Physics 2D Lecture Slides
Lecture 19: Feb 14th 2005

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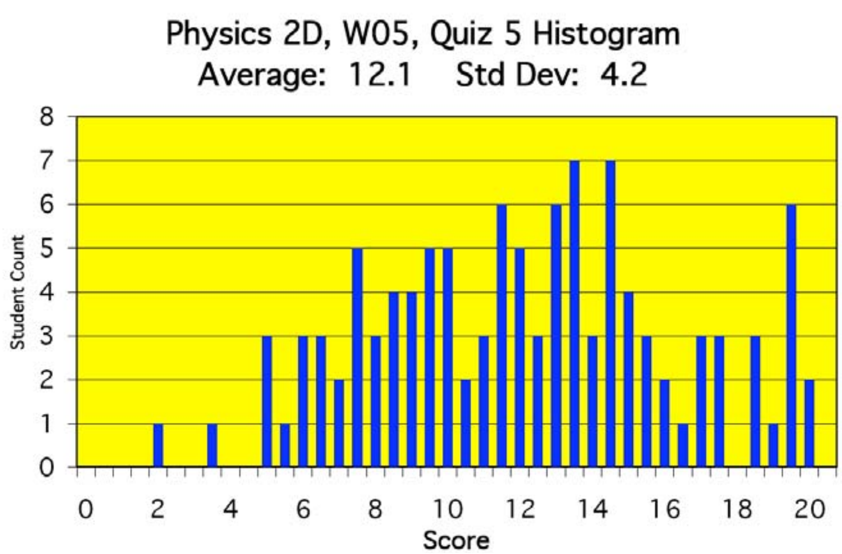
UCSD Physics



Heisenberg's Uncertainty Principles : Reprise

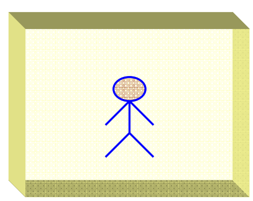
- $\Delta x. \Delta p \geq h/4\pi \Rightarrow$
  - If the measurement of the position of a particle is made with a precision Δx and a SIMULTANEOUS measurement of its momentum p_x in the X direction , then the product of the two uncertainties (measurement errors) can never be smaller than $\cong h/4\pi$ irrespective of how precise the measurement tools

- $\Delta E. \Delta t \geq h/4\pi \Rightarrow$
  - If the measurement of the energy E of a particle is made with a precision ΔE and it took time Δt to make that measurement, then the product of the two uncertainties (measurement errors) can never be smaller than $\cong h/4\pi$ irrespective of how precise the measurement tools

Many many wonderful ways to interpret these laws

Implications of Uncertainty Principles

A bound "particle" is one that is confined in some finite region of space.



One of the cornerstones of Quantum mechanics is that bound particles can not be stationary – <u>even at Zero absolute temperature !</u>

There is a non-zero limit on the kinetic energy of a bound particle

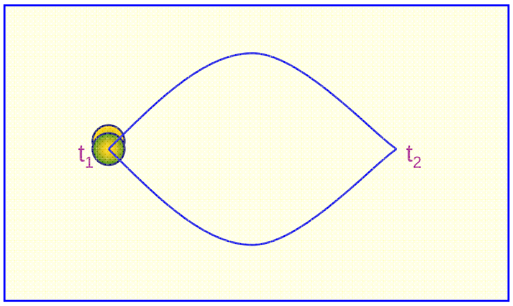
Fluctuations In The Vacuum : Breaking Energy Conservation Rules

Vaccum, at any energy, is bubbling with particle creation and annihilation

$\Delta E \cdot \Delta t \approx h/2\pi$ implies that you can (in principle) pull out an **elephant + anti-elephant** from NOTHING (Vaccum) but for a very very short time Δt !!

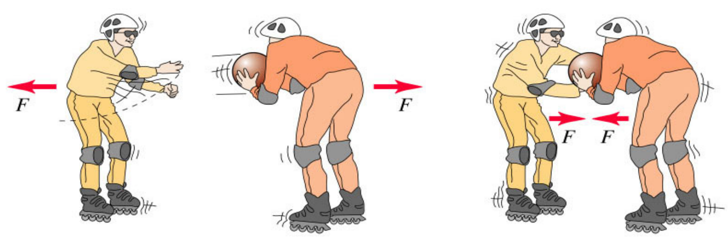
$$\text{How Much Time : } \Delta t = \frac{\hbar}{2Mc^2}$$

How cool is that !



How far can the virtual particles propagate ? Depends on their mass

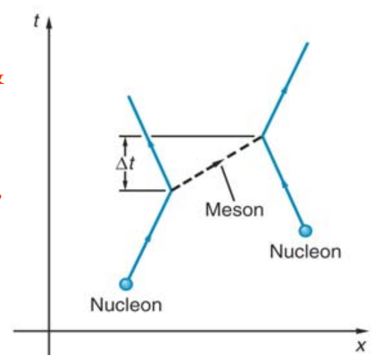
Strong Force Within Nucleus ➔ Exchange Force and Virtual Particles



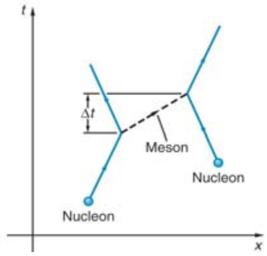
repulsive force: skaters exchange ball

attractive: grab ball from each other's hand

- Strong Nuclear force can be modeled as exchange of virtual particles called $\pi^{\pm}$ mesons by nucleons (protons & neutrons)
- $\pi^{\pm}$ mesons are emitted by proton and reabsorbed by a neutron
- The short range of the Nuclear force is due to the "large" mass of the exchanged meson
- $M_{\pi} = 140$ MeV/c^2



Range of Nuclear Exchange Force



How long can the emitted virtual particle last?

