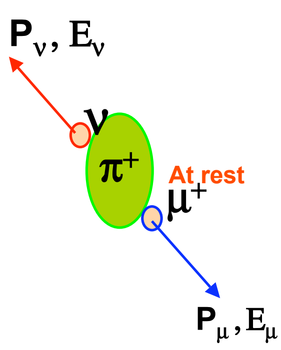


Sequence of events in a matter-antimatter collision:

$$e^{+} + e^{-} \rightarrow \gamma \rightarrow \mu^{+} + \mu^{-}$$

Relativistic Kinematics of Subatomic Particles

Reconstructing Decay of a π Meson



The decay of a stationary $\pi^{+} \rightarrow \mu^{+}\upsilon$ happens quickly,

υ is invisible, has m $\cong 0$; μ^{+} leaves a trace in a B field

μ^{+} mass=106 MeV/c^{2}, KE = 4.6 MeV

What was mass of the fleeting π^{+}?

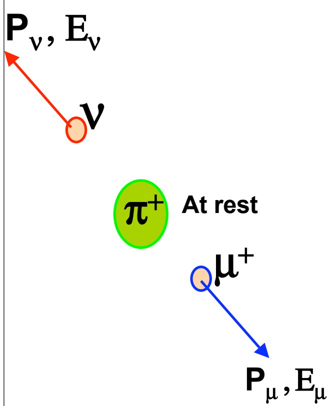
Energy Conservation:

$$E_{\pi} = E_{\mu} + E_{\nu} \;\Rightarrow\; m_{\pi}c^{2} = \sqrt{(m_{\mu}c^{2})^{2} + p_{\mu}^{2}c^{2}} + p_{\nu}c$$

Momentum Conservation : $p_{\mu} = p_{\nu}$

$$\Rightarrow m_{\pi}c^{2} = \sqrt{(m_{\mu}c^{2})^{2} + p_{\mu}^{2}c^{2}} + p_{\mu}c$$

Relativistic Kinematics of Subatomic Particles



Energy Conservation:

$$E_\pi = E_\mu + E_\nu \;\Rightarrow\; m_\pi c^2 = \sqrt{(m_\mu c^2)^2 + p_\mu^2 c^2} + p_\nu c$$

Momentum Conservation : $p_\mu = p_\nu$

$$\Rightarrow m_\pi c^2 = \sqrt{(m_\mu c^2)^2 + p_\mu^2 c^2} + p_\mu c$$

$$\text{also } p_\mu^2 c^2 = E_\mu^2 - (m_\mu c^2)^2 = (K_\mu + m_\mu c^2)^2 - (m_\mu c^2)^2$$

$$= K_\mu^2 + 2K_\mu m_\mu c^2$$

Substituting for $p_\mu^2 c^2 \Rightarrow$

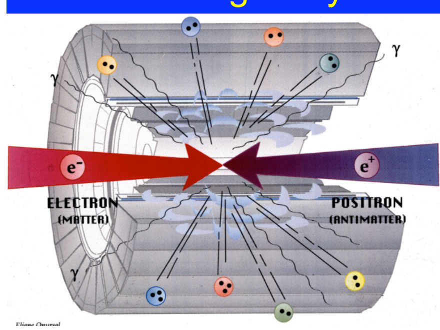
$$m_\pi c^2 = \sqrt{m_\mu^2 c^4 + K_\mu^2 + 2K_\mu m_\mu c^2} + \sqrt{K_\mu^2 + 2K_\mu m_\mu c^2}$$

