Department of Physics
University of California
San Diego

Modern Physics (2D)
Prof. V. Sharma
Quiz # 8 (Mar 7 2003)

Some Relevant Formulae, Constants and Identities

$\lambda = \frac{h}{p}$; $\Delta x . \Delta p \geq \frac{h}{4\pi}$; $\Delta E . \Delta t \geq \frac{h}{4\pi}$

Time Dep. S. Eq: $-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} + U(x)\Psi(x,t) = i\hbar \frac{\partial \Psi(x,t)}{\partial t}$

Time Indep. S. Eq: $-\frac{\hbar^2}{2\mathrm{m}}\frac{\partial^2 \psi(x)}{\partial x^2} + U(x)\psi(x) = E\ \psi(x)$

$[\hat{p}] = \frac{\hbar}{i}\frac{d}{dx}$; $[p^2] = -\hbar^2 \frac{\partial^2}{\partial \mathrm{x}^2}$, $[\hat{\mathrm{K}}] = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial \mathrm{x}^2}$; $[\hat{\mathrm{E}}] = i\hbar \frac{\partial}{\partial \mathrm{t}}$

What to expect when expecting: $<Q> = \int_{-\infty}^{\infty} \Psi^*(x,t)[\hat{Q}]\Psi(x,t)dx$

Uncertainty in Observable Q : $\Delta Q = \sqrt{<Q^2> - <Q>^2}$

Quantum Oscillator in Ground State $\psi(x) = \left[\frac{m\omega}{\pi\hbar}\right]^{\frac{1}{4}} e^{\frac{m\omega x^2}{2\hbar}}$

Energy of Quantum Oscillator $\mathrm{E_n} = (n + \frac{1}{2})\hbar\omega$

Pl. write you answer in the Blue Book in indelible ink. Make sure your code number is prominently displayed on each page.

If you have trouble understanding the question, pl. ask the proctor

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Problem 1 : Go Jump Off A Quantum Cliff [12 pts] :

A beam of particles, each with energy E >0 is incident from the left (of x=0) on to a potential described by

U(x) = 0 for region I (x < 0)
U(x) = $-V_0$ for region II (x > 0)

(a) sketch the potential as a function of x. (b) Write down the expression for the time independent Schrodinger Eqn in each region I and II in terms of wavenumbers $k = \frac{\sqrt{2m(E-U(x))}}{\hbar}$ **(c) Find the general expression for** $\psi_I(x)$ & $\psi_{II}(x)$ in terms of k_1 & k_2 respectively. **(d) Reviewing the physics of the situation which component of** $\psi_{II}(x)$ **can be thrown out? Sketch the wave function in the two regions. Is the particle wavelength the same in the two regions? (e) Using the continuity conditions for** ψ & $\frac{d\psi}{dx}$**, calculate the expression for relative rates at which the particles are reflected (R) and transmitted (T) across the potential step in terms of** k_1 & k_2**. (f) Is the reflection rate the same in the quantum picture as in the classical picture? Explain the difference.**

Problem 2: "Lazy 'R Us" Is The Physics Mantra [8 pts]:

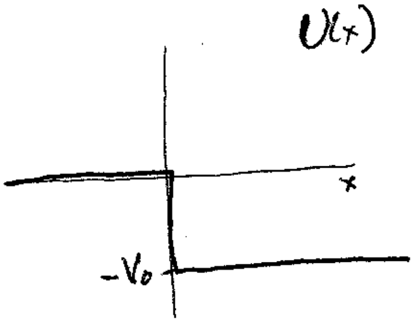
Consider a quantum Harmonic oscillator, of mass m under potential U(x)= $\frac{1}{2}m\omega^2x^2$**, in its ground state. For such a system you learnt in homework that** $\langle x \rangle = 0$ **and** $\langle x^2 \rangle = \frac{\hbar}{2m\omega}$ **. (a) Calculate the uncertainty Δx in its location x. (b) Now estimate <p>. (c) Write the expression for the total non-relativistic energy for this system and use it to relate** $\langle p^2 \rangle$ **to** $\langle x^2 \rangle$**. (d) Finally, calculate the uncertainty Δp and the value of the product Δx . Δp . How well does your calculation agree with Heisenberg's Uncertainty relation?**

"Mantra": A sacred word repeated in prayer or meditation.