$\Delta E \times \Delta t \geq \hbar$

The virtual particle has rest mass + kinetic energy

$\Rightarrow$ Its energy $\boxed{\Delta E \geq Mc^2}$

$\Rightarrow$ Particle can not live for more than $\boxed{\Delta t \leq \hbar / Mc^2}$

Range R of the meson (and thus the exchange force)

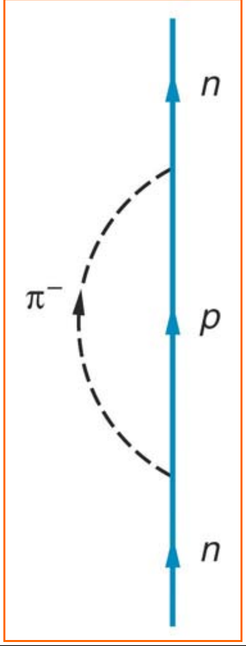
$R = c\Delta t = c\hbar / Mc^2 = \hbar / Mc$

For M=140 MeV/c^2 $\Rightarrow R \simeq \dfrac{1.06\times10^{-34}\, J.s}{(140 MeV / c^2)\times c^2 \times (1.60\times10^{-13}\, J / MeV)}$

$\boxed{R \simeq 1.4\times10^{-15} m = 1.4\, fm}$

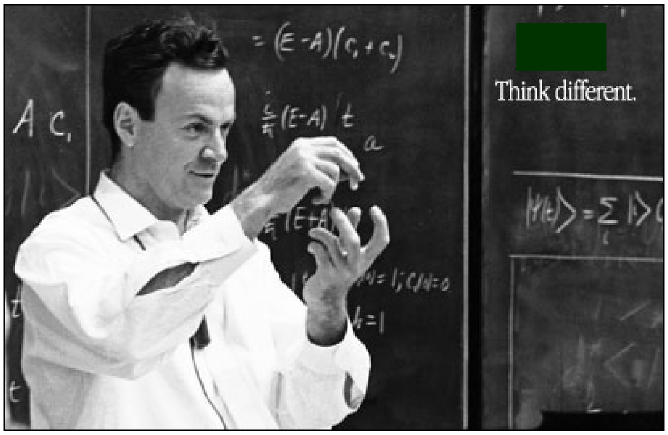
Subatomic Cinderella Act !

- Neutron emits a charged pion for a time Δt and becomes a (charged) proton
- After time Δt , the proton reabsorbs charged pion particle (π^-) to become neutron again
- But in the time Δt that the positive proton and π^- particle exist, they can interact with other charged particles
- After time Δt strikes , the Cinderella act is over !



This heralds the death of common sense in subatomic world

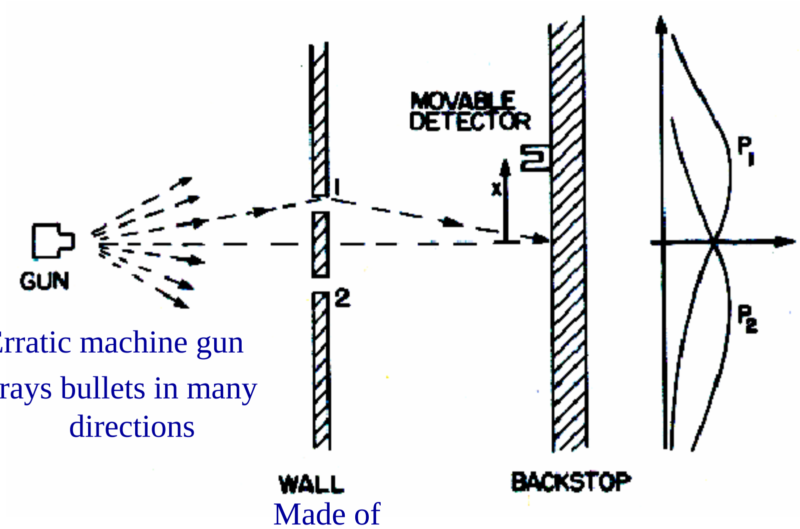
Quantum Behavior : Richard Feynman



See Chapters 1 & 2 of Feynman Lectures in Physics Vol III
Or ***Six Easy Pieces*** by Richard Feynman : Addison Wesley Publishers

Illustrate the Quantum Behavior by comparing and contrasting results of a series of “thought” experiments

