Physics 2D Lecture Slides
Lecture 14: Feb 1st 2005

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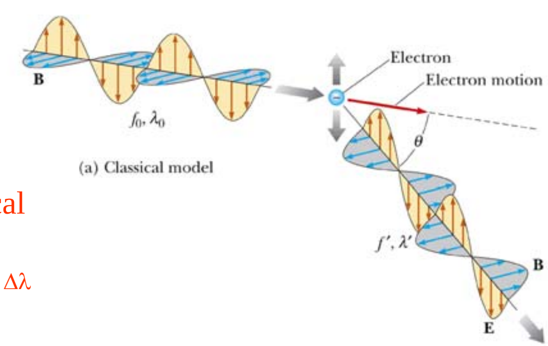
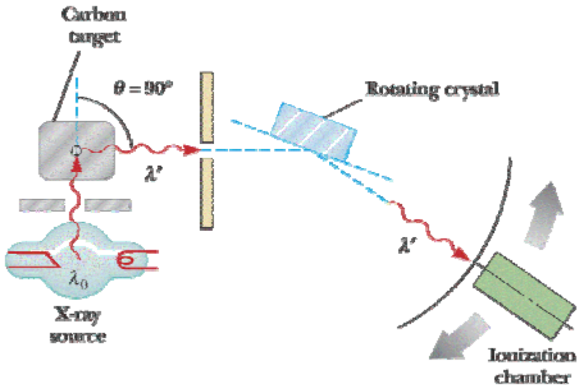
Compton Effect: what should Happen Classically?

- Plane wave [f,λ] incident on a surface with loosely bound electrons → interaction of E field of EM wave with electron: **F** = e**E**
- Electron oscillates with $f = f_{incident}$
- Eventually radiates spherical waves with $f_{radiated} = f_{incident}$
  - At all scattering angles, Δf & Δλ must be zero
- Time delay while the electron gets a "tan" : soaks in radiation



(a) Classical model

Compton Scattering : Summary of Observations

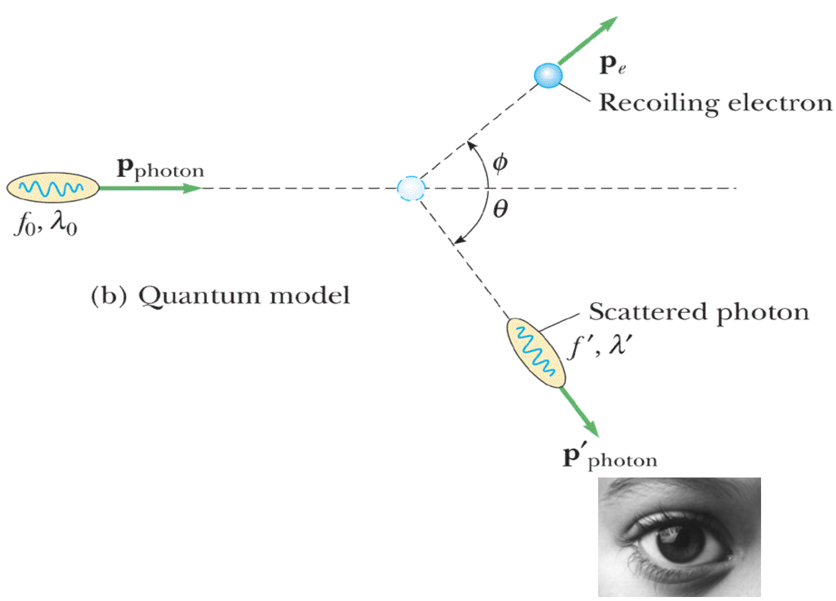


$\Delta\lambda = (\lambda' - \lambda) \propto (1 - \cos\theta)$!

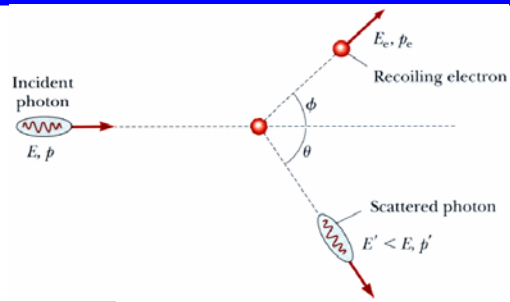
Not isotropy in distribution of scattered radiation

How does one explain this startling anisotropy?

