Physics 2D Lecture Slides
Lecture 18: Feb 9th 2005

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Wave Packets & Uncertainty Principles of Subatomic Physics

in space x: $\boxed{\Delta k . \Delta x = \pi}$ $\Rightarrow$ since $k = \frac{2\pi}{\lambda}$, $p = \frac{h}{\lambda}$

$\Rightarrow$ $\boxed{\Delta p . \Delta x = h/2}$

usually one writes $\boxed{\Delta p . \Delta x \geq \hbar/2}$ approximate relation

In time t : $\boxed{\Delta w . \Delta t = \pi}$ $\Rightarrow$ since $\omega = 2\pi f, E = hf$

$\Rightarrow$ $\boxed{\Delta E . \Delta t = h/2}$

usually one writes $\boxed{\Delta E . \Delta t \geq \hbar/2}$ approximate relation

What do these inequalities mean physically?

Signal Transmission and Bandwidth Theory

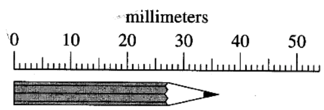
- Short duration pulses are used to transmit digital info
  - Over phone line as brief tone pulses
  - Over satellite link as brief radio pulses
  - Over optical fiber as brief laser light pulses
- Ragardless of type of wave or medium, any wave pulse must obey the fundamental relation
  - » $\Delta\omega\Delta t \cong \pi$
- Range of frequencies that can be transmitted are called bandwidth of the medium
- Shortest possible pulse that can be transmitted thru a medium is $\Delta t_{min} \cong \pi/\Delta\omega$
- Higher bandwidths transmits shorter pulses & allows high data rate

Crucial Concept: Measurement Error

…and ….How well can you know it ?

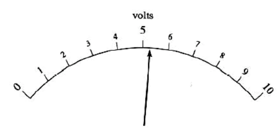
Know the Error of Thy Ways: Measurement Error → Δ

- Measurements are made by observing something : length, time, momentum, energy
- All measurements have some (limited) precision`…no matter the instrument used
- Examples:
  - How long is a desk ? L = (5 ± 0.1) m = L ± ΔL (depends on ruler used)
  - How long was this lecture ? T = (50 ± 1)minutes = T ± ΔT (depends on the accuracy of your watch)
  - How much does Prof. Sharma weigh ? M = (1000 ± 700) kg = m ± Δm
    - Is this a correct measure of my weight ?
      - Correct (because of large error reported) but imprecise
      - My correct weight is covered by the (large) error in observation



Best Estimate Length: 36 mm
Probable Range: 35.5 to 36.5 mm

Length Measure

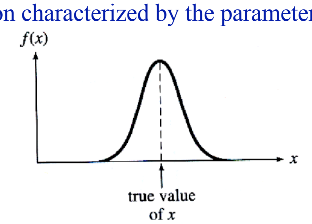


Best Estimate of Voltage: 5.3 V
Estimated Range: 5.2 to 5.4 mm

Voltage (or time) Measure

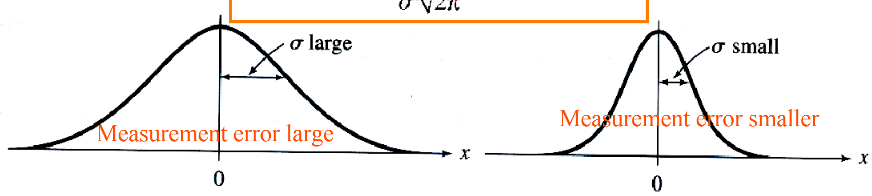
Measurement Error : x ± Δx

- Measurement errors are unavoidable since the measurement procedure is an experimental one
- True value of an measurable quantity is an abstract concept
- In a set of repeated measurements with random errors, the distribution of measurements resembles a Gaussian distribution characterized by the parameter σ or Δ characterizing the width of the distribution



The Gauss, or Normal, Distribution

$$G_{X,\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{(x-X)^2/2\sigma^2}.$$



