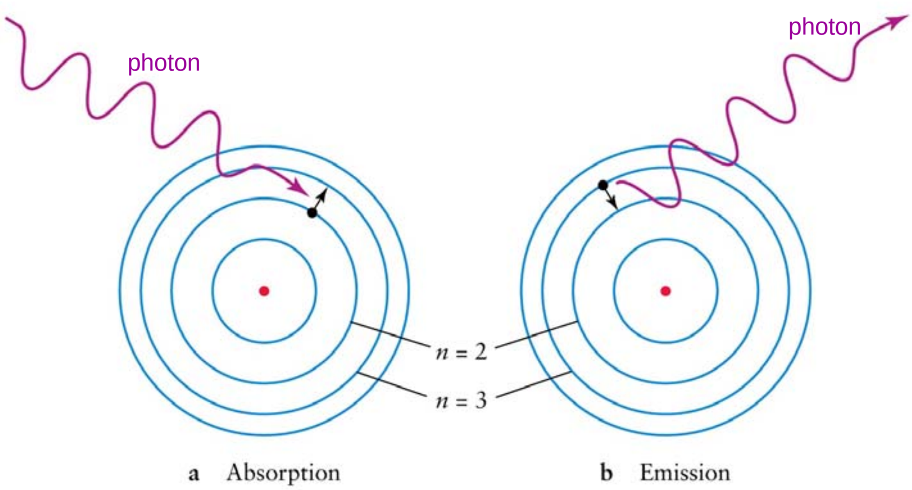
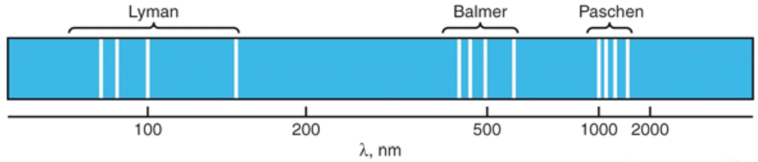
Physics 2D Lecture Slides
Lecture 16: Feb 7th 2005

Vivek Sharma
UCSD Physics

Bohr's Atom: Emission & Absorption Spectra

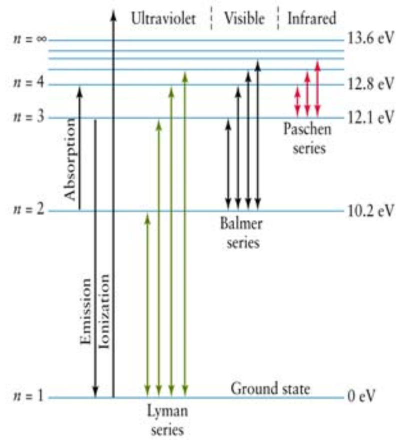


Another Look at the Energy levels



$$E_n = -\left(\frac{ke^2}{2a_0}\right)\frac{Z^2}{n^2}$$

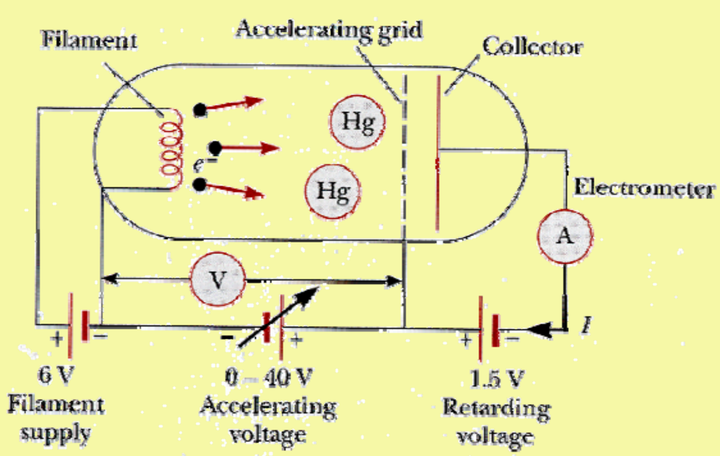
Rydberg Constant



Atomic Excitation by Electrons: Franck-Hertz Expt

Other ways of Energy exchange are also quantized ! Example:

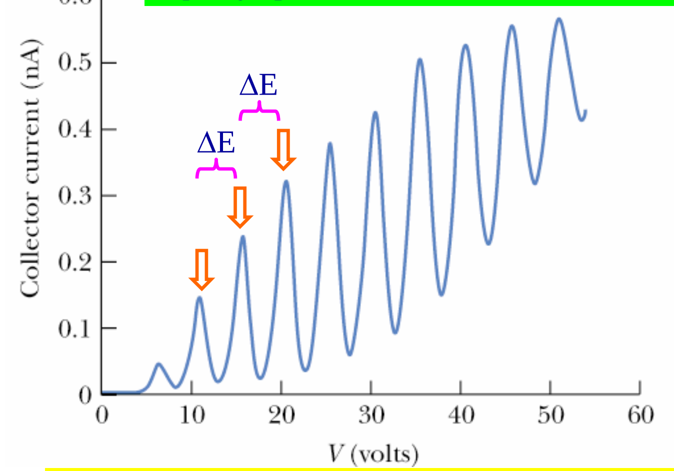
- Transfer energy to atom by colliding electrons on it
- Accelerate electrons, collide with Hg atoms, measure energy transfer in inelastic collision (retarding voltage)



Atomic Excitation by Electrons: Franck-Hertz Expt

Plot # of electrons/time (current) overcoming the retarding potential (V)

Equally spaced Maxima and minima in I-V curve



Atoms accept only discrete amount of Energy,
no matter the fashion in which energy is transffered

Bohr's Explanation of Hydrogen like atoms

- Bohr's Semiclassical theory explained some spectroscopic data → Nobel Prize : 1922
- The "hotch-potch" of clasical & quantum attributes left many (Einstein) unconvinced
  - "appeared to me to be a miracle – and appears to me to be a miracle today ...... One ought to be ashamed of the successes of the theory"
- Problems with Bohr's theory:
  - Failed to predict INTENSITY of spectral lines
  - Limited success in predicting spectra of Multi-electron atoms (He)
  - Failed to provide "time evolution " of system from some initial state
  - Overemphasized Particle nature of matter-could not explain the wave-particle duality of light
  - No general scheme applicable to non-periodic motion in subatomic systems
- "Condemned" as a one trick pony ! Without fundamental insight …raised the question : Why was Bohr successful?

Prince Louise de Broglie & Matter Waves

- Key to Bohr atom was Angular momentum quantization
- Why this Quantization: $mvr = |L| = nh/2\pi$?
- Invoking symmetry in nature, Louise de Broglie (Da Prince of France !) conjectured:

Because photons have wave and particle like nature → particles may have wave like properties !!

Electrons have accompanying "pilot" wave (not EM) which guide particles thru spacetime

A PhD Thesis Fit For a Prince

- Matter Wave !
  - **"Pilot wave" of $\lambda = h/p = h / (\gamma mv)$**
  - **frequency $f = E/h$**
- **Consequence:**
  - **If matter has wave like properties then there would be interference (destructive & constructive)**
    - **Use analogy of standing waves on a plucked string to explain the quantization condition of Bohr orbits**

Matter Waves : How big, how small

1.Wavelength of baseball, m=140g, v=27m/s

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63\times10^{-34}\,J.s}{(.14kg)(27m/s)} = 1.75\times10^{-34}\,m$$

$\Rightarrow$ $\lambda_{baseball}$ <<< size of nucleus

$\Rightarrow$ Baseball "looks" like a particle

2. Wavelength of electron K=120eV (assume NR)

$$K = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mK}$$

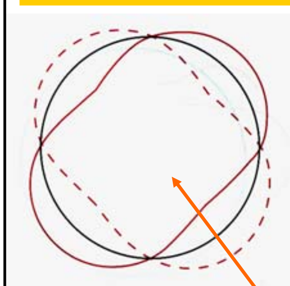
$$= \sqrt{2(9.11\times10^{-31})(120eV)(1.6\times10^{-19})}$$

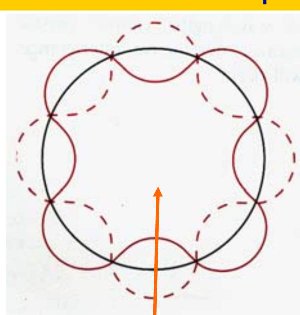
$$= 5.91\times 10^{-24}\,Kg.m/s$$

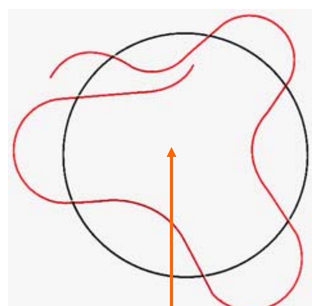
$$\lambda_e = \frac{h}{p} = \frac{6.63\times10^{-34}\,J.s}{5.91\times10^{-24}\,kg.m/s} = 1.12\times10^{-10}\,m$$

$\Rightarrow$ $\lambda_e \simeq$ Size of atom !!