Compton Effect : Quantum (Relativistic) Pool



Compton Scattering: Quantum Picture



Energy Conservation:

$E + m_e c^2 = E' + E_e$

Momentum Conserv:

$p = p'\cos\theta + p_e \cos\phi$

$0 = p'\sin\theta - p_e \sin\phi$

Use these to eliminate electron deflection angle (not measured)

$p_e \cos\phi = p - p'\cos\theta$

$p_e \sin\phi = p'\sin\theta$

Square and add $\Rightarrow$

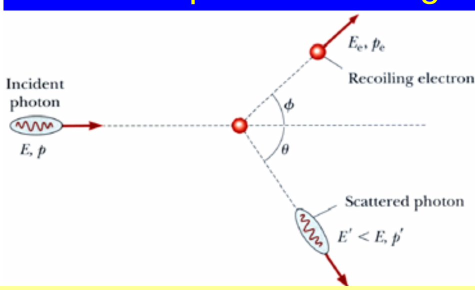
$p_e^2 = p^2 - 2pp'\cos\theta + p'^2$

Eliminate p_e & E_e using

$E_e^2 = p_e^2 c^2 + m_e^2 c^4$ &

$E_e = (E - E') + m_e c^2$

Compton Scattering: The Quantum Picture



Energy Conservation:

$E + m_e c^2 = E' + E_e$

Momentum Conserv:

$p = p'\cos\theta + p_e \cos\phi$

$0 = p'\sin\theta - p_e \sin\phi$

Use these to eliminate electron deflection angle (not measured)

$$\left((E - E') + m_e c^2\right)^2 = \left[p^2 - 2pp'\cos\theta + p'^2\right] + (m_e c^2)^2$$

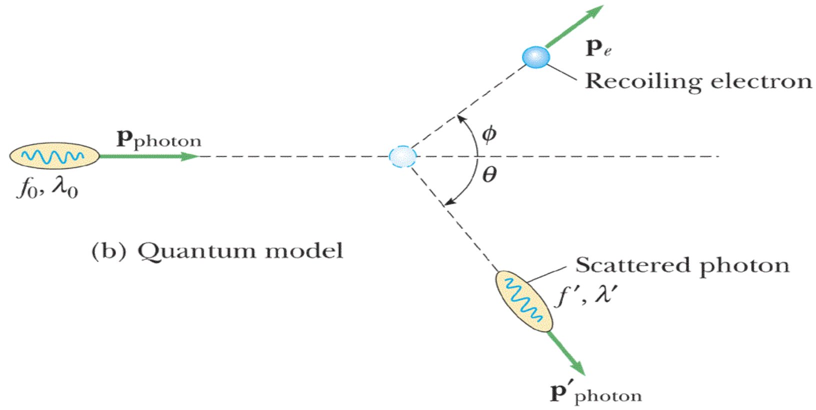
For light $p = \frac{E}{c} \Rightarrow$

$$E^2 + E'^2 - 2EE' + 2(E - E')mc^2 = \left[\frac{E^2}{c^2} - 2\frac{EE'}{c^2}\cos\theta + \frac{E'^2}{c^2}\right]c^2$$

$$\Rightarrow -EE' + (E - E')mc^2 = -EE'\cos\theta$$

$$\Rightarrow \frac{E - E'}{EE'} = -\frac{1}{m_e c^2}(1 - \cos\theta) \Rightarrow \boxed{(\lambda' - \lambda) = \left(\frac{h}{m_e c}\right)(1 - \cos\theta)}$$

Rules of Quantum Pool between Photon and Electron

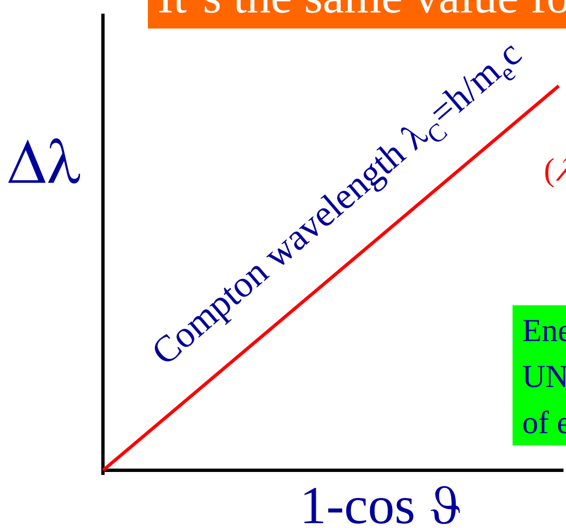


$$(\lambda' - \lambda) = \left(\frac{\boxed{h}}{m_e c}\right)(1 - \cos\theta)$$

Checking for h in Compton Scattering

Plot scattered photon data, calculate slope and measure “h”

It’s the same value for h again !!



$$(\lambda' - \lambda) = \left(\frac{h}{m_e c}\right)(1 - \cos\theta)$$

Energy Quantization is a UNIVERSAL characteristic of energy transactions !