An Experiment with *Indestructible* Bullets

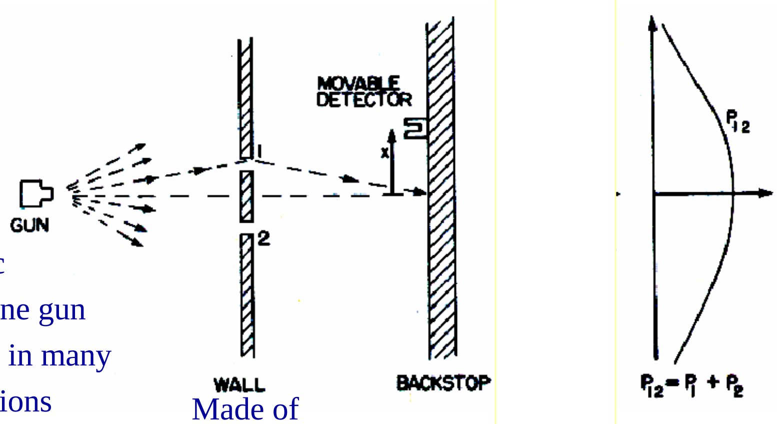


Erratic machine gun sprays bullets in many directions

Made of Armor plate

An Experiment with Indestructible Bullets

Probability P_{12} when Both holes open



Erratic Machine gun sprays in many directions

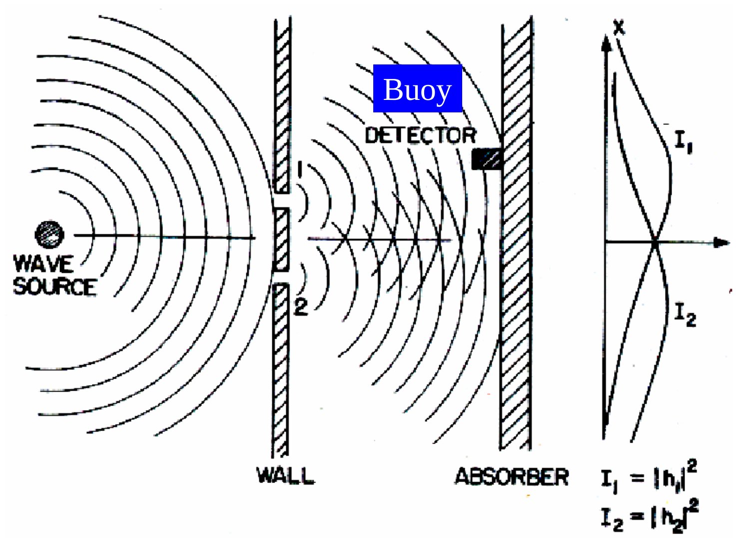
Made of Armor plate

$P_{12} = P_1 + P_2$

An Experiment With Water Waves

open one or the other hole :Measure Intensity of waves

(by measuring amplitude of displacement)

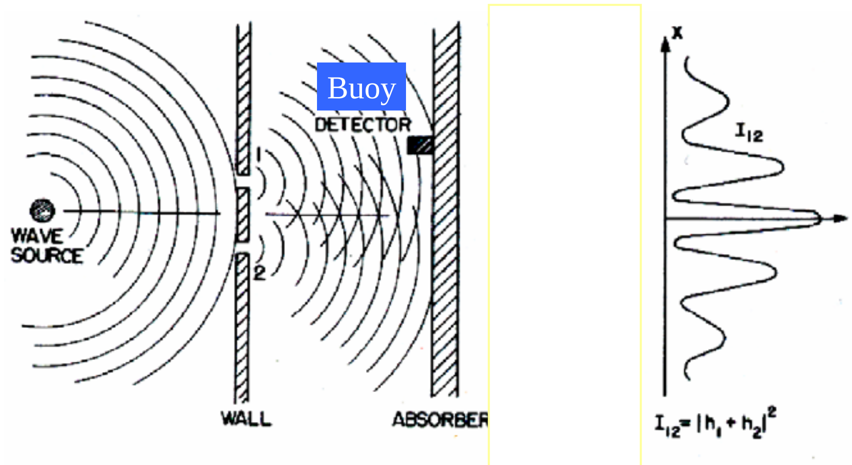


An Experiment With Water Waves

Measure Intensity of Waves

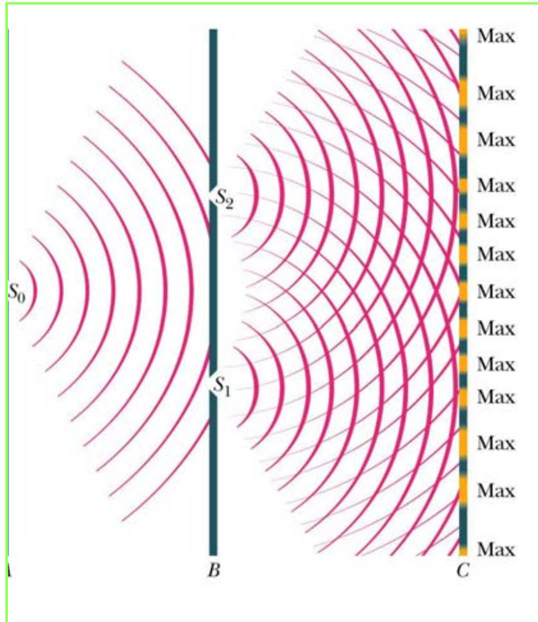
(by measuring amplitude of displacement)