Interpreting Measurements with random Error : Δ

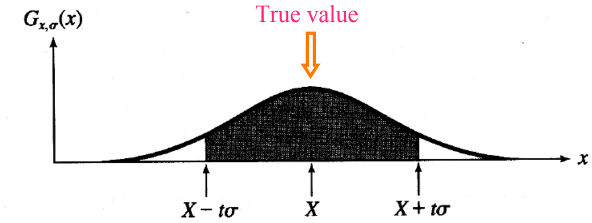
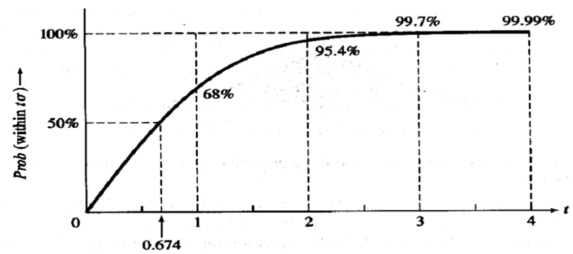


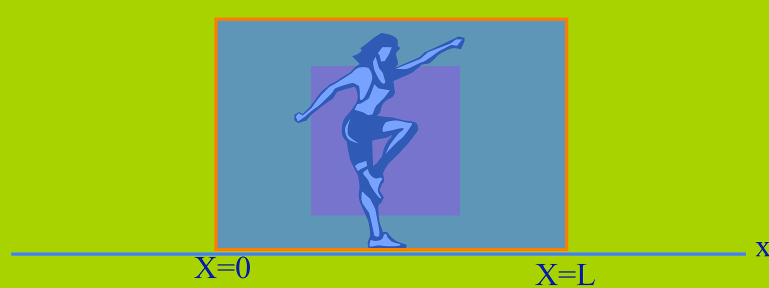
Figure 5.12. The shaded area between $X \pm t\sigma$ is the probability of a measurement within t standard deviations of X.



t	0	0.25	0.5	0.75	1.0	1.25	1.5	1.75	2.0	2.5	3.0	3.5	4.0
Prob (%)	0	20	38	55	68	79	87	92	95.4	98.8	99.7	99.95	99.99

Where in the World is Carmen San Diego?

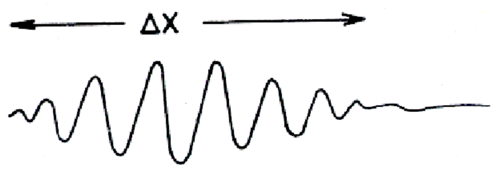
- Carmen San Diego hidden inside a big box of length L
- Suppose you can't see thru the (blue) box, what is you best estimate of her location inside box (she could be anywhere inside the box)



Your best unbiased measure would be $x = L/2 \pm L/2$

There is no perfect measurement, there are always measurement error

Wave Packets & Matter Waves



What is the Wave Length of this wave packet?

$\lambda - \Delta\lambda < \lambda < \lambda + \Delta\lambda$

De Broglie wavelength $\lambda = h/p$

→ Momentum Uncertainty: $p - \Delta p < p < p + \Delta p$

Similarly for frequency ω or f

$\omega - \Delta\omega < \omega < \omega + \Delta\omega$

Planck's condition $E = hf = h\omega/2$

→ $E - \Delta E < E < E + \Delta E$

Back to Heisenberg's Uncertainty Principle & Δ

- $\Delta x . \Delta p \geq h/4\pi \Rightarrow$
  - If the measurement of the position of a particle is made with a precision Δx and a SIMULTANEOUS measurement of its momentum p_x in the X direction , then the product of the two uncertainties (measurement errors) can never be smaller than $\cong h/4\pi$ irrespective of how precise the measurement tools