Saw what light does, Now examine nature of matter

- Fundamental Characteristics of different forms of matter
  - Rest Mass (m)
  - Electric Charge (q)
    - Measurable
      - using some combination of **E** & **B fields interacting with the particle**

Reading Assignment, one problem from here may be on the quiz

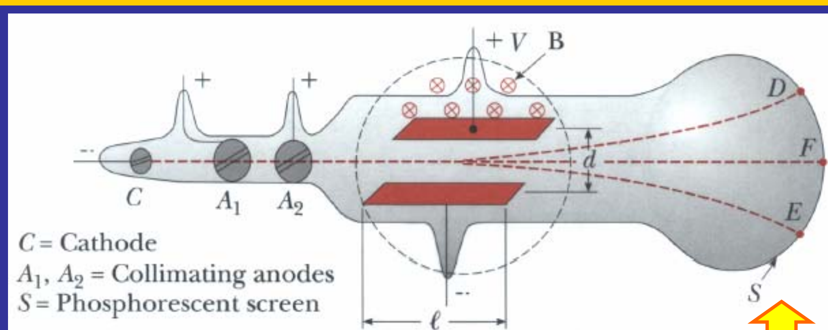
$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

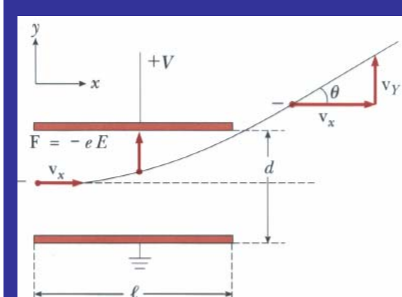
– **Or E/B or some other macroscopic force**

e.g. Drag Force

The “magic” is that one is measuring tiny tiny numbers using Macroscopic devices

Thomson’s Determination of e/m of the Electron

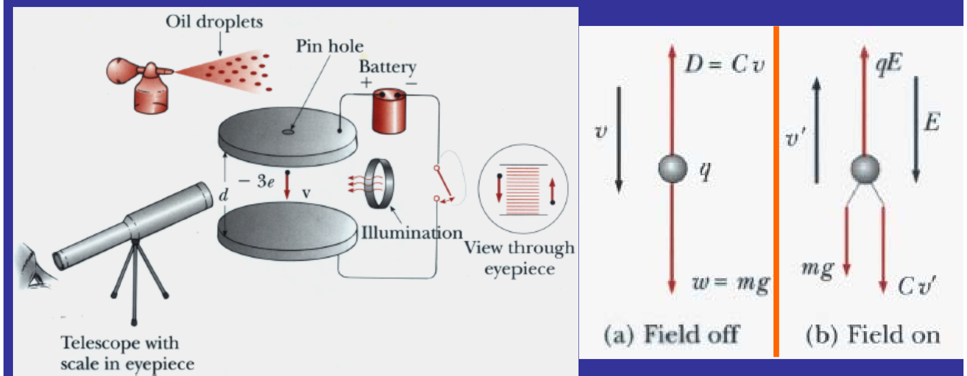




- In E Field alone, electron lands at D
- In B field alone, electron lands at E
- When E and B field adjusted to cancel each other’s force → electron lands at F

→ $e/m = 1.7588 \times 10^{11}$ C/Kg

Millikan's Measurement of Electron Charge



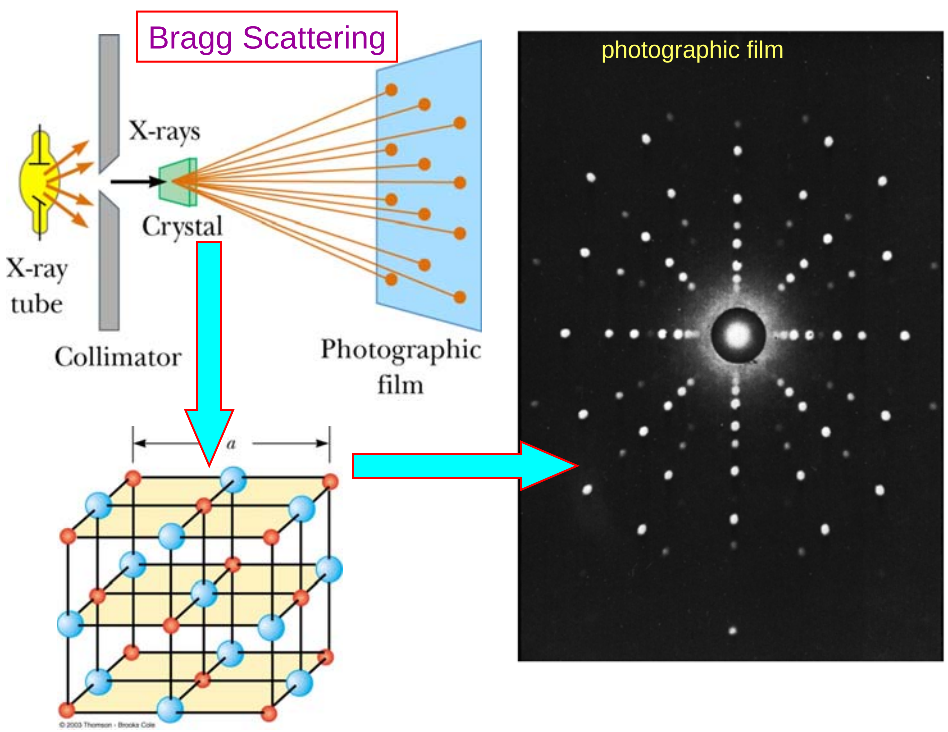
Find charge on oil drop is always in integral multiple of some Q