Intensity I_{12} when Both holes open

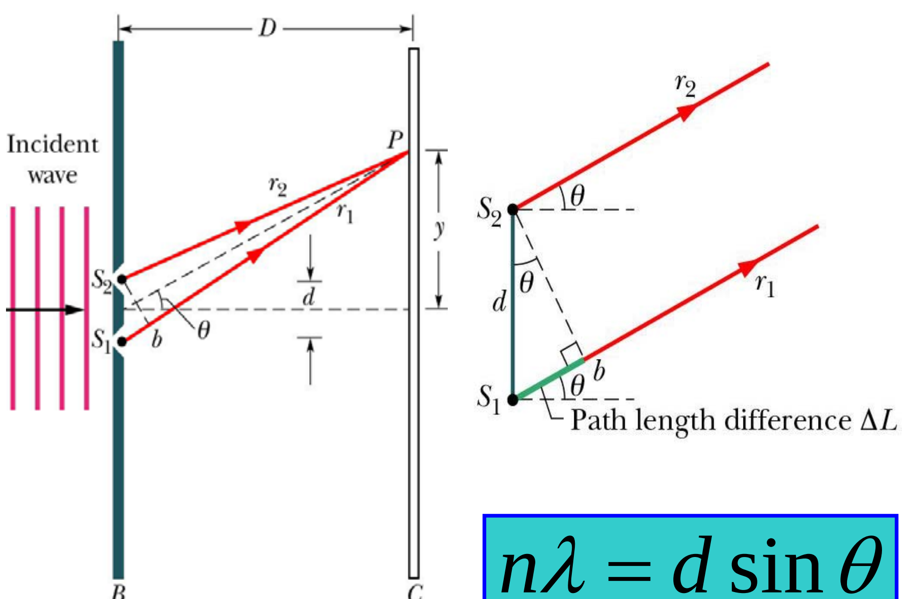


$$I_{12} = |h_1 + h_2|^2 = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

Why ? → Diffraction and Interference In Waves



Interference Phenomenon in Waves

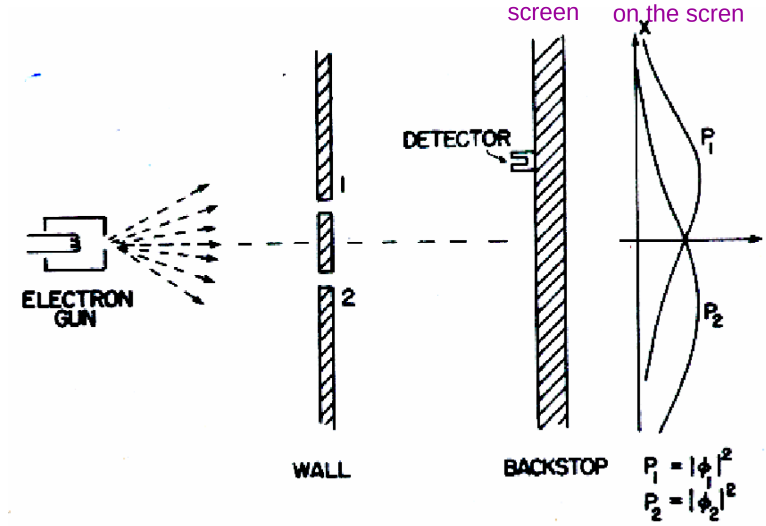


$$n\lambda = d\sin\theta$$

An Experiment With (indestructible) Electrons

Probability P of finding
The electron somewhere
on the scren

screen



An Experiment With (indestructible) Electrons

Probability P_{12} when
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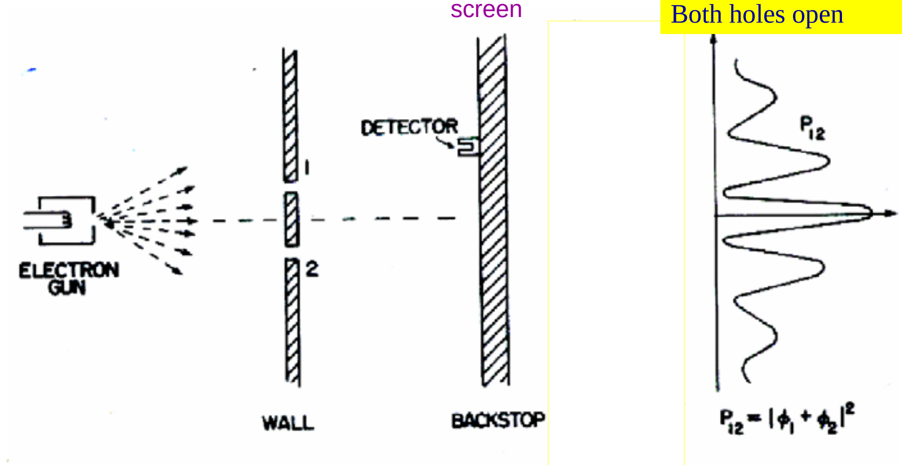
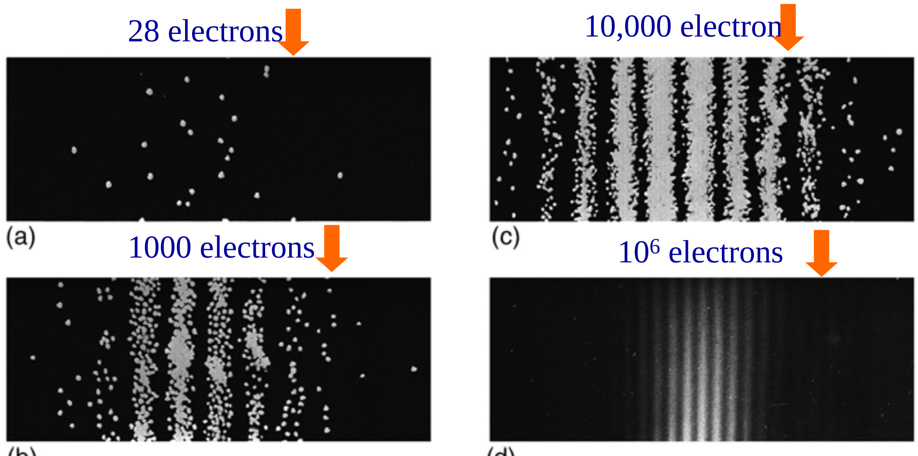