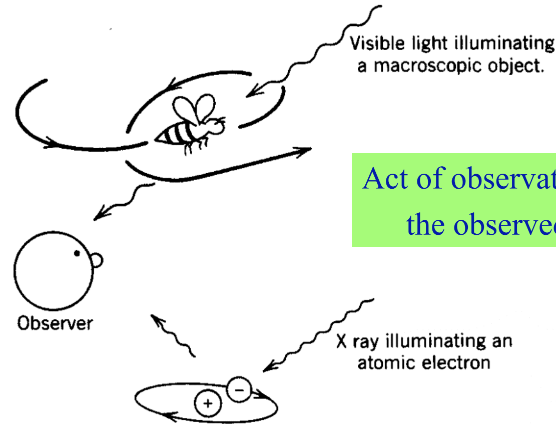
- $\Delta E . \Delta t \geq h/4\pi \Rightarrow$
  - If the measurement of the energy E of a particle is made with a precision ΔE and it took time Δt to make that measurement, then the product of the two uncertainties (measurement errors) can never be smaller than $\cong h/4\pi$ irrespective of how precise the measurement tools

These rules arise from the way we constructed the Wave packets describing Matter "pilot" waves

Perhaps these rules Are bogus, can we verify this with some physical picture ??

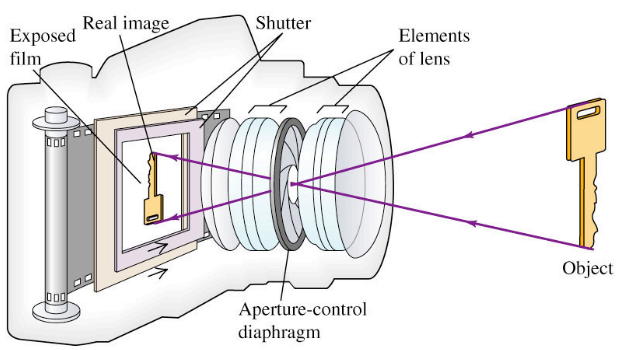
The Act of Observation (Compton Scattering)

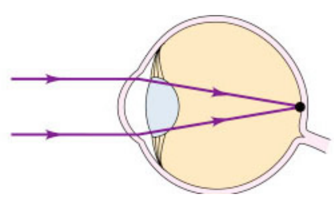
Observations of particle motion by means of scattered illumination. When the incident wavelength is reduced to accommodate the size of the particle, the momentum transferred by the photon becomes large enough to disturb the observed motion.



Act of observation disturbs the observed system

The Act of Observation : Your Eye is a Camera

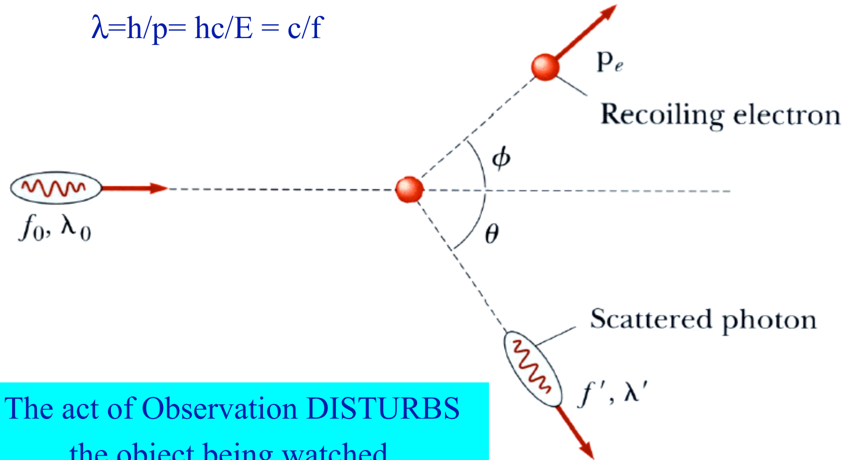




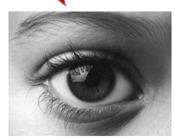
your eye is a camera
pupil is the aperture
retina is the “film”