$q_e = 1.688 \times 10^{-19}$ Coulombs

→ $m_e = 9.1093 \times 10^{-31}$ Kg

→ Fundamental properties (finger print) of electron

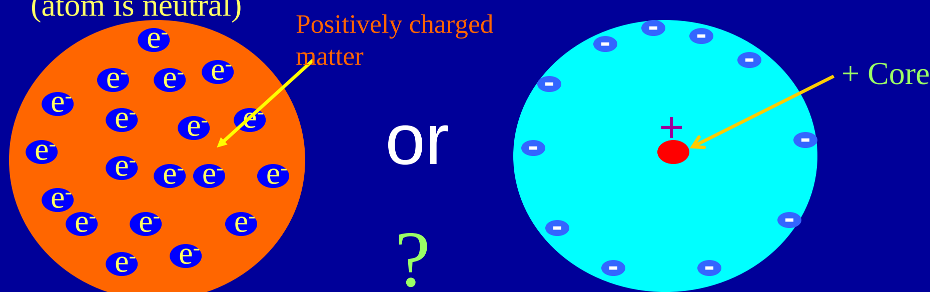
(similarly can measure proton properties etc)



Summary : From X Ray (EM Wave) Scattering data, Size of the Atom was known to be about 10^{-10} m

Where are the electrons inside the atom?

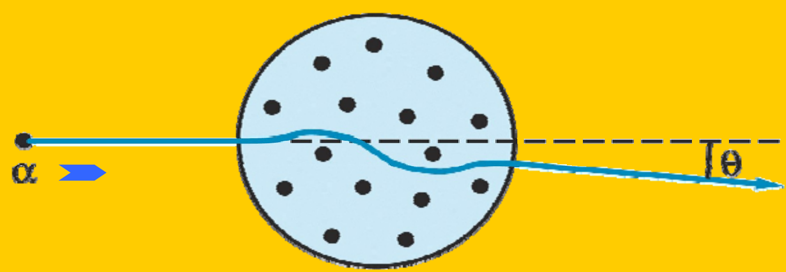
Early Thought: "Plum pudding" model→ Atom has a homogenous distribution of Positive charge with electrons embedded in them (atom is neutral)



- How to test these hypotheses? → Shoot "bullets" at the atom and watch their trajectory. What Kind of bullets ?
  - •Indestructible charged bullets → Ionized He^{++} atom = α^{++} particles
  - •Q = +2e , Mass M_α=4amu >> m_e , V_α= 2 x 10^7 m/s (non-relavistic) [charged to probe charge & mass distribution inside atom]

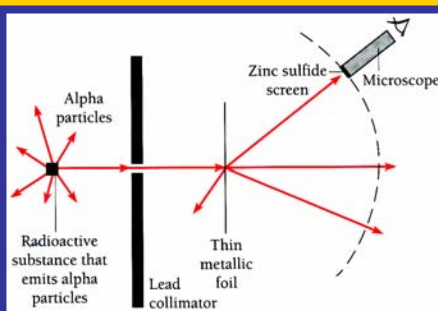
Plum Pudding Model of Atom

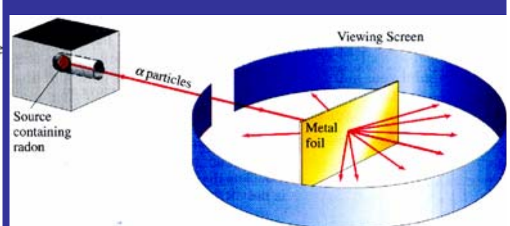
- Non-relativistic mechanics ($V_\alpha/c = 0.1$)
- In Plum-pudding model, α-rays hardly scatter because
  - Positive charge distributed over size of atom (10^{-10}m)
  - $M_\alpha >> M_e$ (like moving truck hits a bicycle)
  - → predict α-rays will pass thru array of atoms with little scatter (~1°)



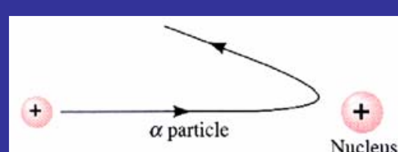
Need to test this hypothesis → Ernest Rutherford