$P_{12} \neq P_1 + P_2$

Interference Pattern of Electrons When Both slits open

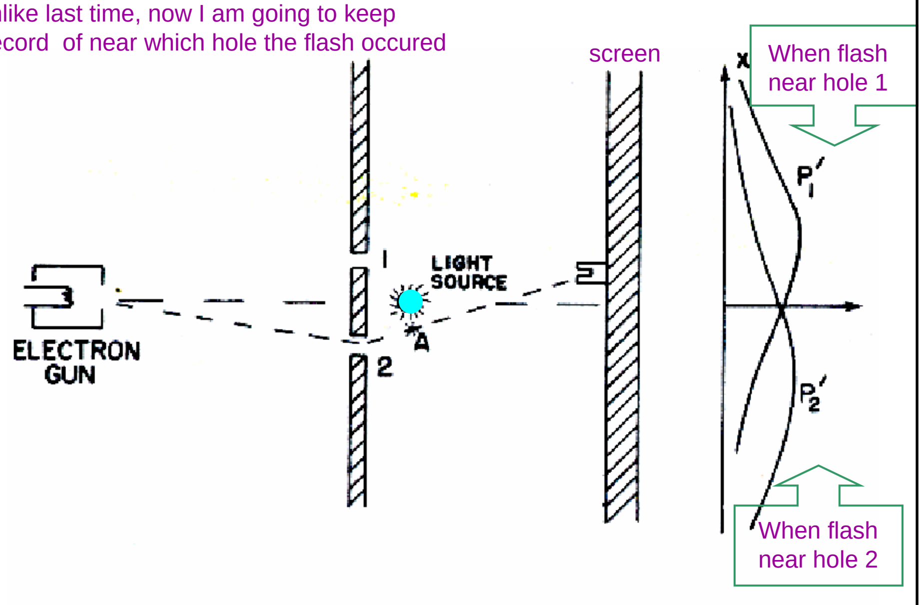
Growth of 2-slit Interference pattern thru different exposure periods
Photographic plate (screen) struck by:



White dots simulate presence of electron
No white dots at the place of destructive Interference (minima)

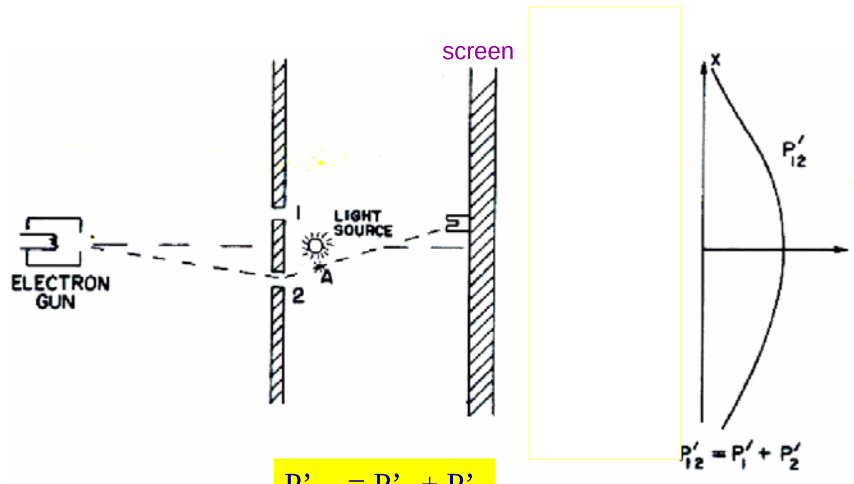
Watching The Electrons By Shining Intense Light

Unlike last time, now I am going to keep
Record of near which hole the flash occured



Watching The Electrons By Shining Intense Light

Probability P_{12} when both holes open and I can ***see*** and keep track of which hole the electron came thru