Compton Scattering: Shining light to observe electron

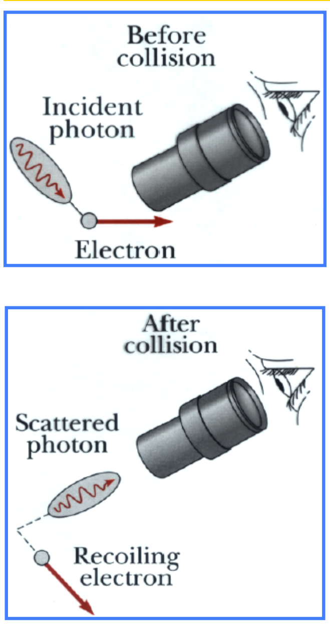
$\lambda = h/p = hc/E = c/f$

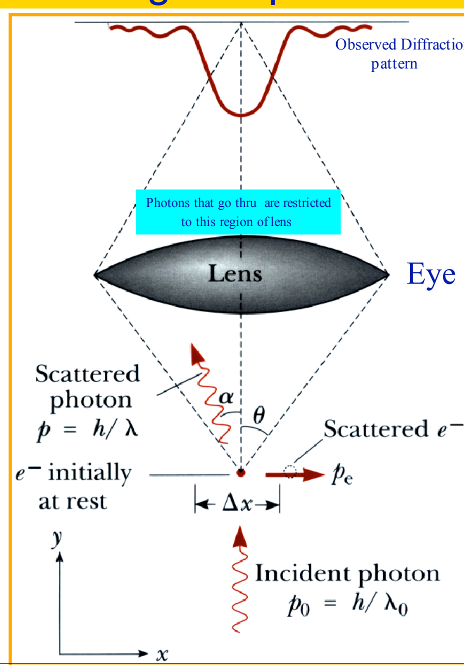


The act of Observation DISTURBS the object being watched, here the electron moves away from where it was originally



Act of Watching: A Thought Experiment





Diffraction By a Circular Aperture (Lens)

See Resnick, Halliday Walker 6th Ed , Ch 37, pages 898-900

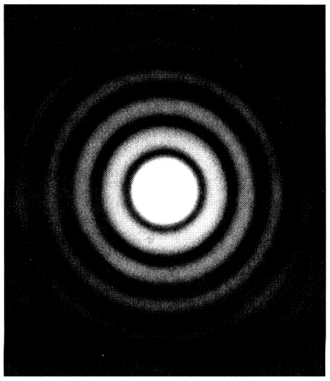


Fig. 37-9 The diffraction pattern of a circular aperture. Note the central maximum and the circular secondary maxima. The figure has been overexposed to bring out these secondary maxima, which are much less intense than the central maximum.

Diffracted image of a point source of light thru a lens (circular aperture of size d)

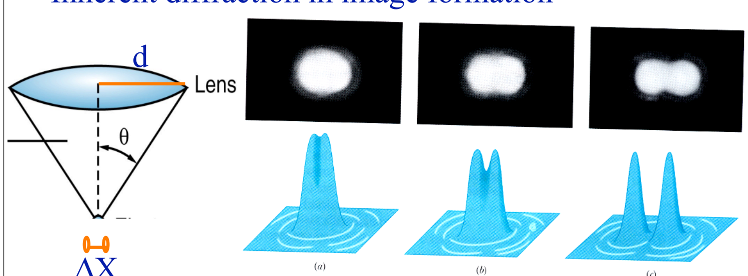
First minimum of diffraction pattern is located by

$$\sin\theta = 1.22\frac{\lambda}{d}$$

See previous picture for definitions of ϑ, λ, d

Resolving Power of Light Thru a Lens

Image of 2 separate point sources formed by a converging lens of diameter d, ability to resolve them depends on λ & d because of the Inherent diffraction in image formation