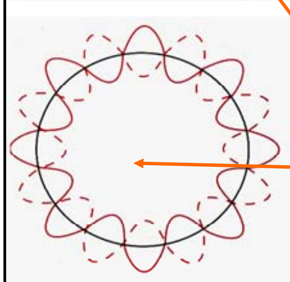
Models of Vibrations on a Loop: Model of e in atom


Circumference = 2 wavelengths


Circumference = 4 wavelengths



Fractional # of waves in a loop can not persist due to destructive interference

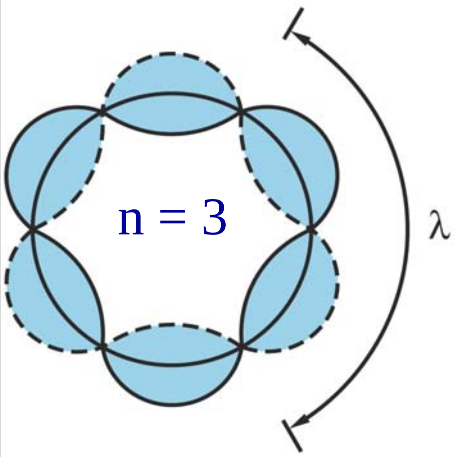

Circumference = 8 wavelengths

Modes of vibration when a integral # of λ fit into loop

(Standing waves)

vibrations continue Indefinitely

De Broglie's Explanation of Bohr's Quantization



Standing waves in H atom:

Constructive interference when

$n\lambda = 2\pi r$

since $\lambda = \frac{h}{p} = \frac{h}{mv}$ ......(NR)

$$\Rightarrow \frac{nh}{mv} = 2\pi r$$

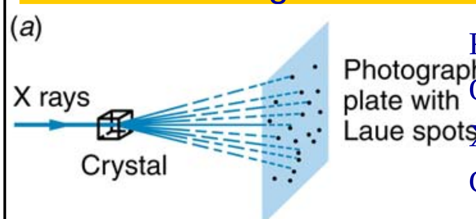
$$\Rightarrow \boxed{n\hbar = mvr}$$

Angular momentum

Quantization condition!

This is too intense ! Must verify such "loony tunes" with experiment

Reminder: Light as a Wave : Bragg Scattering Expt

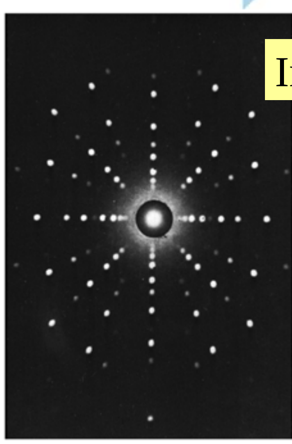


Range of X-ray wavelengths scatter

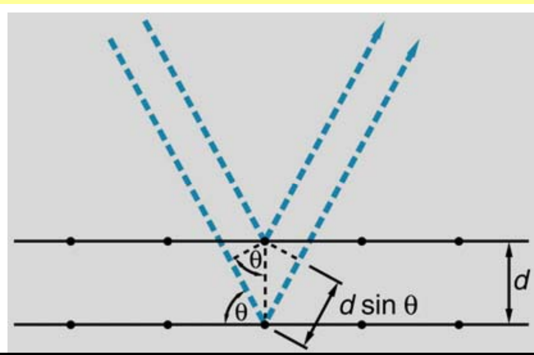
Off a crystal sample

X-rays constructively interfere from

Certain planes producing bright spots



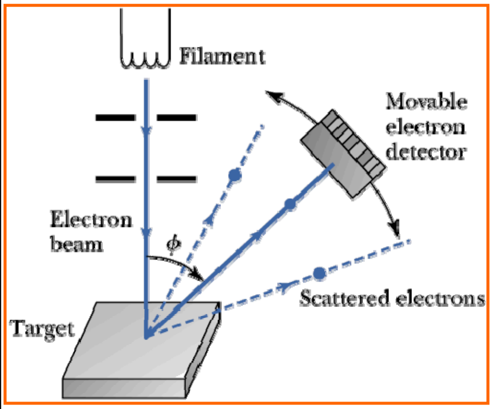
Interference → Path diff=$2d\sin\vartheta = n\lambda$



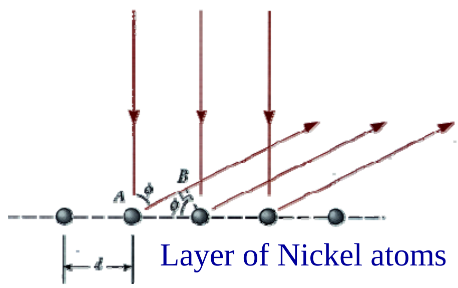
Verification of Matter Waves: Davisson & Germer Expt