Probing Within an Atom with α Particles

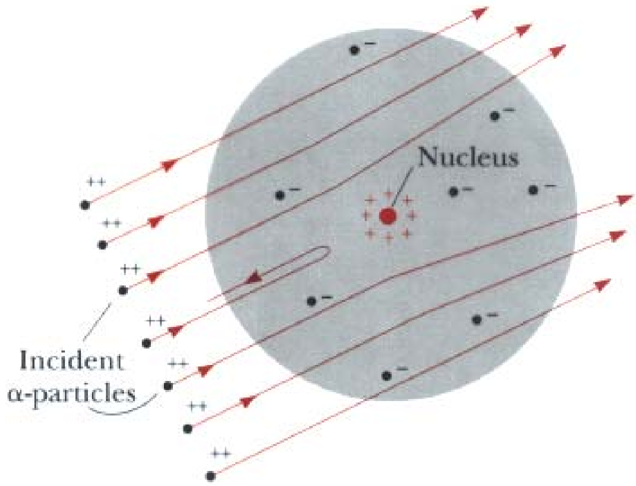




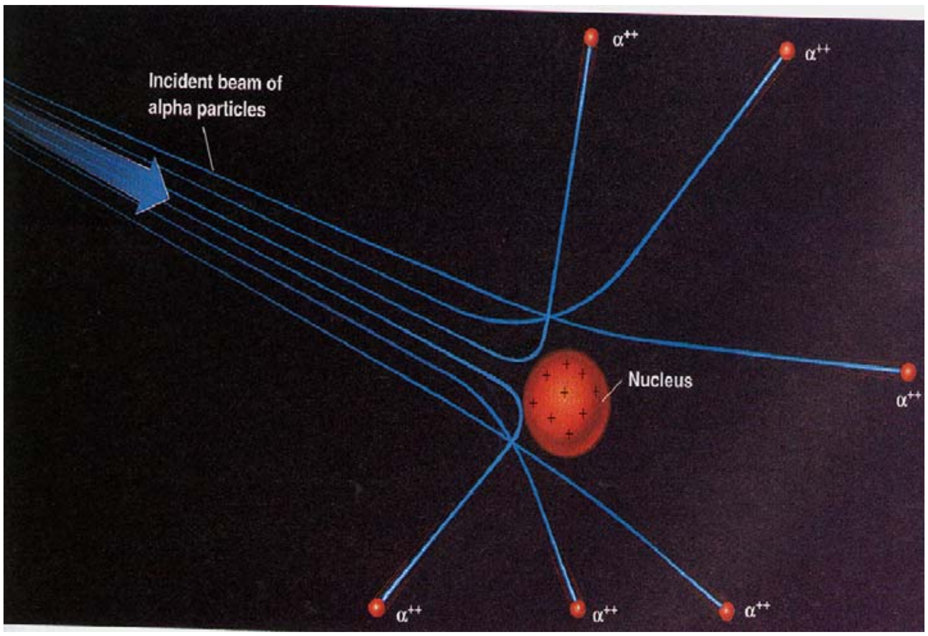
- Most α particles pass thru gold foil with nary a deflection
- SOME ($\cong 10^{-4}$) scatter at LARGE angles Φ
- Even fewer scatter almost backwards → Why



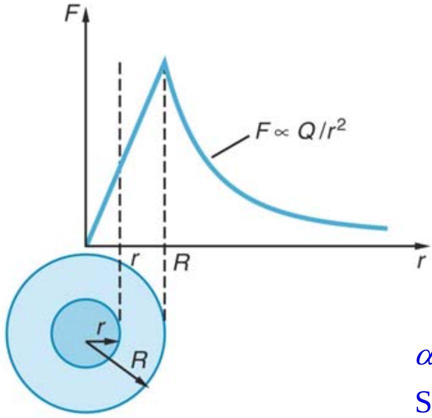
"Rutherford Scattering" discovered by his PhD Student (Marsden)

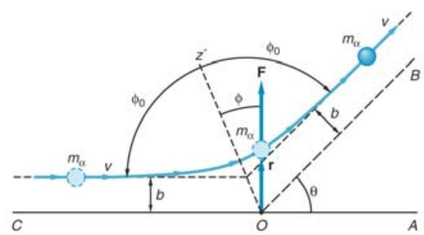


Rutherford Discovers Nucleus (Nobel Prize)



Force on α-particle due to heavy Nucleus





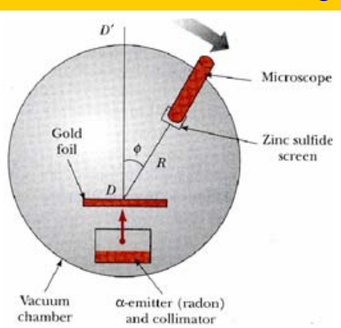
- Outside radius r =R, $F \propto Q/r^2$
- Inside radius r < R, $F \propto q/r^2 = Qr/R^2$
- Maximum force at radius r = R

α particle trajectory is hyperbolic

Scattering angle is related to impact par.