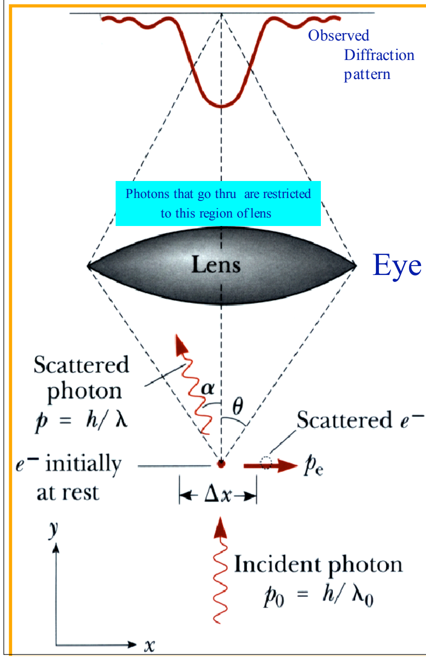


Not resolved barely resolved resolved

Resolving power $\Delta x \simeq \frac{\lambda}{2\sin\theta}$

ϑ Depends on d

Putting it all together: act of Observing an electron



- Incident light (p,λ) scatters off electron
- To be collected by lens → γ must scatter thru angle α
  - $-\vartheta \le \alpha \le \vartheta$
- Due to Compton scatter, electron picks up momentum
  - P_X , P_Y $-\frac{h}{\lambda}\sin\theta \le P_x \le \frac{h}{\lambda}\sin\theta$

    electron momentum uncertainty is

    $\Delta p \cong \frac{\sim 2h}{\lambda}\sin\theta$

- After passing thru lens, photon diffracts, lands somewhere on screen, image (of electron) is fuzzy
- How fuzzy ? Optics says shortest distance between two resolvable points is : $\Delta x = \frac{\lambda}{2\sin\theta}$
- Larger the lens radius, larger the $\vartheta \Rightarrow$ better resolution

$$\Rightarrow \Delta p.\Delta x \simeq \left(\frac{2h\sin\theta}{\lambda}\right)\left(\frac{\lambda}{2\sin\theta}\right) = h$$

$$\Rightarrow \Delta p.\Delta x \ge \hbar / 2$$

Pseudo-Philosophical Aftermath of Uncertainty Principle

- Newtonian Physics & Deterministic physics topples over
  - Newton's laws told you all you needed to know about trajectory of a particle
    - Apply a force, watch the particle go !
      - Know every thing ! X, v, p , F, a
      - Can predict exact trajectory of particle if you had perfect device

- No so in the subatomic world !
  - Of small momenta, forces, energies
  - Cant predict anything exactly
    - Can only predict probabilities
      - There is so much chance that the particle landed here or there
      - Cant be sure !....cognizant of the errors of thy observations

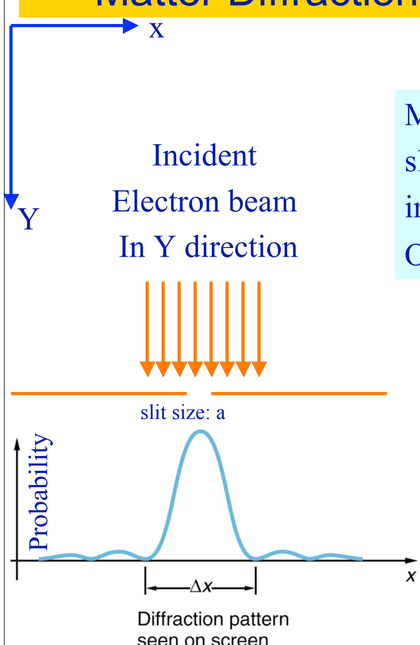
Philosophers went nuts !...what has happened to nature

Philosophers just talk, don't do real life experiments!

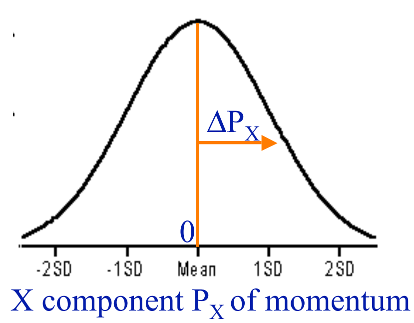
All Measurements Have Associated Errors

- If your measuring apparatus has an intrinsic inaccuracy (error) of amount Δp
- Then results of measurement of momentum p of an object at rest can easily yield a range of values accommodated by the measurement imprecision :
  - $-\Delta p \leq p \leq \Delta p$
- Similarly for all measurable quantities like x, t, Energy !