If electrons have associated wave like properties → expect interference pattern when incident on a layer of atoms (reflection diffraction grating) with inter-atomic separation d such that

path diff AB= $d\sin\vartheta = n\lambda$

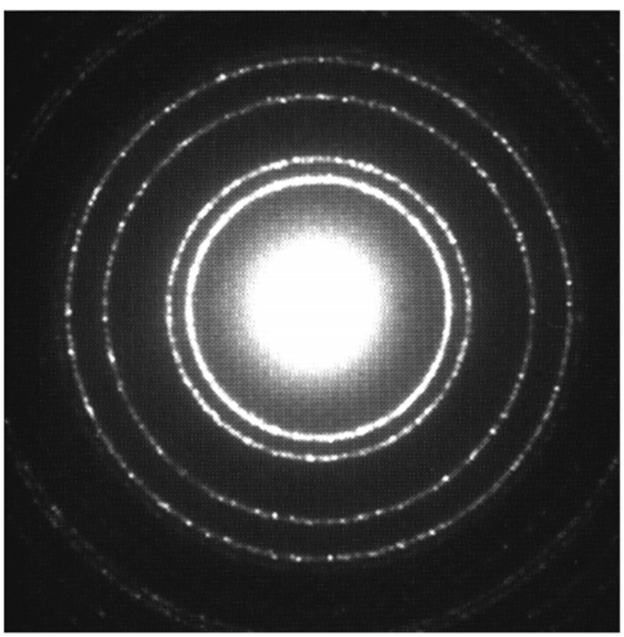


Atomic lattice as diffraction grating



Layer of Nickel atoms

Electrons Diffract in Crystal, just like X-Rays !!

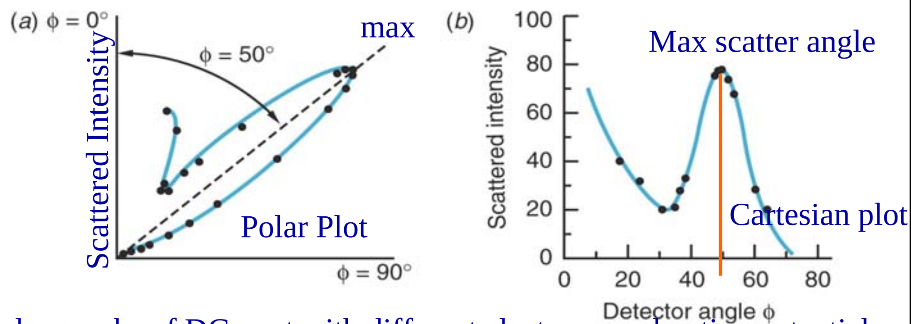


Diffraction pattern produced by 600eV electrons incident on a Al foil target

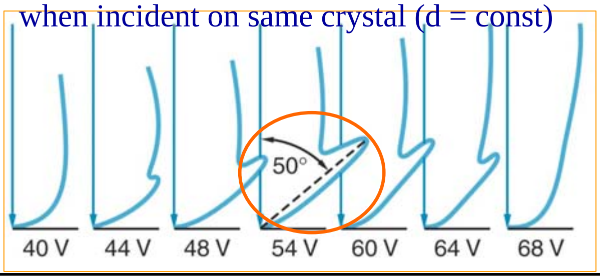
Notice the waxing and waning of scattered electron Intensity.

What to expect if electron had no wave like attribute

Davisson-Germer Experiment: 54 eV electron Beam



Polar graphs of DG expt with different electron accelerating potential when incident on same crystal (d = const)



Peak at $\Phi=50^o$ when $V_{acc} = 54$ V

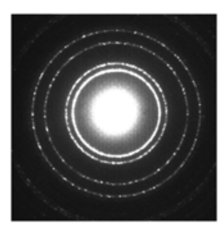
Analyzing Davisson-Germer Expt with de Broglie idea

de Broglie λ for electron accelerated thru V_{acc} =54V

- $\frac{1}{2}mv^2 = K = \frac{p^2}{2m} = eV \Rightarrow v = \sqrt{\frac{2eV}{m}}$; $p = mv = m\sqrt{\frac{2eV}{m}}$

If you believe de Broglie

$$\lambda = \frac{\mathrm{h}}{\mathrm{p}} = \frac{h}{mv} = \frac{h}{m\sqrt{\frac{2eV}{m}}} = \boxed{\frac{h}{\sqrt{2meV}} = \lambda^{predict}}$$



For V_{acc} = 54 Volts $\Rightarrow \lambda = 1.67\times10^{-10} m$ (de Broglie)

Exptal data from Davisson-Germer Observation:

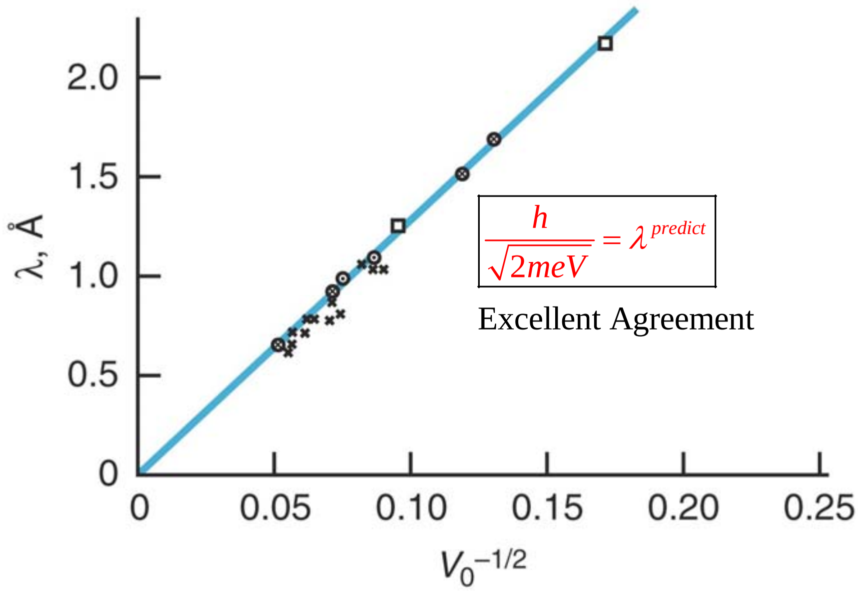
d_{nickel} =2.15 Å =$2.15\times10^{-10}m$ (from Bragg Scattering)

$\theta_{diff}^{max} = 50^o$ (observation from scattering intensity plot)

Diffraction Rule : d sinϕ = nλ

$\Rightarrow$ For Principal Maxima (n=1); λ^{meas} =(2.15 Å)(sin 50°) $\begin{array}{l}\lambda^{predict} = 1.67 \dot{A} \\ \lambda^{observ} = 1.65 \dot{A}\end{array}$ *Excellent* agreement

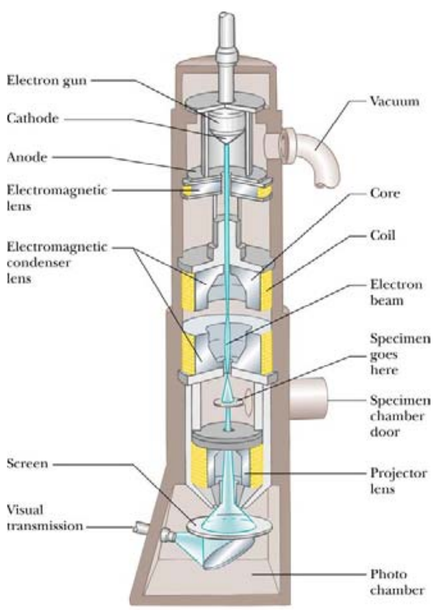
Davisson Germer Experiment: Matter Waves !

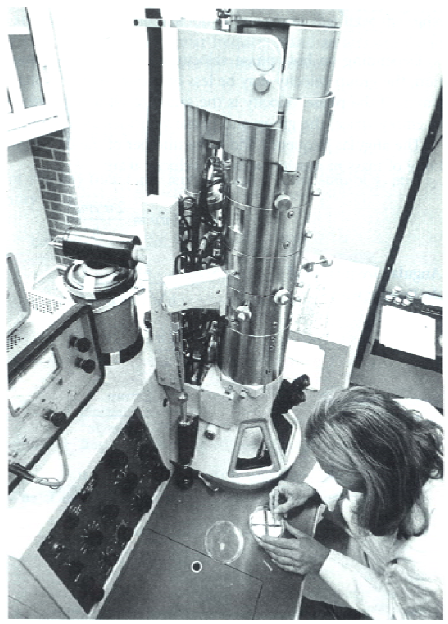


$$\frac{h}{\sqrt{2meV}} = \lambda^{predict}$$

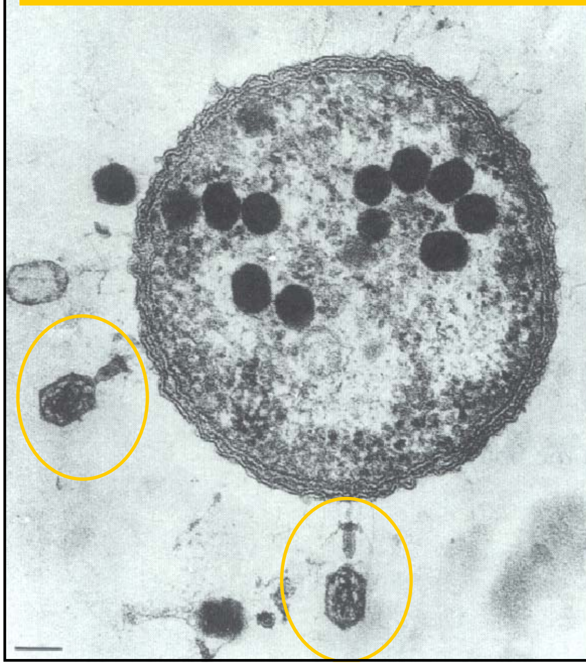
Excellent Agreement

Practical Application → Electron Microscope !