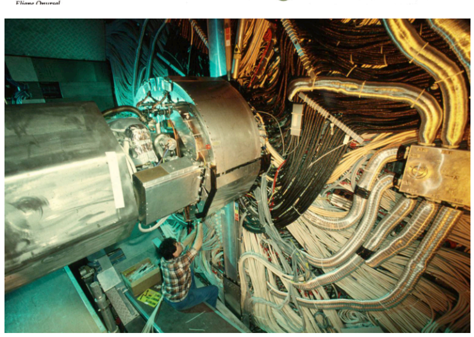
Put in all the known #s

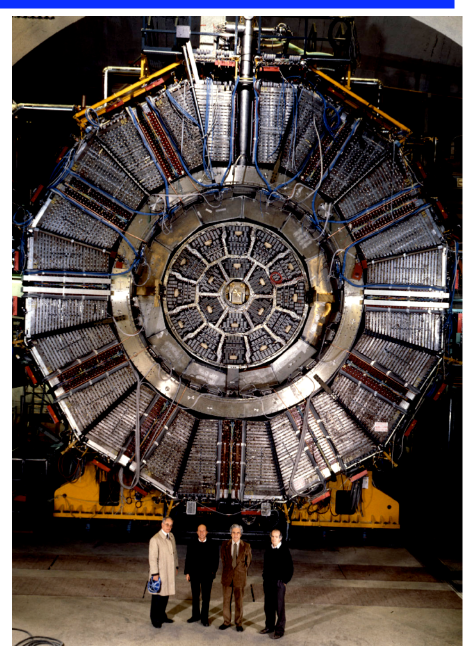
$$\Rightarrow m_\pi c^2 = 111 MeV + 31 MeV = 141 MeV$$

$$\Rightarrow m_\pi = 141 MeV / c^2$$

Detecting Baby Universes : Need a "Camera"



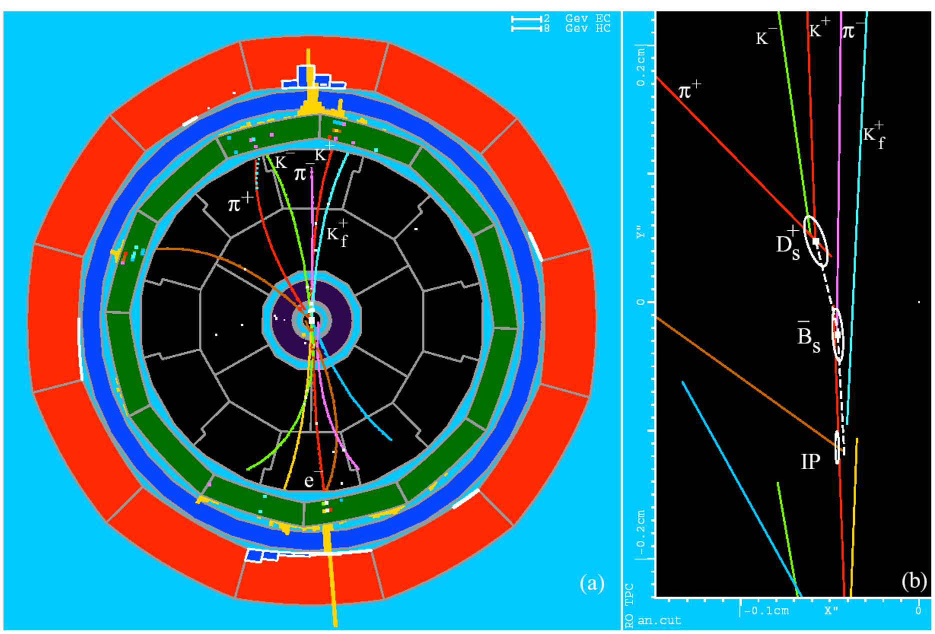




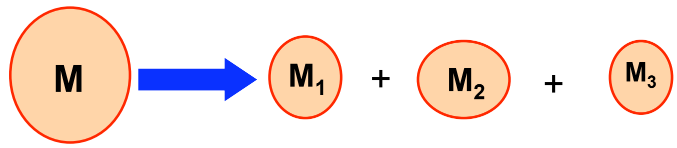
Two Faced Particle : Beauty With Strangeness (Bs)

Sometimes Matter
Sometimes Antimatter

My Discovery (1993): Beauty With Strangeness



Conservation of Mass-Energy: Nuclear Fission



Nuclear Fission

$$Mc^2 = \underbrace{\frac{M_1c^2}{\sqrt{1-\frac{u_1^2}{c^2}}}}_{<1} + \underbrace{\frac{M_2c^2}{\sqrt{1-\frac{u_2^2}{c^2}}}}_{<1} + \underbrace{\frac{M_3c^2}{\sqrt{1-\frac{u_3^2}{c^2}}}}_{<1} \Rightarrow M > M_1 + M_2 + M_3$$