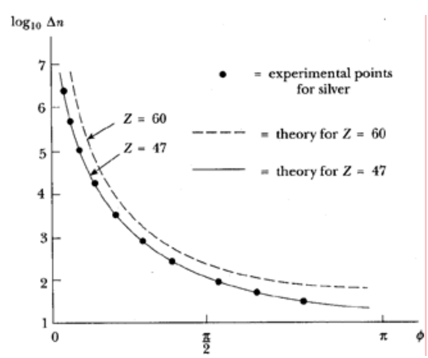
Impact Parameter $b = \left(\frac{kq_\alpha Q}{m_\alpha v_\alpha^2}\right)\left(\cot\frac{\theta}{2}\right)$

Rutherford Scattering: Prediction and Experimental Result

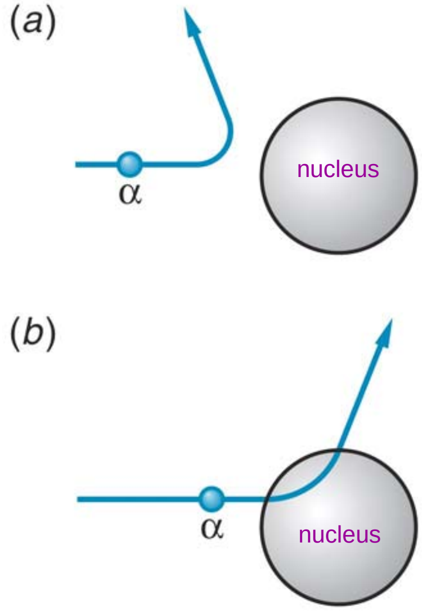


$$\Delta n = \frac{k^2 Z^2 e^4 NnA}{4R^2\left(\frac{1}{2}m_\alpha v_\alpha^2\right)^2 Sin^4(\varphi/2)}$$

- # scattered Vs ϕ depends on :
  - n = # of incident alpha particles
  - N = # of nuclei/area of foil
  - Ze = Nuclear charge
  - K_α of incident alpha beam
  - A= detector area



Rutherford Scattering & Size of Nucleus



distance of closest appoach $\propto$ r size of nucleus

Kinetic energy of α = $K_\alpha = \frac{1}{2} m_\alpha v_\beta^2$

α particle will penetrate thru a radius r until all its kinetic energy is used up to do work AGAINST the Coulomb potential of the Nucleus:

$$K_\alpha = \frac{1}{2} m_\alpha v_\beta^2 = 8MeV = k\frac{(Ze)(2e)}{r}$$

$$\Rightarrow \quad r = \frac{2kZe^2}{K_\alpha}$$

For K_α =7.7.MeV, Z_{Al} =13

$$\Rightarrow \quad r = \frac{2kZe^2}{K_\alpha} = 4.9\times10^{-15} m$$

Size of Nucleus = $10^{-15} m$

Size of Atom = $10^{-10} m$

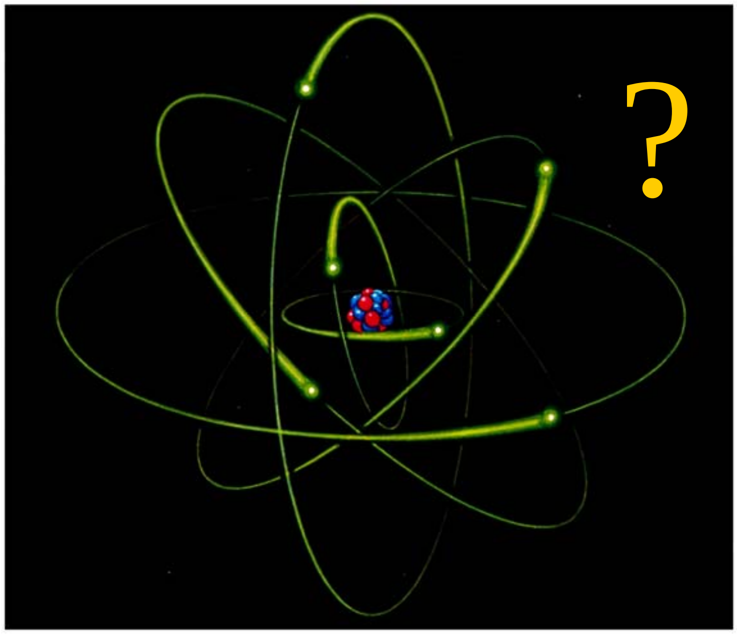
Dimension Matters !

