Physics 2D Lecture Slides
Lecture 6 : Jan 11th 2005

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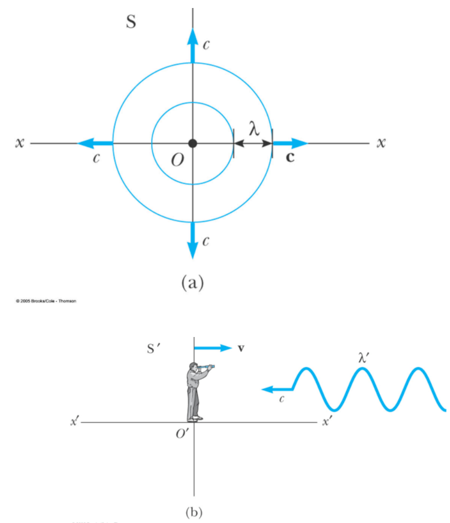
First Quiz This Friday !

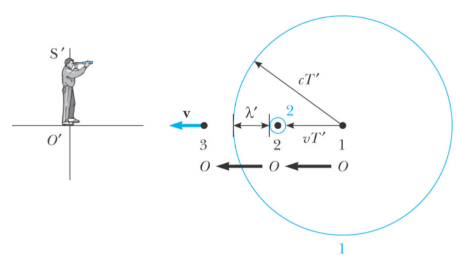
- Bring a Blue Book, calculator; check battery
  - Make sure you remember the code number for this couse given to you (record it some place safe!)
- No “cheat Sheet” please, I will give you equations and constants that I think you need
- When you come for the quiz, pl. occupy seats in the front first.
- Pl. observe one seat distance in the back rows (there is plenty of space)
- Academic Honesty is for you to observe and for me to enforce:
  - Be a good citizen, in this course and forever !

Time Dilation Example: Relativistic Doppler Shift

- Light : velocity $c = f\lambda$, $f=1/T$
- A source of light S at rest
- Observer S'approches S with velocity v
- S' measures f' or λ', $c = f'\lambda'$
- Expect f' > f since more wave crests are being crossed by Observer S' due to its approach direction than if it were at rest w.r.t source S



Relativistic Doppler Shift



Examine two successive wavefronts emitted by S at location 1 and 2

In S' frame, T' = time between two wavefronts

In time T', the Source moves by cT' w.r.t 1

Meanwhile Light Source moves a distance vT'

Distance between successive wavefront

$\lambda' = cT' - vT'$

$\lambda' = cT' - vT'$, now use $f = c/\lambda$

$$\Rightarrow f' = \frac{c}{(c-v)T'} \text{ ; but } T' = \frac{T}{\sqrt{1-(v/c)^2}} = \gamma T$$

substituting for T', use f = 1/T

$$\Rightarrow f' = \frac{\sqrt{1-(v/c)^2}}{1-(v/c)}$$

$$\Rightarrow f' = \frac{\sqrt{1+(v/c)}}{\sqrt{1-(v/c)}} f$$

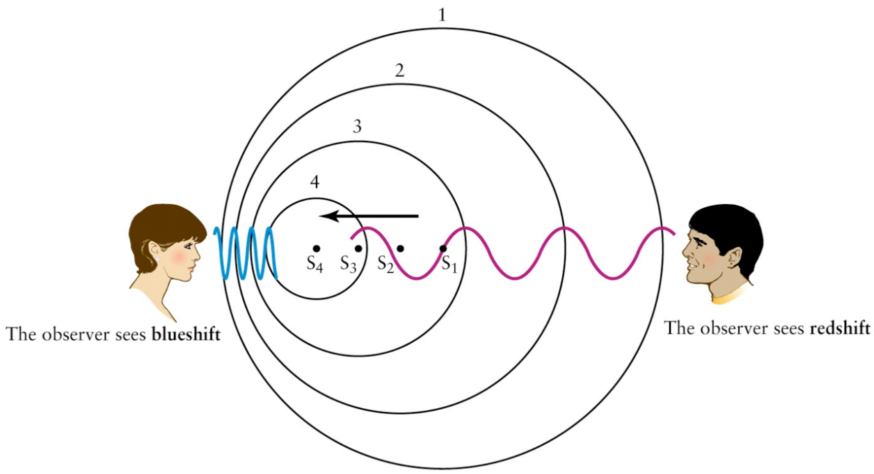
better remembered as:

$$f_{obs} = \frac{\sqrt{1+(v/c)}}{\sqrt{1-(v/c)}} f_{source}$$

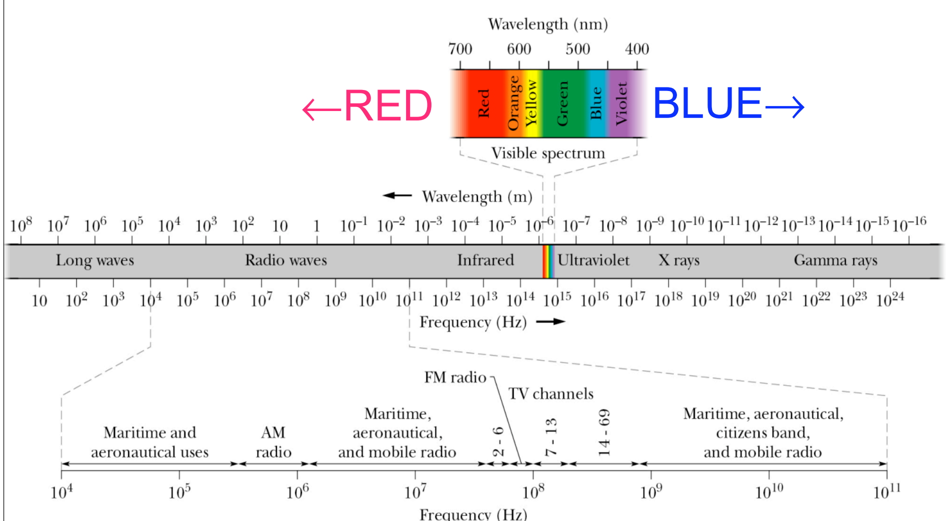
f_{obs} = Frequency measured by observer approching light source

Relativistic Doppler Shift

$$f_{obs} = \frac{\sqrt{1+(v/c)}}{\sqrt{1-(v/c)}} f_{source}$$

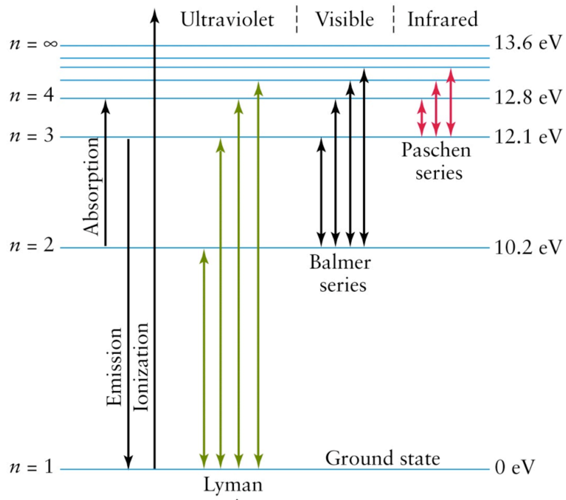


Doppler Shift & Electromagnetic Spectrum



Fingerprint of Elements: Emission & Absorption Spectra

Example : The Atomic Energy levels of Hydrogen



Doppler Shift in Spectral Lines and Motion of Stellar Objects

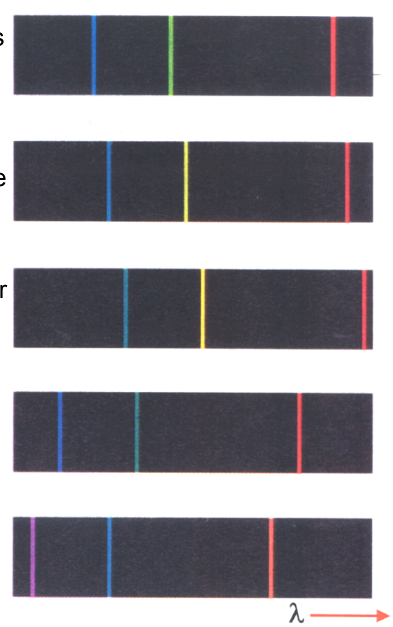
Laboratory Spectrum, lines at rest wavelengths