Loss of mass shows up as kinetic energy of final state particles

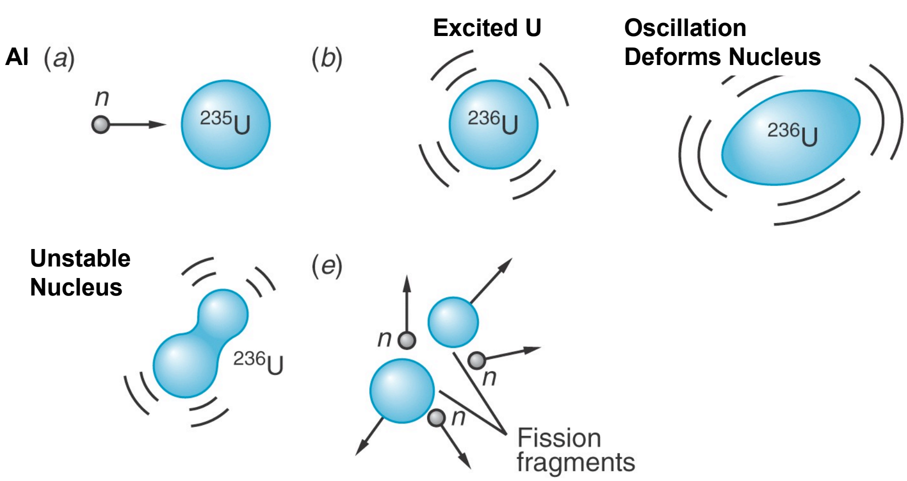
Disintegration energy per fission $Q=(M-(M_1+M_2+M_3))c^2 = \Delta Mc^2$

${}^{236}_{92}U \rightarrow {}^{143}_{55}Cs + {}^{90}_{92}Rb + 3\,{}^{1}_{0}n$ (1 AMU= $1.6605402\times10^{-27}kg = 931.49$ MeV)

$\Delta m=0.177537u=2.9471\times10^{-28}kg = 165.4$ MeV= energy release/fission =peanuts

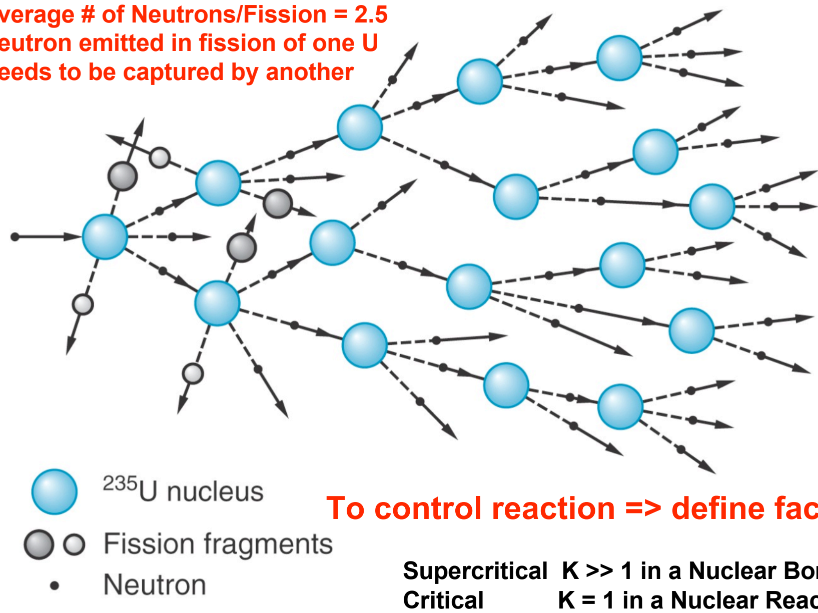
What makes it explosive is 1 mole of Uranium = 6.023×10^{23} Nuclei !!

Nuclear Fission Schematic : "Tickling" a Nucleus



Sustaining Chain Reaction: 1st three Fissions

Average # of Neutrons/Fission = 2.5
Neutron emitted in fission of one U
Needs to be captured by another