Size of Nucleus = $10^{-15} m$

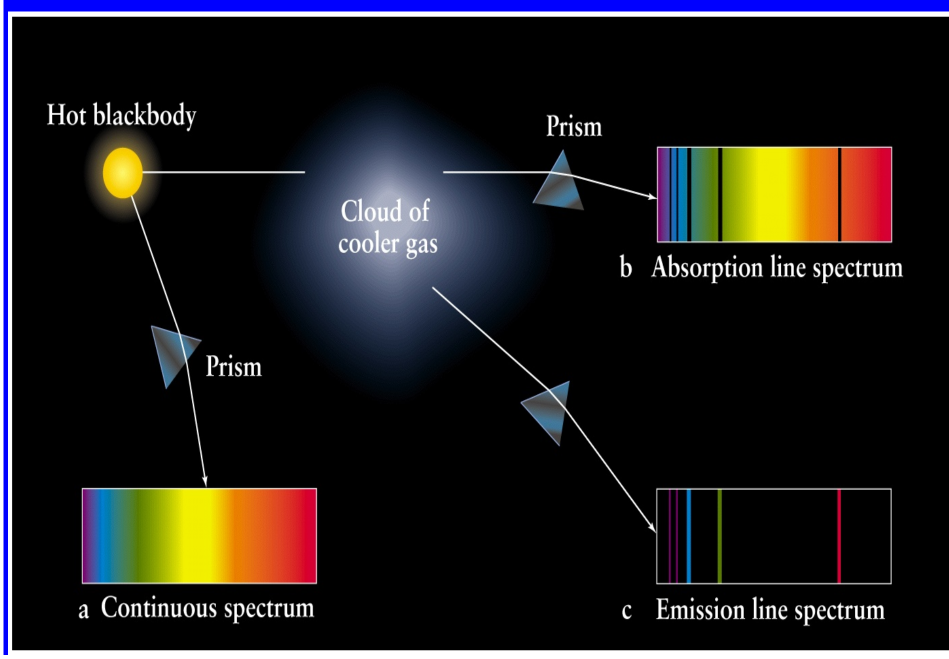
Size of Atom = $10^{-10} m$

- how are the electrons located inside an atom
- How are they held in a stable fashion
  - necessary condition for us to exist !
  - All these discoveries will require new experiments and observations

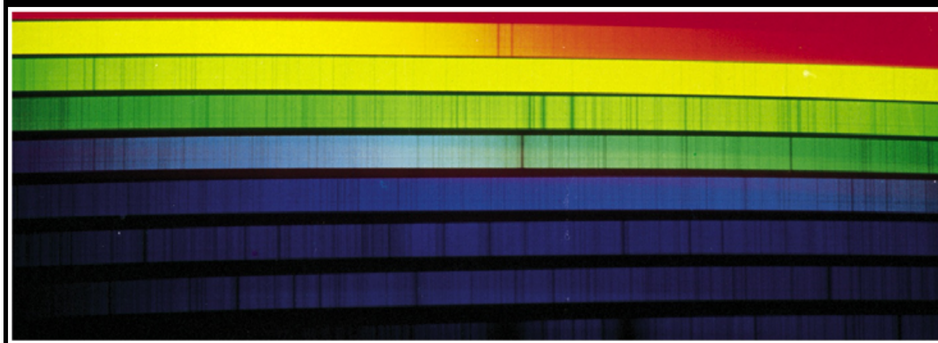
Rutherford Atom & Classical Physics



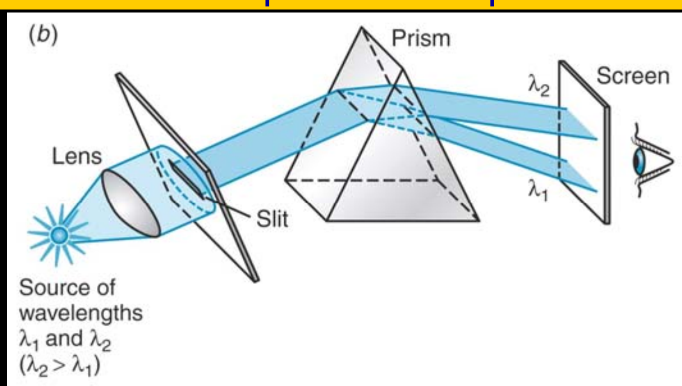
Continuous & Discrete spectra of Elements