Lines Redshifted, Object moving away from me

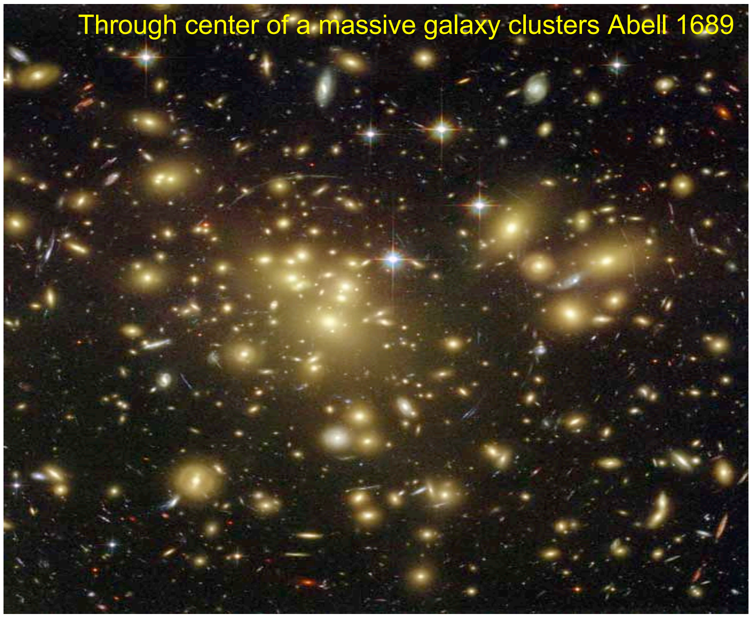
Larger Redshift, object moving away even faster

Lines blueshifted, Object moving towards me

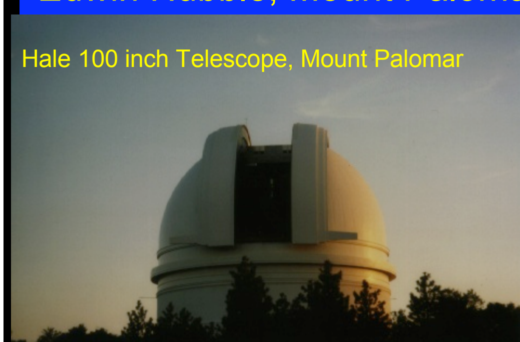
Larger blueshift, object approaching me faster



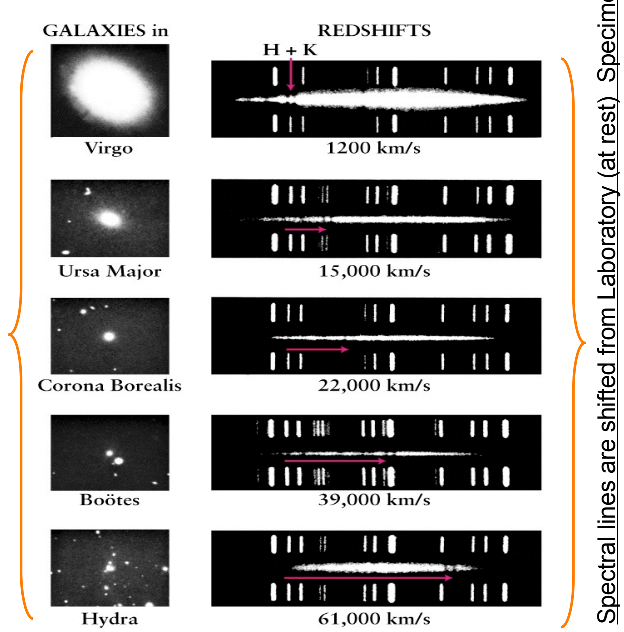
Seeing Distant Galaxies Through Hubble Telescope