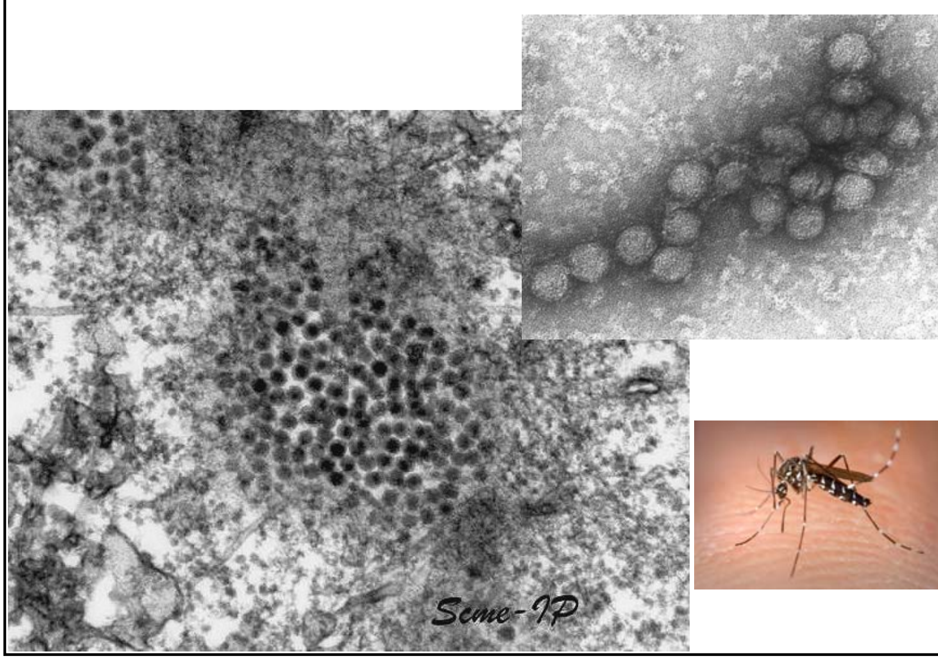
Electron Microscope : Excellent Resolving Power



Electron Micrograph Showing Bacteriophage Viruses in E. Coli bacterium

The bacterium is ≅ 1μ size

West Nile Virus extracted from a crow brain



So Just What is Waving in Matter Waves ?

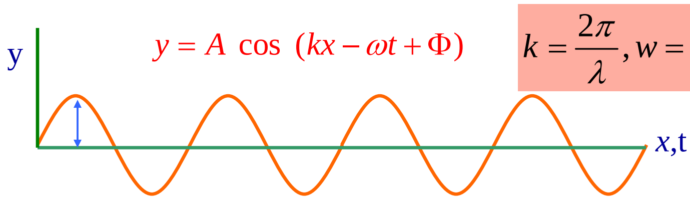
- For waves in an ocean, it's the water that "waves"
- For sound waves, it's the molecules in medium
- For light it's the **E** & **B** vectors
- What's waving for matter waves ?
  - **It's the PROBABLILITY OF FINDING THE PARTICLE that waves !**
  - **Particle can be represented by a wave packet in**
    - **Space**
    - **Time**
    - **Made by superposition of many sinusoidal waves of different λ**
    - **It's a "pulse" of probability**

Imagine Wave pulse moving along a string: its localized in time and space (unlike a pure harmonic wave)

What Wave Does Not Describe a Particle

$y = A\ \cos\ (kx - \omega t + \Phi)$

$k = \frac{2\pi}{\lambda}, w = 2\pi f$



- What wave form can be associated with particle's pilot wave?
- A traveling sinusoidal wave? $y = A\ \cos\ (kx - \omega t + \Phi)$
- Since de Broglie "pilot wave" represents particle, it must travel with same speed as particle ……(like me and my shadow)

Phase velocity (v_p) of sinusoidal wave: $v_p = \lambda f$

In Matter:

(a) $\lambda = \frac{h}{p} = \frac{h}{\gamma m v}$

(b) $f = \frac{E}{h} = \frac{\gamma m c^2}{h}$

$\Rightarrow v_p = \lambda f = \frac{E}{p} = \frac{\gamma m c^2}{\gamma m v} = \frac{c^2}{v} > c!$