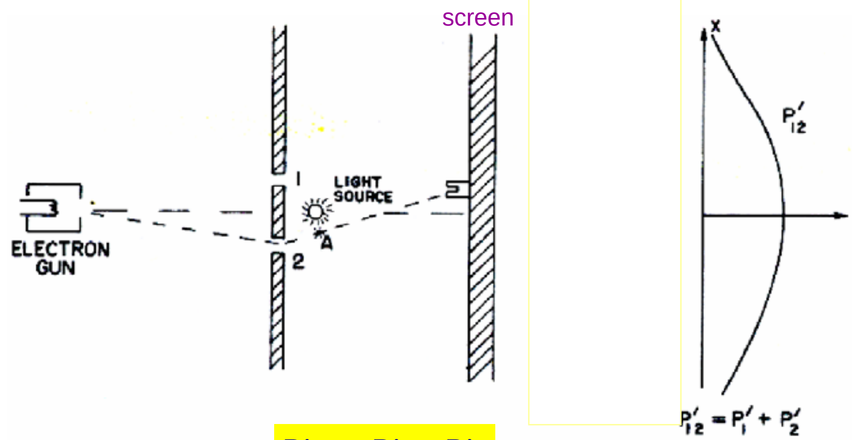
$P'_{12} = P'_1 + P'_2$

Watching Electrons And Hearing Them Land on Screen

- Maybe I should dim the intensity of light, perhaps electrons get all confused when then see a "mob" of photons streaming their way
- Try decreasing the Intensity of light→ fewer photons incident
  - Problem now is that some time electrons wont get scattered by the illumination as they pass thru one of the holes, so I wont see a Flash everytime the electron gun goes off….but the electrons do land somewhere on the screen, so I will hear the "click" of their landing on the screen

Watching The Electrons By Shining *Faint* Light

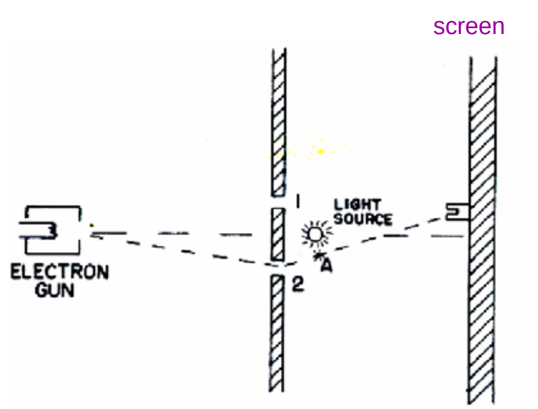
Probability P_{12} when both holes open and I see flash thru one hole or the other and thus can keep track of which hole the electron came thru when it lands on screen



$P'_{12} = P'_1 + P'_2$

Watching electrons with dim light: <u>don't see</u> flash of light but hear detector clicks

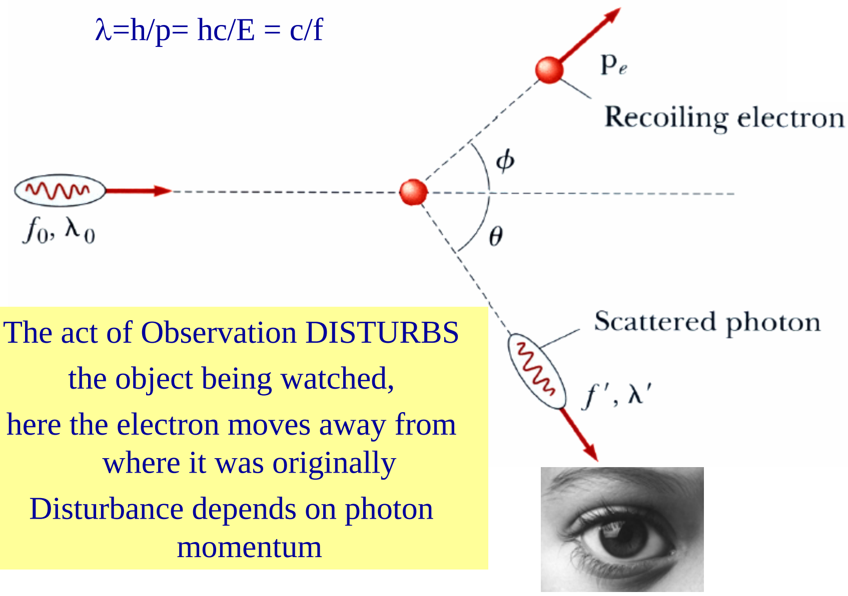
Probability P_{12} when both holes open and I Don't see (so don't know) which hole the electron came thru



!

What is Happening ? Shining light to observe electron → Compton Scattering

$\lambda = h/p = hc/E = c/f$



The act of Observation DISTURBS the object being watched, here the electron moves away from where it was originally

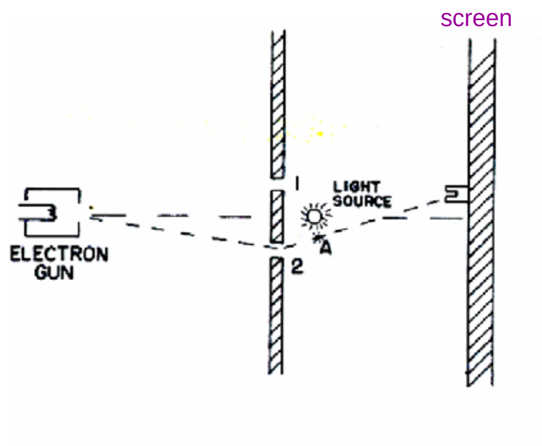
Disturbance depends on photon momentum

Compton Scattering → Oops !

