Physics 2D Lecture Slides
Lecture 7 : Jan 12th 2005

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First Quiz This Friday !

- Bring a Blue Book, calculator; check battery
  - Make sure you remember the code number for this couse given to you (record it some place safe!)
- No "cheat Sheet" please, I will give you equations and constants that I think you need
- When you come for the quiz, pl. occupy seats in the front first.
- Pl. observe one seat distance in the back rows (there is plenty of space)
- Academic Honesty is for you to observe and for me to enforce:
  - Be a good citizen, in this course and forever !

Lorentz Transformation Between Ref Frames

Lorentz Transformation

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

Inverse Lorentz Transformation

$$x = \gamma(x' + vt')$$

$$y = y'$$

$$z = z'$$

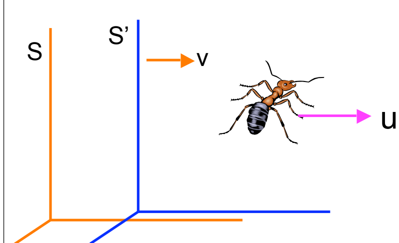
$$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

As v→0 , Galilean Transformation is recovered, as per requirement

Notice : SPACE and TIME Coordinates mixed up !!!

Lorentz Velocity Transformation Rule

S and S' are measuring ant's speed u along x, y, z axes



In S' frame, $u_{x'} = \dfrac{x_2' - x_1'}{t_2' - t_1'} = \dfrac{dx'}{dt'}$

$$dx' = \gamma(dx - vdt), \quad dt' = \gamma\left(dt - \frac{v}{c^2}dx\right)$$

$$u_{x'} = \frac{dx - vdt}{dt - \frac{v}{c^2}dx}, \quad \text{divide by dt'}$$

$$u_{x'} = \frac{u_x - v}{1 - \frac{vu_x}{c^2}}$$

For v << c, $u_{x'} = u_x - v$

(Galilean Trans. Restored)

Velocity Transformation Perpendicular to S-S' motion

$$dy' = dy, \quad dt' = \gamma(dt - \frac{v}{c^2}dx)$$

$$u_y' = \frac{dy'}{dy'} = \frac{dy}{\gamma(dt - \frac{v}{c^2}dx)}$$

divide by dt on RHS

$$u_y' = \frac{u_y}{\gamma(1 - \frac{v}{c^2}u_x)}$$

There is a change in velocity in the direction $\perp$ to S-S' motion !

Similarly Z component of Ant's velocity transforms as

$$u_z' = \frac{u_z}{\gamma(1 - \frac{v}{c^2}u_x)}$$

Inverse Lorentz Velocity Transformation

Inverse Velocity Transform:

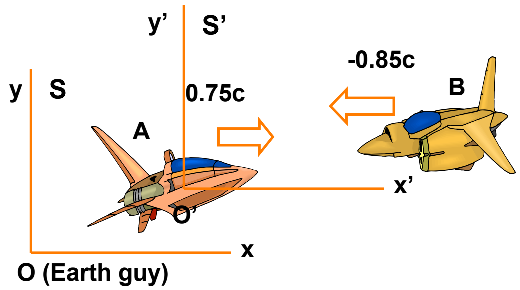
$$\mathrm{u_x} = \frac{u_{x'} + v}{1 + \frac{vu_{x'}}{c^2}}$$

$$u_y = \frac{u_y'}{\gamma(1 + \frac{v}{c^2}u_x')}$$

$$u_z = \frac{u_z'}{\gamma(1 + \frac{v}{c^2}u_x')}$$

As usual, replace $v \Rightarrow -v$

Does Lorentz Transform "work" For Topgun ?