Edwin Hubble, Mount Palomar & Expanding Universe



Edwin Hubble 1920

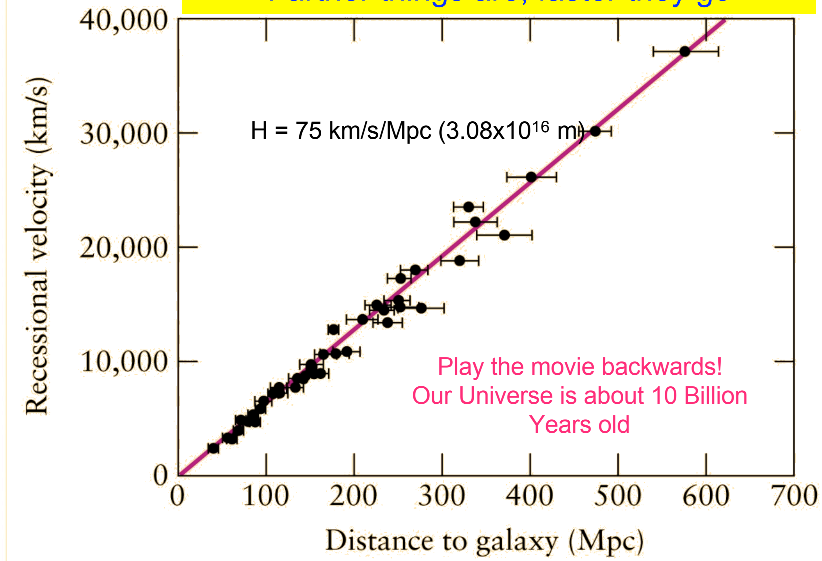


Galaxies at different locations in Universe moving away at different velocities

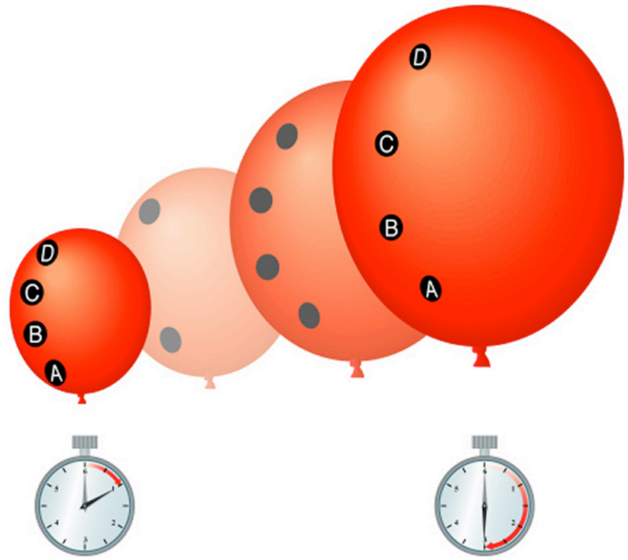
Hubble's Measurement of Recessional Velocity of Galaxies

Reccesional Velocity $V \propto$ distance; V = H d

Farther things are, faster they go



Cosmological Redshift & Discovery of the Expanding Universe:
[Space itself is Expanding]



New Rules of Coordinate Transformation Needed

- The Galilean/Newtonian rules of transformation could not handles frames of refs or objects traveling fast
  - $V \approx C$ (like v = 0.1 c or 0.8c or 1.0c)
- Einstein's postulates led to
  - Destruction of concept of simultaneity ($\Delta t \neq \Delta t'$)
  - Moving clocks run slower
  - Moving rods shrink
- Lets formalize this in terms of <u>general rules</u> of coordinate transformation : Lorentz Transformation
  - Recall the Galilean transformation rules
    - $x' = (x-vt)$
    - $t' = t$
      - These rules that work ok for ferraris now must be modified for rocket ships with $v \approx c$