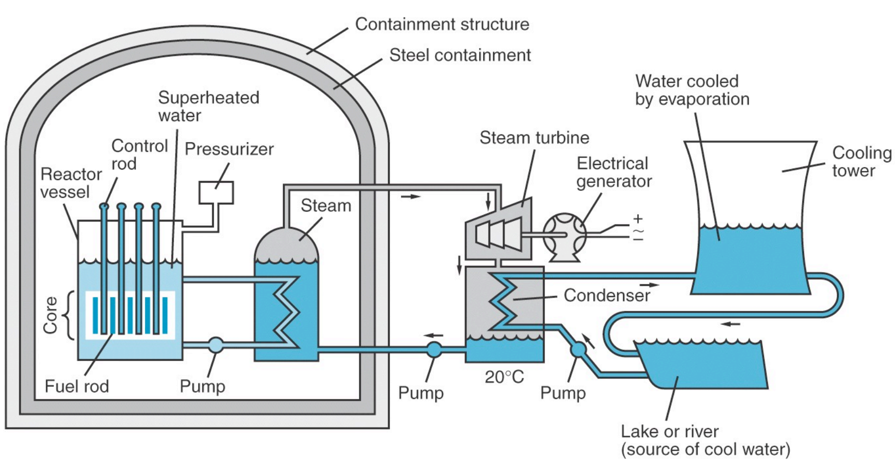


To control reaction => define factor K

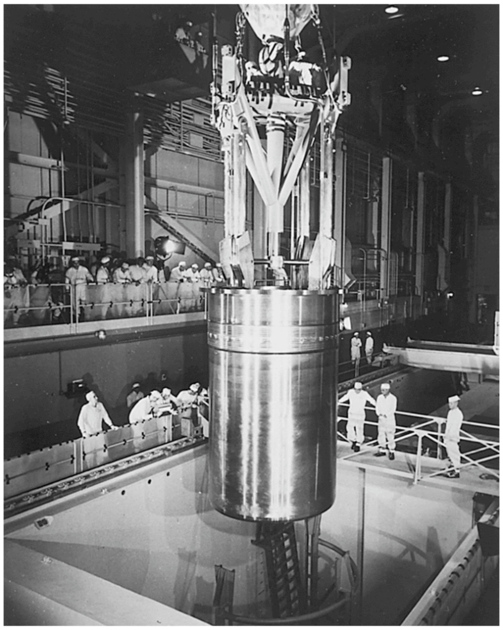
Supercritical K >> 1 in a Nuclear Bomb
Critical K = 1 in a Nuclear Reactor

Schematic of a Pressurized-Water Reactor

Water in contact with reactor core serves as a moderator and heat transfer Medium. Heat produced in fission drives turbine



Lowering Fuel Core in a Nuclear Reactor



First Nuclearr reactor :Pennsylvania 1957

Pressure Vessel contains :
14 Tons of Natural Uranium
+ 165 lb of enriched Uranium

Power plant rated at 90MW, Retired (82)

Pressure vessel packed with Concrete now sits in Nuclear Waste Facility in Hanford, Washington

Nuclear Fusion : What Powers the Sun

Opposite of Fission

Mass of a Nucleus < mass of its component protons+Neutrons

Nuclei are stable, bound by an attractive "Strong Force"

Think of Nuclei as molecules and proton/neutron as atoms making it

Binding Energy: Work/Energy required to pull a bound system (M) apart leaving its components (m) free of the attractive force and at rest:

$$Mc^2 + BE = \sum_{i=1}^{n} m_i c^2$$

$$ {}^{2}_{1}H + {}^{2}_{1}H = {}^{4}_{2}He + 23.9\ \text{MeV} = $$

Deuterium + Deuterium = Helium + Released Energy

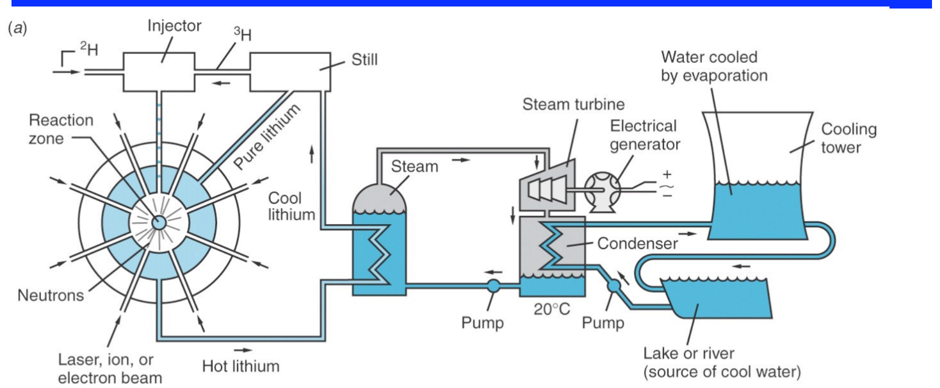
Think of energy released in Fusion as Dissociation energy of Chemistry

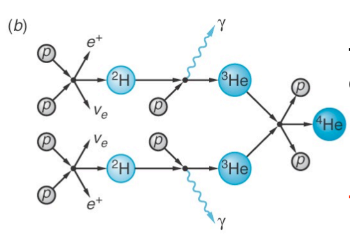
Sun's Power Output = 4×10^{26} Watts $\Rightarrow 10^{38}$ Fusion/Second !!!!