Two rockets A &B travel in opposite directions

An observer on earth (S) measures speeds = 0.75c And 0.85c for A & B respectively

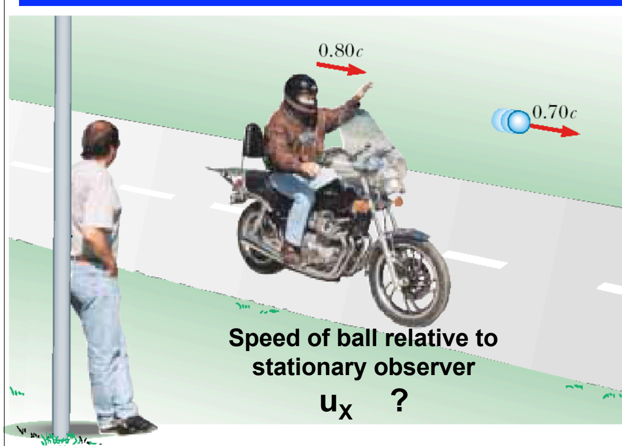
What does A measure as B's speed?

Place an imaginary S' frame on Rocket A ⇒ v = 0.75c relative to Earth Observer S

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{-0.850c - 0.750c}{1 - \frac{(-0.850c)(0.750c)}{c^2}} = -0.977c$$

Consistent with Special Theory of Relativity

Example of Inverse Velocity Transformation



Biker moves with speed = 0.8c past stationary observer

Throws a ball forward with speed = 0.7c

What does stationary observer see as velocity of ball ?

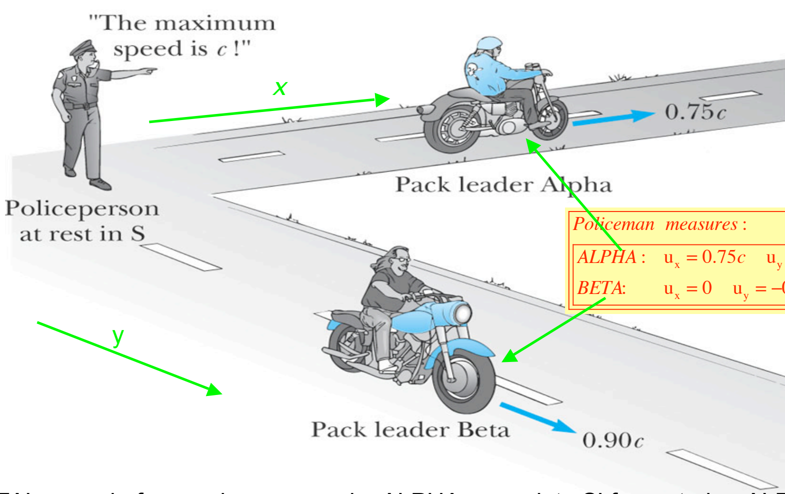
Place S' frame on biker
Biker sees ball speed
$u_{X'}$ =0.7c

$$u_x = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}} = \frac{0.70c + 0.80c}{1 + \frac{(0.70c)(0.80c)}{c^2}} = 0.96c$$

Velocity Transformation Perpendicular to S-S’ motion

2 bike gang leaders racing at relativistic speeds along *perpendicular* paths

How fast does BETA recede over right shoulder of ALPHA as seen by ALPHA



Policeman measures:

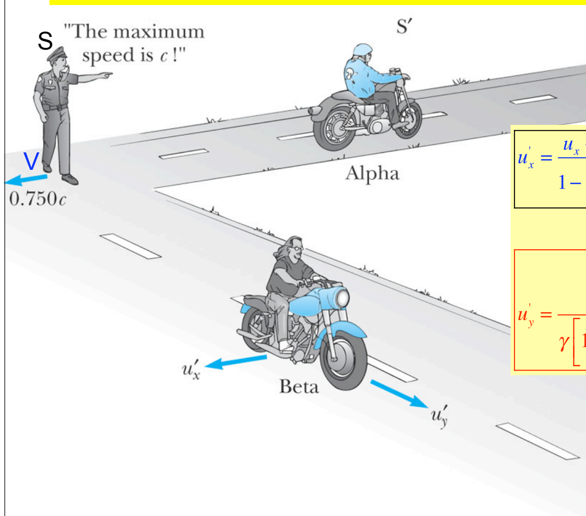
ALPHA: $u_x = 0.75c \quad u_y = 0$

BETA: $u_x = 0 \quad u_y = -0.90c$

To get BETA’s speed of recession as seen by ALPHA, associate S’ frame to be ALPHA’s make Policeman’s frame to be S (measuring u_x and u_y) and calculate u'_x and u'_y