- May be the problem is that I am whacking the electron very hard with high frequency, low wavelength photons (X rays for example)
- …and this is "stunning" the electron thus confusing their trajectory
- May be we should shine "gentler" light → very low frequency, low momentum (p=E/c) and high wavelength photon …say radio waves. Go back to high intensity light since that is not the problem
- Lets do this experiment and see what happens

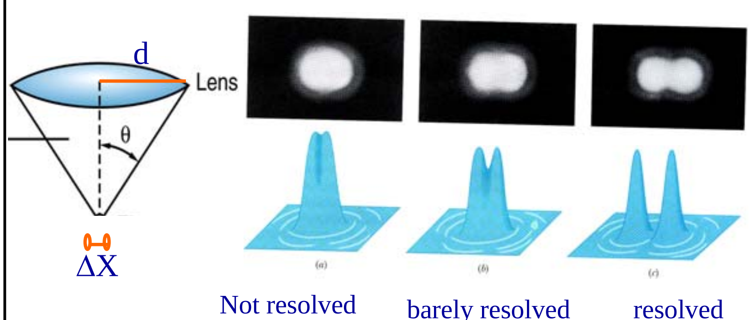
Watching Electrons With Light of λ >> slit size but High Intensity

Probability P_{12} when both holes open but can't tell anymore, from the location of the fuzzy flash, which hole the electron came thru…I know it comes thru because I hear the "click" of it landing on the screen



Why Fuzzy Flash? → Resolving Power of Light

Remember: Image of 2 separate point sources formed by a converging lens of diameter d, ability to resolve them depends on λ & d because of the inherent diffraction in image formation



$$\text{Resolving power } \Delta x \simeq \frac{\lambda}{2\sin\theta}$$

Summary of Experiments So Far

1. Probability of an event is given by the square of amplitude of a complex # Ψ: Probability Amplitude
2. When an event occurs in several alternate ways, probability amplitude for the event is sum of probability amplitudes for each way considered separately. There is interference:

   $\Psi = \Psi_1 + \Psi_2$

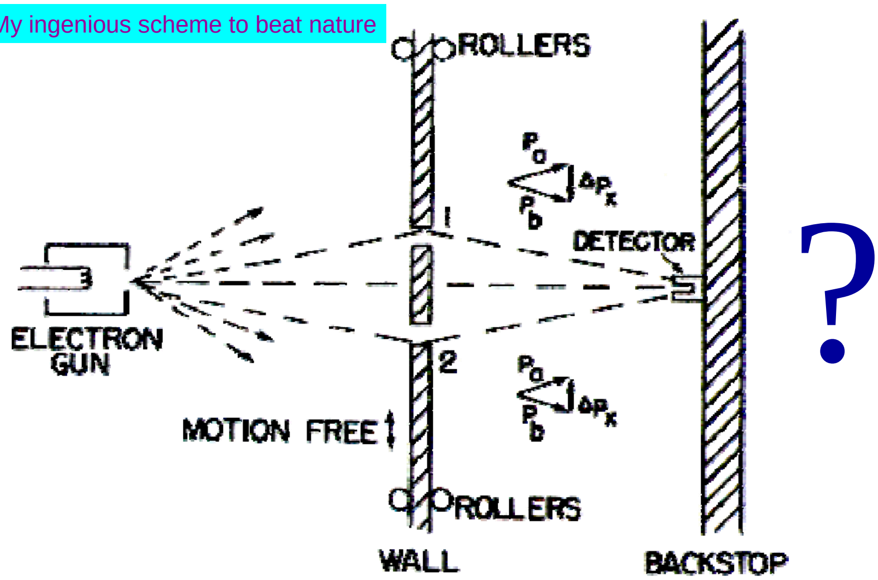
   $P_{12} = |\Psi_1 + \Psi_2|^2$
3. If an experiment is done which is capable of determining whether one or other alternative is actually taken, probability for the event is just the sum of each alternative → Interference pattern is LOST !

Is There No Way to Beat The Uncertainty Principle?

- How about NOT watching the electrons!
- Let's be a bit crafty !!
- Since this is a *thought* experiment → ideal conditions
- Make up a contraption which does not violate any law
  - Mount the wall on rollers, put a lot of *grease* → frictionless
  - Wall will move when electron hits it
  - Watch recoil of the wall containing the slits when the electron hits it
  - By watching whether wall moved up or down I can tell
    - Electron went thru hole # 1
    - Electron went thru hole #2
- Will my ingenious plot succeed? After all I am so smart!

Measuring The Recoil of The Wall ➔ Not Watching Electron !

My ingenious scheme to beat nature



Think About it and Tell me Now If I will Succeed ?

Losing Out To Uncertainty Principle

- To measure the RECOIL of the wall ⇒
  - must know the initial momentum of the wall before electron hit it
  - Final momentum after electron hits the wall
  - Calculate vector sum = recoil
- Uncertainty principle :
  - To do this ⇒ must know momentum at all times exactly so $\Delta P = 0$ → knowledge of wall location is imprecise, $\Delta X = \infty$ [so can not know the position of wall exactly]
  - If don't know the wall location, ***then dont know where the holes were***
  - Holes will be in different place for every electron that goes thru
  - → The center of interference pattern will have different (random) location (interference pattern) for each electron
  - Such random shift is just enough to *smear out the I. pattern* so that no interference is observed !
- Uncertainty Principle Protects Quantum Mechanics !