Conflicts with Relativity → Unphysical

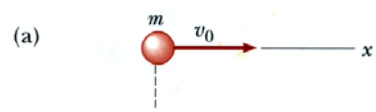
Single sinusoidal wave of infinite extent does not represent particle localized in space

Need "wave packets" localized Spatially (x) and Temporally (t)

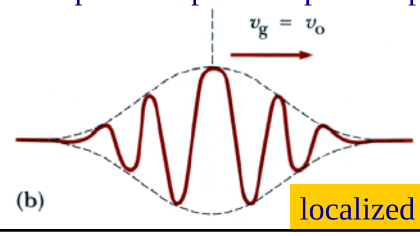
Wave Group or Wave Pulse

- Wave Group/packet:
  - Superposition of many sinusoidal waves with different wavelengths and frequencies
  - Localized in space, time
  - Size designated by
    - Δx or Δt
  - Wave groups travel with the speed $v_g = v_0$ of particle
- Constructing Wave Packets
  - Add waves of diff λ,
  - For each wave, pick
    - Amplitude
    - Phase
  - Constructive interference over the space-time of particle
  - Destructive interference elsewhere !

Imagine Wave pulse moving along a string: its localized in time and space (unlike a pure harmonic wave)



Wave packet represents particle prob



Resulting wave's "displacement" $y = y_1 + y_2$:

$$y = A[\cos(k_1 x - w_1 t) + \cos(k_2 x - w_2 t)]$$

Trignometry : $\cos A + \cos B = 2\cos(\frac{A+B}{2})\cos(\frac{A-B}{2})$

$$\therefore y = 2A\left[\left(\cos(\frac{k_2 - k_1}{2}x - \frac{w_2 - w_1}{2}t)\right)\left(\cos(\frac{k_2 + k_1}{2}x - \frac{w_2 + w_1}{2}t)\right)\right]$$

since $k_2 \cong k_1 \cong k_{ave}$, $w_2 \cong w_1 \cong w_{ave}, \Delta k \ll k, \Delta w \ll w$

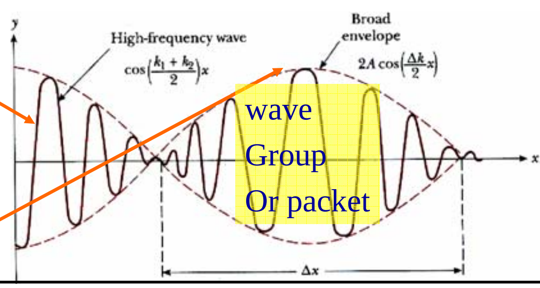
$$\therefore \boxed{y = 2A\left[\left(\cos(\frac{\Delta k}{2}x - \frac{\Delta w}{2}t)\right)\cos(kx - wt)\right]} \equiv y = A' \cos(ks - wt),$$ A' oscillates in x,t

$$A' = 2A\left(\cos(\frac{\Delta k}{2}x - \frac{\Delta w}{2}t)\right) = \text{modulated amplitude}$$

Phase Vel $\quad V_p = \frac{w_{ave}}{k_{ave}}$

Group Vel $\quad V_g = \frac{\Delta w}{\Delta k}$

V_g : Vel of envelope= $\frac{dw}{dk}$



Wave Packet : Localization

•Finite # of diff. Monochromatic waves always produce INFINTE sequence of repeating wave groups → can't describe (localized) particle

•To make localized wave packet, add " infinite" # of waves with Well chosen Ampl A, Wave# k, ang.

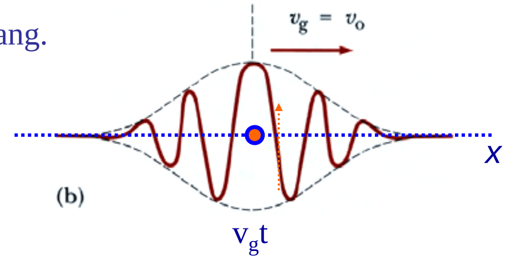
$$\psi(x,t) = \int_{-\infty}^{\infty} A(k)\, e^{i(kx-wt)}dk$$