Velocity Transformation Perpendicular to S-S’ motion

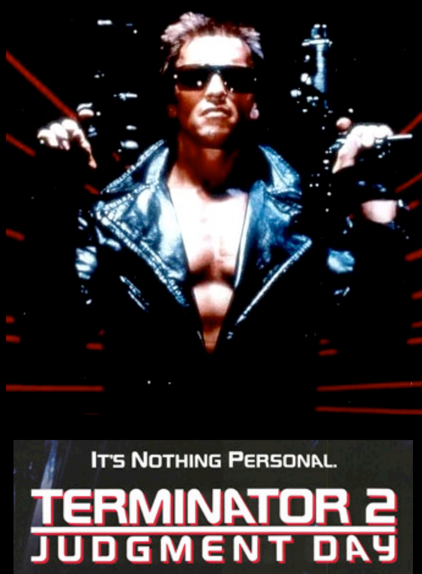
Make Alpha the S’ frame at rest, Policeman is frame S and we know what he (S) measures as the x,y components of BETA’s velocity in his frame.



$$u'_x = \frac{u_x - V}{1 - \frac{u_x V}{c^2}} = \frac{0 - 0.75c}{1 - \frac{0.0c \times 0.75c}{c^2}} = -0.75c$$

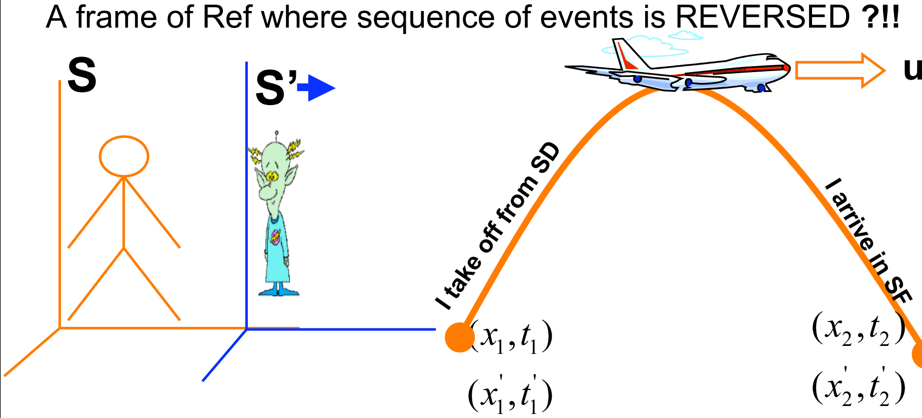
$$u'_y = \frac{u_y}{\gamma\left[1 - \frac{u_x V}{c^2}\right]} = \frac{-0.90c \times \sqrt{1 - \left(\frac{0.75c}{c}\right)^2}}{1 - 0} = -0.595c$$

Hollywood Yarns Of Time Travel !



Terminator : Can you be **seen** to be born before your mother?