Summary

- Probability of an event in an ideal experiment is given by the square of the absolute value of a complex number Ψ which is call probability amplitude
  - P = probability
  - Ψ= probability amplitude,
  - $P=|\Psi|^2$
- When an even can occur in several alternative ways, the probability amplitude for the event is the sum of the probability amplitudes for each way considered separately. There is interference:
  - $\Psi= \Psi_1+ \Psi_2$
  - $P=|\Psi_1+ \Psi_2|^2$
- If an experiment is performed which is capable of determining whether one or other alternative is actually taken, the probability of the event is the sum of probabilities for each alternative. The interferenence is lost: $P = P_1 + P_2$

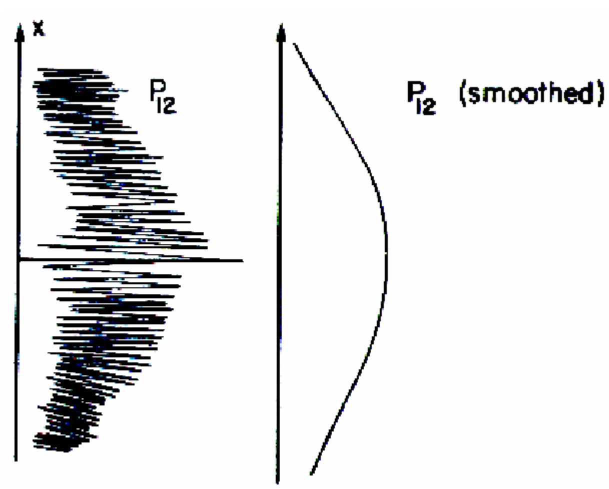
The Lesson Learnt From These Experiments

- In trying to determine which slit the particle went through, we are examining particle-like behavior
- In examining the interference pattern of electron, we are using wave like behavior of electron

Bohr's Principle of Complementarity:

It is not possible to simultaneously determine physical observables in terms of both particles and waves

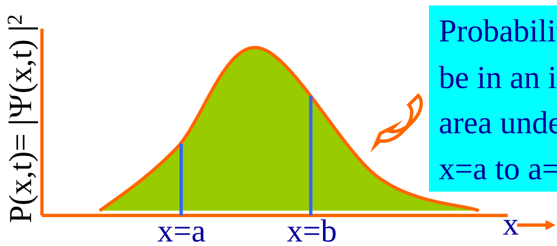
The Bullet Vs The Electron: Each Behaves the Same Way



Quantum Mechanics of Subatomic Particles

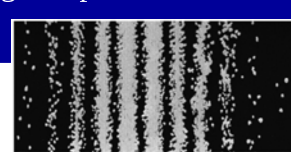
- Act of Observation destroys the system (No watching!)
- If can't watch then all conversations can only be in terms of Probability P
- Every particle under the influence of a force is described by a Complex wave function $\Psi(x,y,z,t)$
- Ψ is the ultimate DNA of particle: contains all info about the particle under the force (in a potential e.g Hydrogen)
- Probability of per unit volume of finding the particle at some point (x,y,z) and time t is given by
  - $P(x,y,z,t) = \Psi(x,y,z,t) \,.\, \Psi^*(x,y,z,t) = |\Psi(x,y,z,t)|^2$
- When there are more than one path to reach a final location then the probability of the event is
  - $\Psi = \Psi_1 + \Psi_2$
  - $P = |\Psi^* \Psi| = |\Psi_1|^2 + |\Psi_2|^2 + 2 |\Psi_1| |\Psi_2| \cos\phi$

Wave Function of "Stuff" & Probability Density



Probability of a particle to be in an interval $a \leq x \leq b$ is area under the curve from x=a to a=b

- Although not possible to specify with certainty the location of particle, its possible to assign probability P(x)dx of finding particle between x and x+dx
- $P(x)\, dx = |\Psi(x,t)|^2\, dx$
- E.g intensity distribution in light diffraction pattern is a measure of the probability that a photon will strike a given point within the pattern



Ψ: The Wave function Of A Particle

- The particle must be some where

$$\int_{-\infty}^{+\infty} |\psi(x,t)|^2 \, dx = 1$$

- Any Ψ satisfying this condition is NORMALIZED
- Prob of finding particle in finite interval

$$P(a \le x \le b) = \int_a^b \psi^*(x,t) \; \psi(x,t) dx$$

- Fundamental aim of Quantum Mechanics
  - Given the wavefunction at some instant (say t=0) find Ψ at some subsequent time t
  - Ψ(x,t=0) → Ψ(x,t) …evolution
  - Think of a probabilistic view of particle's "newtonian trajectory"
    - We are replacing Newton's 2nd law for subatomic systems

The Wave Function is a mathematical function that describes a physical object → Wave function must have some rigorous properties :

- Ψ must be finite
- Ψ must be continuous fn of x,t
- Ψ must be single-valued
- Ψ must be smooth fn →

$\frac{d\psi}{dx}$ must be continuous

WHY ?

Bad (Mathematical) Wave Functions Of a Physical System : You Decide Why