Discovering The Correct Transformation Rule

$$x' = x - vt \qquad \text{guess} \rightarrow \quad x' = G(x - vt)$$

$$x = x' + vt' \qquad \text{guess} \rightarrow \quad x = G(x' + vt')$$

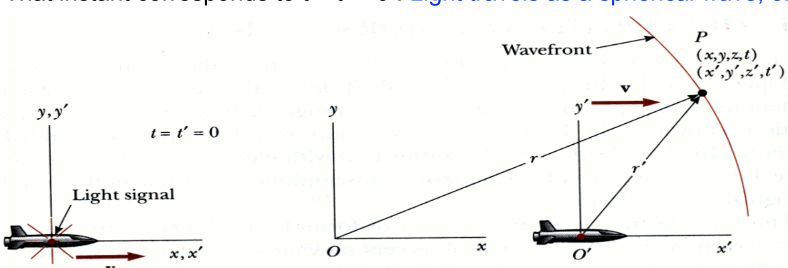
Need to figure out the functional form of G !0

- G must be dimensionless
- G does not depend on x,y,z,t
- But G depends on v/c
- G must be symmetric in velocity v
- As v/c→0 , G →1

Guessing The Lorentz Transformation

Do a Thought Experiment : Watch Rocket Moving along x axis

Rocket in S' (x',y',z',t') frame moving with velocity v w.r.t observer on frame S (x,y,z,t)
Flashbulb mounted on rocket emits pulse of light at the instant origins of S,S' coincide
That instant corresponds to t = t' = 0 . Light travels as a spherical wave, origin is at O,O'



Speed of light is c for both observers: Postulate of SR

Examine a point P (at distance r from O and r' from O') on the Spherical Wavefront

Clearly t and t' must be different
$t \neq t'$

The distance to point P from O : r = ct
The distance to point P from O : r' = ct'

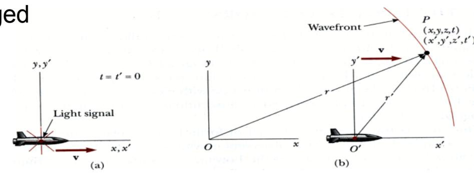
Discovering Lorentz Transfromation for (x,y,z,t)

Motion is along x-x' axis, so y, z unchanged

y'=y,   z' = z

Examine points x or x' where spherical wave

r , x' =r'



$$x = ct = G(x'+vt')$$

$$x' = ct' = G(x - vt),$$

$$\Rightarrow t' = \frac{G}{c}(x - vt)$$

$$\therefore x = ct = G(ct'+vt')$$

$$\therefore ct = G^2\left[(ct - vt) + vt - \frac{v^2}{c}t\right]$$

$$\Rightarrow c^2 = G^2[c^2 - v^2]$$

$$\text{or } G = \frac{1}{\sqrt{1-(v/c)^2}} = \gamma$$

$$\therefore \quad x' = \gamma(x - vt)$$

$$x' = \gamma(x - vt), \; x = \gamma(x'+vt')$$

$$\Rightarrow \quad x = \gamma(\gamma(x - vt) + vt')$$

$$\therefore \; x - \gamma^2 x + \gamma^2 vt = \gamma vt'$$

$$\therefore \; t' = \left[\frac{x}{\gamma v} - \frac{\gamma^2 x}{\gamma v} + \frac{\gamma^2 vt}{\gamma v}\right] = \gamma\left[\frac{x}{\gamma^2 v} - \frac{x}{v} + t\right]$$

$$\therefore \; t' = \gamma\left[t + \frac{x}{v}\left(\frac{1}{\gamma^2} - 1\right)\right], \text{ since } \left(\frac{1}{\gamma^2} - 1\right) = -\left(\frac{v}{c}\right)^2$$

$$\Rightarrow t' = \gamma\left[t + \frac{x}{v}[1 - \left(\frac{v}{c}\right)^2 - 1\right] = \gamma\left[t - \left(\frac{vx}{c^2}\right)\right]$$

Lorentz Transformation Between Ref Frames

Lorentz Transformation	Inverse Lorentz Transformation
$x' = \gamma(x - vt)$	$x = \gamma(x'+vt')$
$y' = y$	$y = y'$
$z' = z$	$z = z'$
$t' = \gamma\left(t - \frac{vx}{c^2}\right)$	$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$

As v→0 , Galilean Transformation is recovered, as per requirement

Notice : SPACE and TIME Coordinates mixed up !!!