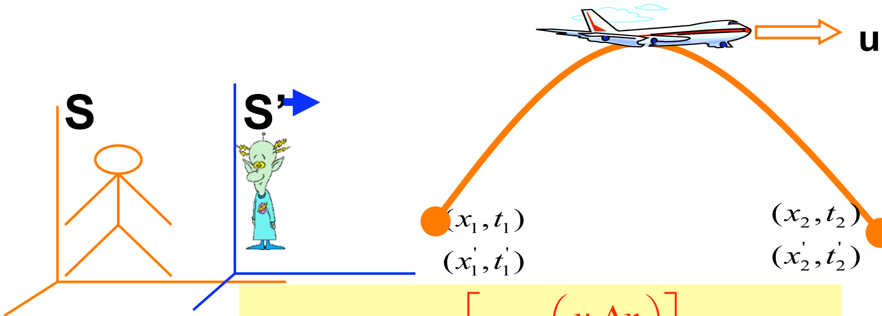
A frame of Ref where sequence of events is REVERSED **?!!**



$$\Delta t' = t_2' - t_1' = \gamma\left[\Delta t - \left(\frac{v\,\Delta x}{c^2}\right)\right]$$

Reversing sequence of events $\Rightarrow \Delta t' < 0$

I Can't be seen to arrive in SF before I take off from SD



$$\Delta t' = t_2' - t_1' = \gamma\left[\Delta t - \left(\frac{v\,\Delta x}{c^2}\right)\right]$$

For what value of v can $\Delta t' < 0$

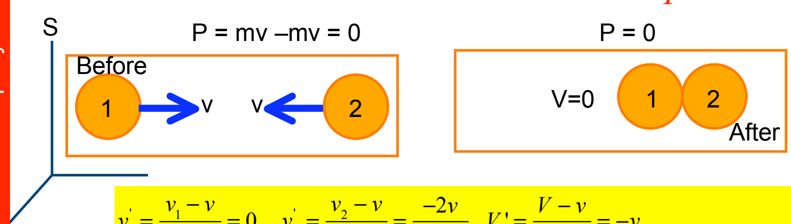
$$\Delta t' < 0 \Rightarrow \Delta t < \frac{v\,\Delta x}{c^2} \quad \Rightarrow 1 < \frac{v\,\Delta x}{c^2 \Delta t} = \frac{v\ \mathrm{u}}{c^2}$$

$$\Rightarrow \frac{\mathrm{v}}{\mathrm{c}} > \frac{c}{u} \qquad \Rightarrow \boxed{v > c : \text{Not allowed !!}}$$

Relativistic Momentum and Revised Newton's Laws

Need to generalize the laws of Mechanics & Newton to confirm to Lorentz Transform and the Special theory of relativity: Example : $\vec{p} = m\vec{u}$

Watching an Inelastic Collision between two putty balls

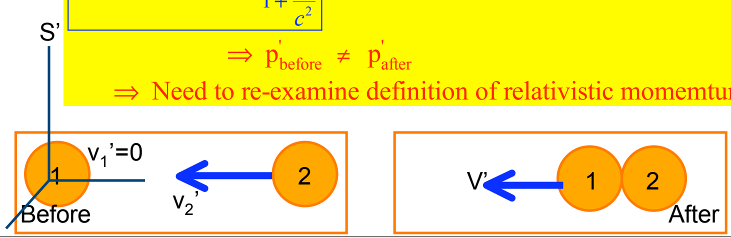


$$v_1' = \frac{v_1 - v}{1 - \frac{v_1 v}{c^2}} = 0, \quad v_2' = \frac{v_2 - v}{1 - \frac{v_1 v}{c^2}} = \frac{-2v}{1 + \frac{v^2}{c^2}}, \quad V' = \frac{V - v}{1 - \frac{Vv}{c^2}} = -v$$

$$\boxed{p_{before}' = mv_1' + mv_2' = \frac{-2mv}{1 + \frac{v^2}{c^2}}}, \qquad \boxed{p_{after}' = 2mV' = -2mv}$$

$\Rightarrow p_{\text{before}}' \neq p_{\text{after}}'$

$\Rightarrow$ Need to re-examine definition of relativistic momemtum



Definition (without proof) of Relativistic Momentum

$$\vec{p} = \frac{m\vec{u}}{\sqrt{1-(u/c)^2}} = \gamma m\vec{u}$$

With the new definition relativistic momentum is conserved in all frames of references : Do the exercise