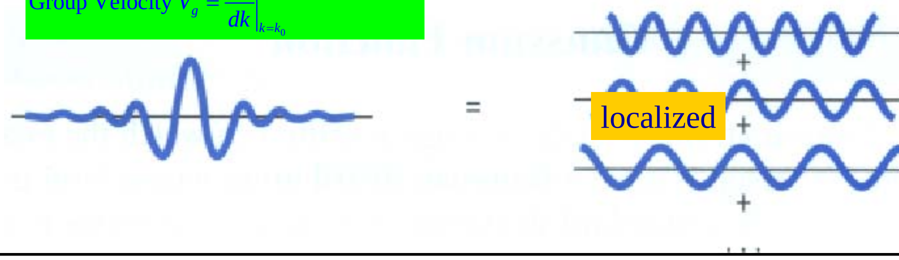
$A(k)$ = Amplitude Fn

⇒ diff waves of diff k have different amplitudes A(k)

w = w(k), depends on type of wave, media

Group Velocity $V_g = \left.\frac{dw}{dk}\right|_{k=k_0}$





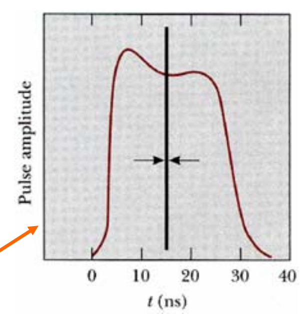
Group, Velocity, Phase Velocity and Dispersion

In a Wave Packet: $w = w(k)$

Group Velocity $V_g = \left.\frac{dw}{dk}\right|_{k=k_0}$

Since $V_p = \frac{w}{k}$ (def) $\Rightarrow w = kV_p$

$$\therefore V_g = \left.\frac{dw}{dk}\right| = V_p\big|_{k=k_0} + k\left.\frac{dV_p}{dk}\right|_{k=k_0}$$



1ns laser pulse disperse By x30 after travelling 1km in optical fiber

usually $V_p = V_p(k \text{ or } \lambda)$

Material in which V_p varies with λ are said to be Dispersive

Individual harmonic waves making a wave pulse travel at different V_p thus changing shape of pulse and become spread out

In non-dispersive media, $V_g = V_p$

In dispersive media $V_g \neq V_p$, depends on $\frac{dV_p}{dk}$

Matter Wave Packets

Consider An Electron:

mass = m velocity = v, momentum = p

Energy $E = hf = \gamma mc^2$; $\omega = 2\pi f = \frac{2\pi}{h}\gamma mc^2$

Wavelength $\lambda = \frac{h}{p}$; $k = \frac{2\pi}{\lambda} \Rightarrow k = \frac{2\pi}{h}\gamma mv$

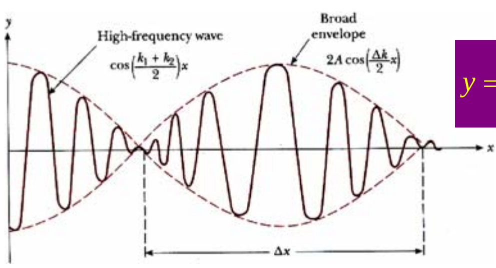
Group Velocity : $V_g = \frac{dw}{dk} = \frac{dw/dv}{dk/dv}$

$$\frac{dw}{dv} = \frac{d}{dv}\left[\frac{\frac{2\pi}{h}mc^2}{[1-(\frac{v}{c})^2]^{1/2}}\right] = \frac{2\pi mv}{h[1-(\frac{v}{c})^2]^{3/2}} \quad \& \quad \frac{dk}{dv} = \frac{d}{dv}\left[\frac{2\pi}{h[1-(\frac{v}{c})^2]^{1/2}}mv\right] = \frac{2\pi m}{h[1-(\frac{v}{c})^2]^{3/2}}$$

$V_g = \frac{dw}{dk} = \frac{dw/dv}{dk/dv} = v \Rightarrow$ Group velocity of electron Wave packet "pilot wave" is same as electron's physical velocity

But velocity of individual waves making up the wave packet $V_p = \frac{w}{k} = \frac{c^2}{v} > c!$ (not physical)

Wave Packets & Uncertainty Principle



$$y = 2A\left[\left(\cos\left(\frac{\Delta k}{2}x - \frac{\Delta w}{2}t\right)\right)\cos(kx - wt)\right]$$

- Distance ΔX between adjacent minima = $(X_2)_{node} - (X_1)_{node}$
- Define $X_1 = 0$ then phase diff from $X_1 \rightarrow X_2 = \pi$

Node at y = 0 = 2A cos $(\frac{\Delta w}{2}t - \frac{\Delta k}{2}x)$

$\Rightarrow$ $\boxed{\Delta k.\Delta x = 2\pi}$ $\Rightarrow$ Need to combine more k to make small Δx packet

also implies $\Rightarrow$ $\boxed{\Delta p.\Delta x = h}$

and

$\boxed{\Delta w.\Delta t = 2\pi}$ $\Rightarrow$ Need to combine more ω to make small Δt packet

also $\Rightarrow$ $\boxed{\Delta E.\Delta t = h}$

What does This mean?