Not just Space, Not just Time

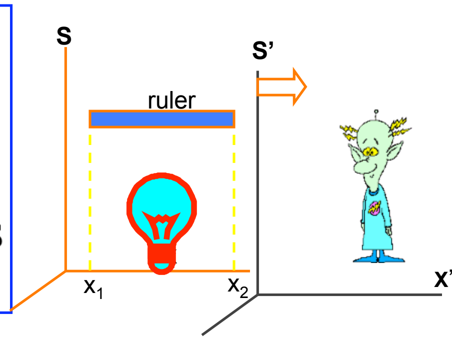
New Word, new concept !

SPACETIME

Lorentz Transform for Pair of Events

$$\left.\begin{aligned}\Delta x' &= \gamma(\Delta x - v\,\Delta t)\\ \Delta t' &= \gamma\left(\Delta t - \frac{v}{c^2}\,\Delta x\right)\end{aligned}\right\}\ \mathrm{S} \rightarrow \mathrm{S}'$$

$$\left.\begin{aligned}\Delta x &= \gamma(\Delta x' + v\,\Delta t')\\ \Delta t &= \gamma\left(\Delta t' + \frac{v}{c^2}\,\Delta x'\right)\end{aligned}\right\}\ \mathrm{S}' \rightarrow \mathrm{S}$$



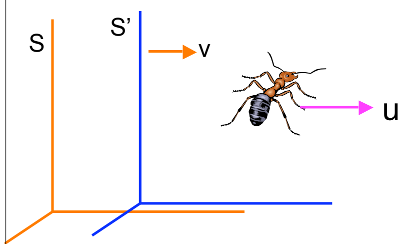
Can understand Simultaneity, Length contraction & Time dilation formulae from this

Time dilation: Bulb in S frame turned on at t_1 & off at t_2 : What $\Delta t'$ did S' measure ?
two events occur at same place in S frame => $\Delta x = 0$
$\Delta t' = \gamma\,\Delta t$ (in this example Δt = **proper time**)

Length Contraction: Ruler measured in S between x_1 & x_2 : What $\Delta x'$ did S' measure ?
two ends measured at same time in S' frame => $\Delta t' = 0$
$\Delta x = \gamma\,(\Delta x' + 0)$ => $\Delta x' = \Delta x / \gamma$ (*in this example* Δx = **proper length**)

Lorentz Velocity Transformation Rule

S and S' are measuring ant's speed u along x, y, z axes



In S' frame, $u_{x'} = \dfrac{x_2' - x_1'}{t_2' - t_1'} = \dfrac{dx'}{dt'}$

$dx' = \gamma(dx - vdt)$, $dt' = \gamma(dt - \dfrac{v}{c^2}dx)$

$u_{x'} = \dfrac{dx - vdt}{dt - \dfrac{v}{c^2}dx}$, divide by dt'

$$u_{x'} = \frac{u_x - v}{1 - \dfrac{vu_x}{c^2}}$$

For v << c, $u_{x'} = u_x - v$

(Galilean Trans. Restored)

Velocity Transformation Perpendicular to S-S' motion

$dy' = dy$, $dt' = \gamma(dt - \dfrac{v}{c^2}dx)$

$$u_y' = \frac{dy'}{dy'} = \frac{dy}{\gamma(dt - \dfrac{v}{c^2}dx)}$$

divide by dt on RHS

$$u_y' = \frac{u_y}{\gamma(1 - \dfrac{v}{c^2}u_x)}$$

There is a change in velocity in the direction ⊥ to S-S' motion !

Similarly Z component of Ant's velocity transforms as

$$u_z' = \frac{u_z}{\gamma(1 - \dfrac{v}{c^2}u_x)}$$

Inverse Lorentz Velocity Transformation

Inverse Velocity Transform:

$$u_x = \frac{u_{x'} + v}{1 + \frac{v u_{x'}}{c^2}}$$

$$u_y = \frac{u_y^{'}}{\gamma(1 + \frac{v}{c^2} u_x^{'})}$$

$$u_z = \frac{u_z^{'}}{\gamma(1 + \frac{v}{c^2} u_x^{'})}$$

As usual, replace

$v \Rightarrow -v$