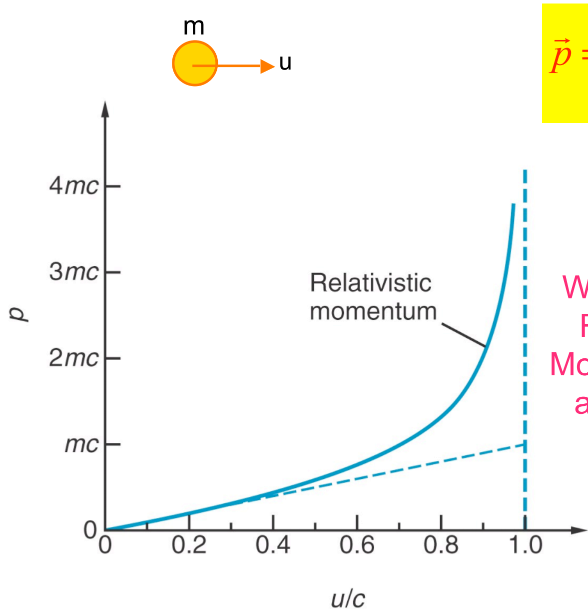
New Concepts

Rest mass = mass of object measured
In a frame of ref. where object is at rest

$$\gamma = \frac{1}{\sqrt{1-(u/c)^2}}$$

u is velocity of the object
NOT of a reference frame !

Nature of Relativistic Momentum



$$\vec{p} = \frac{m\vec{u}}{\sqrt{1-(u/c)^2}} = \gamma m\vec{u}$$

With the new definition of Relativistic momentum Momentum is conserved in all frames of references

Relativistic Force & Acceleration

$$\vec p = \frac{m\vec u}{\sqrt{1-(u/c)^2}} = \gamma m\vec u$$

Relativistic
Force
And
Acceleration

$$\vec F = \frac{d\vec p}{dt} = \frac{d}{dt}\left(\frac{m\vec u}{\sqrt{1-(u/c)^2}}\right) \quad use \quad \frac{d}{dt} = \frac{du}{dt}\frac{d}{du}$$

$$F = \left[\frac{m}{\sqrt{1-(u/c)^2}} + \frac{mu}{\left(1-(u/c)^2\right)^{3/2}} \times \left(\frac{-1}{2}\right)\left(\frac{-2u}{c^2}\right)\right]\frac{du}{dt}$$

$$F = \left[\frac{mc^2 - mu^2 + mu^2}{c^2\left(1-(u/c)^2\right)^{3/2}}\right]\frac{du}{dt}$$

$$F = \left[\frac{m}{\left(1-(u/c)^2\right)^{3/2}}\right]\frac{du}{dt} \quad \text{: Relativistic Force}$$

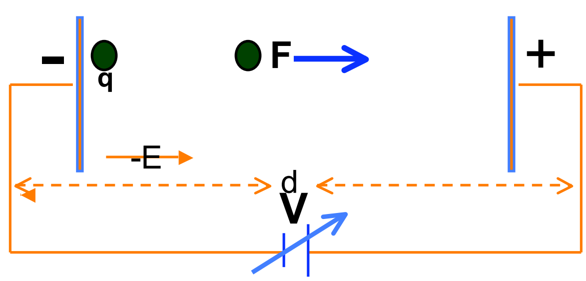
Since Acceleration $\vec a = \frac{d\vec u}{dt}$, [rate of change of velocity]

$$\Rightarrow \vec a = \frac{\vec F}{m}\left[1-(u/c)^2\right]^{3/2}$$

Reason why you cant quite get up to the speed of light no matter how hard you try!