Matter Diffraction & Uncertainty Principle



Momentum measurement beyond slit show particle not moving exactly in Y direction, develops a X component Of motion $\Delta P_X = h/(2\pi\ a)$



Heisenberg's Uncertainty Principles

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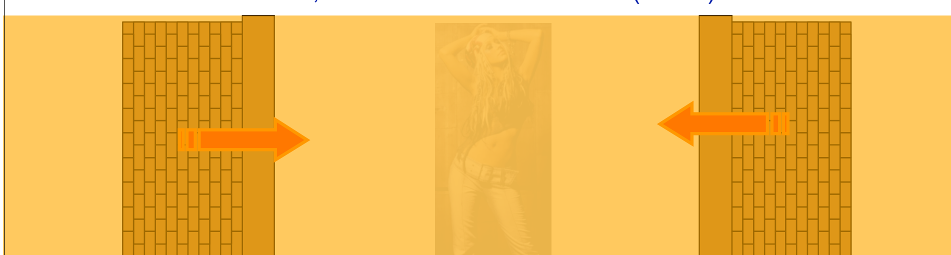
What do these simple equations mean ?

The Quantum Mechanics of Christina Aguilera!

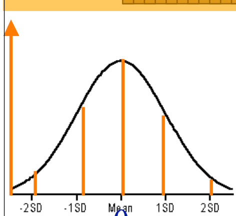
Christina at rest between two walls originally at infinity: Uncertainty in her location $\Delta X = \infty$.
At rest means her momentum P=0 , $\Delta P=0$ (Uncertainty principle)

Slowly two walls move in from infinity on each side, now $\Delta X = L$, so $\Delta p \neq 0$
She is not at rest now, in fact her momentum $P \approx \pm (h/2\pi L)$



L

X Axis



Christina's Momentum p

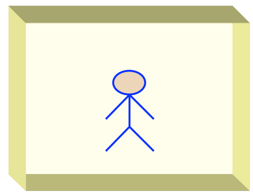
On average, measure <p> = 0
but there are quite large fluctuations!
Width of Distribution $= \Delta P$

$\Delta P = \sqrt{(P^2)_{ave} - (P_{ave})^2}; \quad \Delta P \sim \frac{\hbar}{L}$

Bottomline : Christina dances to the tune of Uncertainty Principle!

Implications of Uncertainty Principles

A bound "particle" is one that is confined in some finite region of space.



One of the cornerstones of Quantum mechanics is that bound particles can not be stationary – <u>even at Zero absolute temperature !</u>

There is a non-zero limit on the kinetic energy of a bound particle

Matter-Antimatter Collisions and Uncertainty Principle

Look at Rules of Energy and Momentum Conservation : Are they ?

$E_{before} = mc^2 + mc^2$ and $E_{after} = 2mc^2$

$P_{before} = 0$ but since photon produced in the annihilation → $P_{after} = 2mc$!

Such violation are allowed but must be consumed instantaneously !
Hence the name "virtual" particles

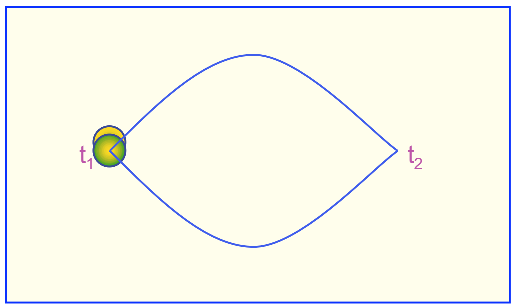
Fluctuations In The Vacuum : Breaking Energy Conservation Rules

Vaccum, at any energy, is bubbling with particle creation and annihilation

$\Delta E \cdot \Delta t \approx h/2\pi$ implies that you can (in principle) pull out an **elephant + anti-elephant** from NOTHING (Vaccum) but for a very very short time Δt !!