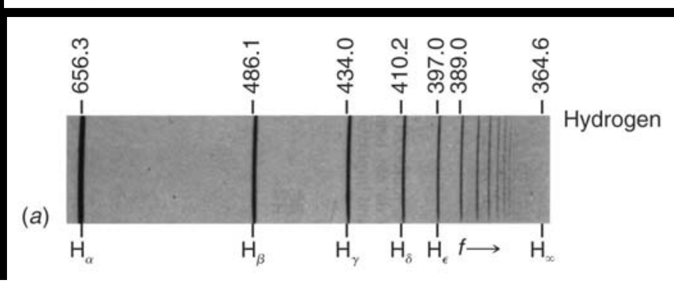


Visible Spectrum of Sun Through a Prism

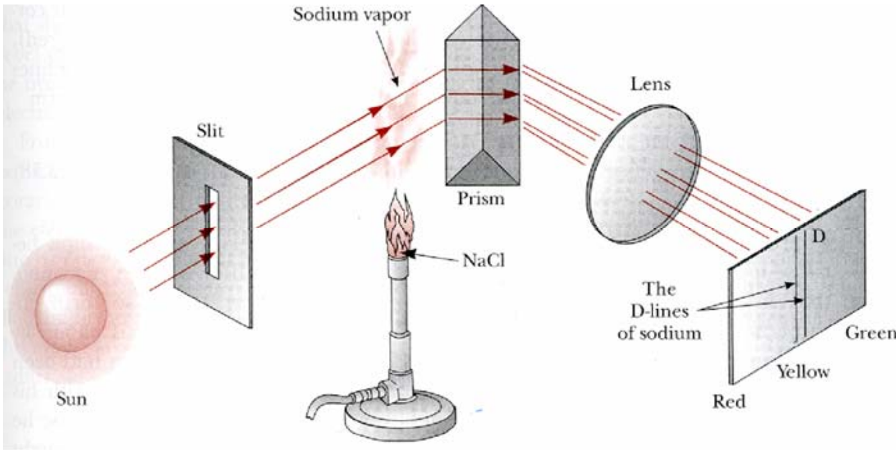


Emission & Absorption Line Spectra of Elements





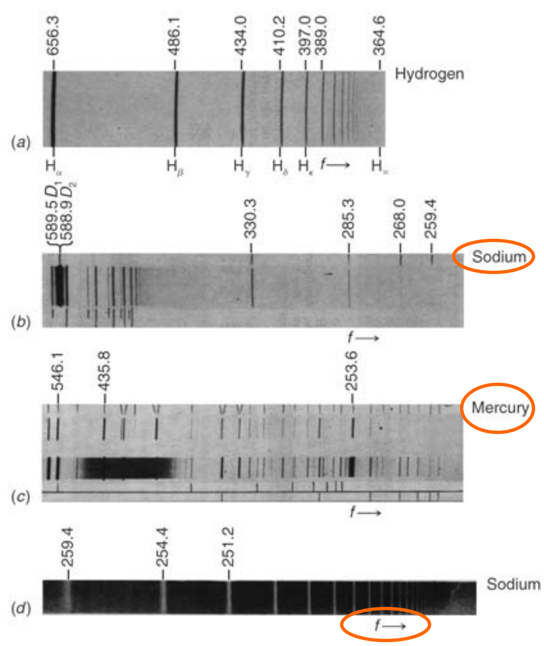
Kirchhoff' Experiment : "D" Lines in Na



D lines darken noticeably when Sodium vapor introduced
Between slit and prism

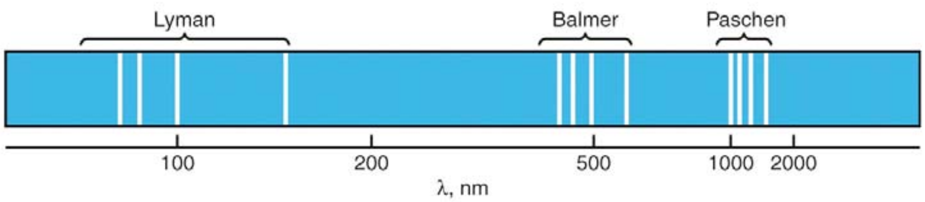
Emission & Absorption Line Spectrum of Elements

•Emission line appear dark because of photographic exposure



Absorption spectrum of Na
While light passed thru Na vapor
is absorbed at specific λ

Spectral Observations : series of lines with a pattern



- Empirical observation (by trial & error)
- All these series can be summarized in a simple formula

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right), n_f > n_i, n_i = 1,2,3,4..$$

Fitting to spectral line series data

$$R = 1.09737 \times 10^7 m^{-1}$$

How does one explain this ?

The Rapidly Vanishing Atom: A Classical Disaster !

Not too hard to draw analogy with dynamics under another Central Force

Think of the Gravitational Force between two objects and their circular orbits.

Perhaps the electron rotates around the Nucleus and is bound by their electrical charge

$$F = G\frac{M_1 M_2}{r^2} \Rightarrow k\frac{Q_1 Q_2}{r^2}$$

Laws of E&M destroy this equivalent picture : Why ?