Note: As $u/c \to 1$, $\vec a \to 0$!!!!
Its harder to accelerate when you get closer to speed of light

A Linear Particle Accelerator



Parallel Plates

E= V/d
F= eE

Charged particle q moves in straight line in a uniform electric field $\vec E$ with speed $\vec u$ accelarates under force $\vec F = q\vec E$

$$|\vec a| = \left|\frac{d\vec u}{dt}\right| = \left|\frac{\vec F}{m}\right|\left(1-\frac{u^2}{c^2}\right)^{3/2} = \left|\frac{q\vec E}{m}\right|\left(1-\frac{u^2}{c^2}\right)^{3/2}$$

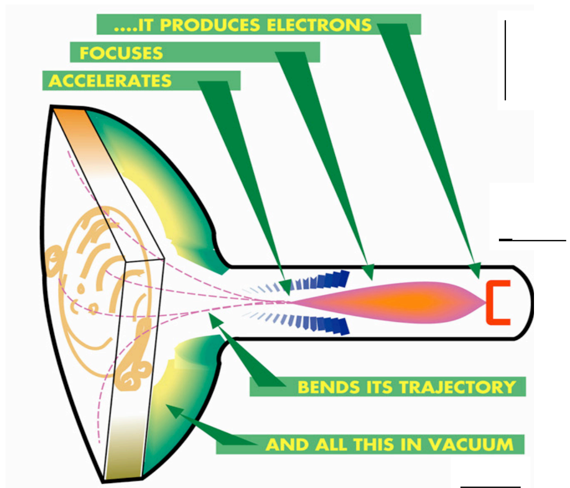
larger the potential difference V across plates, larger the force on particle

Under force, work is done on the particle, it gains Kinetic energy

<u>New Unit of Energy</u>

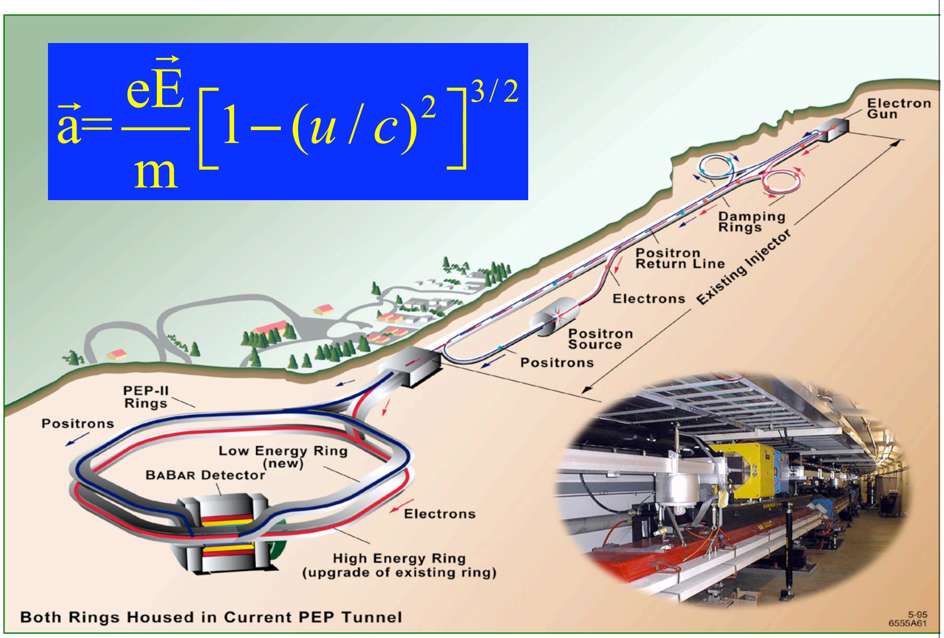
1 eV = 1.6×10^{-19} Joules
1 MeV = 1.6×10^{-13} Joules
1 GeV = 1.6x10-10 Joules

Your Television (the CRT type) is a Small Particle Accelerator !



Linear Particle Accelerator : 50 GigaVolts Accelating Potential

$$\vec{a} = \frac{e\vec{E}}{m}\left[1-(u/c)^2\right]^{3/2}$$



PEP-II accelerator schematic and tunnel view

Hollywood Yarns !