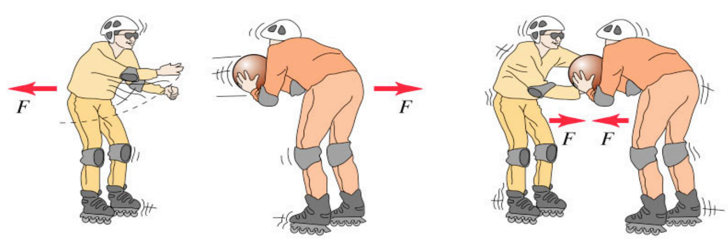
$$\text{How Much Time : } \Delta t = \frac{\hbar}{2Mc^2}$$

How cool is that !



How far can the virtual particles propagate ? Depends on their mass

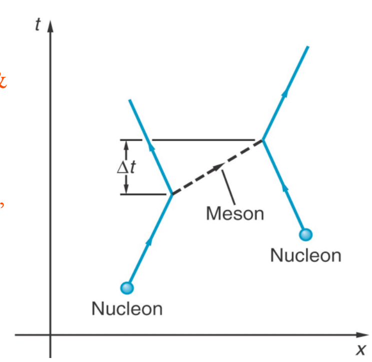
Strong Force Within Nucleus ➔ Exchange Force and Virtual Particles



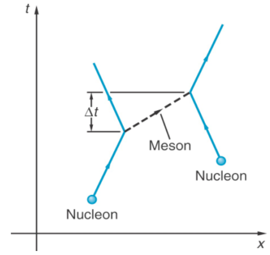
repulsive force: skaters exchange ball

attractive: grab ball from each other's hand

- Strong Nuclear force can be modeled as exchange of virtual particles called $\pi^{\pm}$ mesons by nucleons (protons & neutrons)
- $\pi^{\pm}$ mesons are emitted by proton and reabsorbed by a neutron
- The short range of the Nuclear force is due to the "large" mass of the exchanged meson
- $M_{\pi} = 140\ \text{MeV}/c^2$



Range of Nuclear Exchange Force



How long can the emitted virtual particle last?

$\Delta E \times \Delta t \geq \hbar$

The virtual particle has rest mass + kinetic energy

$\Rightarrow$ Its energy $\boxed{\Delta E \geq Mc^2}$

$\Rightarrow$ Particle can not live for more than $\boxed{\Delta t \leq \hbar / Mc^2}$

Range R of the meson (and thus the exchange force)

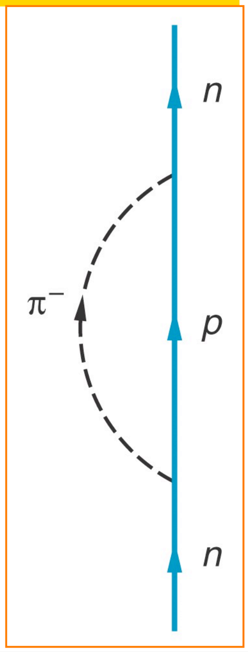
$R = c\Delta t = c\hbar / Mc^2 = \hbar / Mc$

For M=140 MeV/c^2 $\Rightarrow R \simeq \dfrac{1.06\times10^{-34}\,J.s}{(140\,MeV/c^2)\times c^2 \times (1.60\times10^{-13}\,J/MeV)}$

$\boxed{R \simeq 1.4\times10^{-15}\,m = 1.4\,fm}$

Subatomic Cinderella Act !

- Neutron emits a charged pion for a time Δt and becomes a (charged) proton
- After time Δt , the proton reabsorbs charged pion particle (π^-) to become neutron again
- But in the time Δt that the positive proton and π^- particle exist, they can interact with other charged particles
- After time Δt strikes , the Cinderella act is over !



This heralds the death of common sense in subatomic world