Nuclear Fusion: Wishing For The Star

- Fusion is eminently desirable because
  - More Energy/Nucleon
    - (3.52 MeV in fusion Vs 1 MeV in fission)
    - $^{2}H + {}^{3}H \rightarrow {}^{4}He + n + 17.6$ MeV
  - Relatively abundant fuel supply, No danger like nuclear reactor going supercritical
- Unfortunately technology not commercially available
  - What's inside nuclei => protons and Neutrons
  - Need Large KE to overcome Coulomb repulsion between nuclei
    - About 1 MeV needed to bring nuclei close enough together for Strong Nuclear Attraction → fusion
    - Need to
      - heat particle to high temp such that thermal energy $E = kT \approx 10$keV → tunneling thru coulomb barrier
      - Implies heating to $T \approx 10^{8}$ K (like in stars)
      - Confine Plasma (± ions) long enough for fusion
        - » In stars, enormous gravitational field confines plasma

Inertial Fusion Reactor : Schematic





Pellet of frozen-solid Deuterium & tritium bombarded from all sides with intense pulsed laser beam with energy ≈10^{6} Joules lasting 10^{-8} S

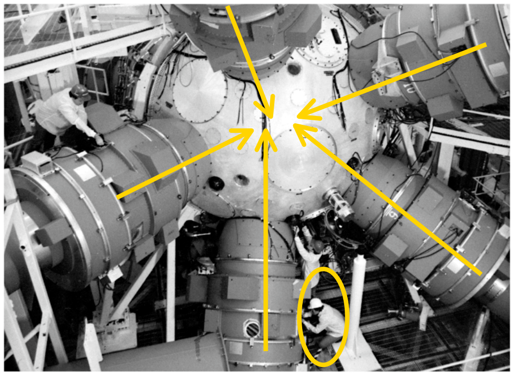
Momentum imparted by laser beam compresses pellet by 1/10000 of normal density and heats it to temp $T \approx 10^{8}$ K for 10^{-10} S

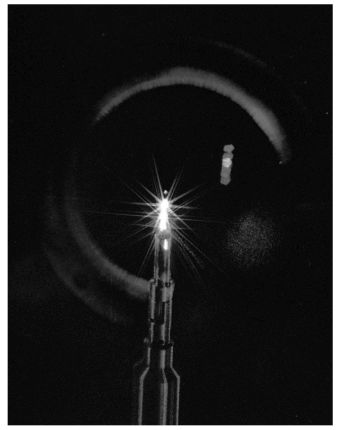
Burst of fusion energy transported away by liquid Li

World’s Most Powerful Laser : NOVA @ LLNL

Size of football field, 3 stories tall

Generates 1.0 x 10^{14} watts (100 terawatts)

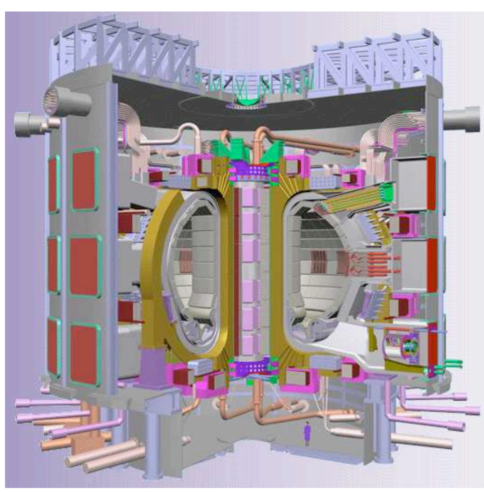




10 laser beams converge onto H pellet (0.5mm diam)

Fusion reaction is visible as a starlight lasting 10^{-10} S
Releasing 10^{13} neutrons

ITER: The Next Big Step in Nuclear Fusion



Visit www.iter.org for Details of this mega Science & Engineering Project
This may be future of cheap, clean Nuclear Energy for Earthlings