Phys 2D Quiz 8 Solns

1]

a]



b] $k_I = \frac{\sqrt{2mE}}{\hbar}$, $k_{II} = \frac{\sqrt{2m(E+V_0)}}{\hbar}$

So Region I: $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = E\psi \Rightarrow \boxed{\frac{\partial^2\psi}{\partial x^2} = -k_I^2\psi}$

Region II: $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} - V_0\psi = E\psi \Rightarrow \boxed{\frac{\partial^2\psi}{\partial x^2} = -k_{II}^2\psi}$

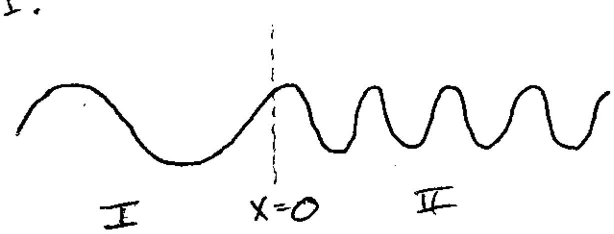
c] $$\boxed{\begin{aligned}\psi_I &= Ae^{ik_Ix} + Be^{-ik_Ix}\\ \psi_{II} &= Ce^{ik_{II}x} + De^{-ik_{II}x}\end{aligned}}$$

d] Can lose the $De^{-ik_{II}x}$ component, since nothing's reflected in II.

The wavelength is <u>not</u> the same in the 2 regions – since $k_{II} > k_I$,

$\lambda_{II} < \lambda_I$.

Sketch:



e] Continuous: $\psi_I(0)=\psi_{II}(0) \Rightarrow A+B=C$

Smoove: $\left.\frac{\partial\psi_I}{\partial x}\right|_{x=0}=\left.\frac{\partial\psi_{II}}{\partial x}\right|_{x=0} \Longrightarrow ik_I A - ik_I B = ik_{II} C$

So $B=\frac{k_I-k_{II}}{k_I+k_{II}}A$, $C=\frac{2k_I}{k_I+k_{II}}A$

$$\boxed{R=\frac{|B|^2}{|A|^2}=\frac{(k_I-k_{II})^2}{(k_I+k_{II})^2}} \qquad \boxed{T=1-R=\frac{4k_Ik_{II}}{(k_I+k_{II})^2}}$$

f] No, in the classical picture $R=0$!
So not even sorta the same.

2] a] $\Delta x=\sqrt{\langle x^2\rangle-\langle x\rangle^2}=\boxed{\left(\frac{\hbar}{2m\omega}\right)^{1/2}}$

b] $\boxed{\langle p\rangle=0}$ since the thing is oscillating
Could calculate this too.

c] $\frac{p^2}{2m}+\frac{1}{2}m\omega^2x^2=E$, so $\boxed{\langle p^2\rangle=2m\left(\langle E\rangle-\frac{1}{2}m\omega^2\langle x^2\rangle\right)}$

d] $\langle p^2\rangle=2mE-m^2\omega^2\cdot\frac{\hbar}{2m\omega}=2m\cdot\frac{1}{2}\hbar\omega-\frac{1}{2}\hbar m\omega=\frac{1}{2}\hbar m\omega$

So $\Delta p=\sqrt{\langle p^2\rangle-\langle p\rangle^2}=\boxed{\left(\frac{\hbar}{2}m\omega\right)^{1/2}} \Rightarrow \boxed{\Delta x\Delta p=\frac{\hbar}{2}}$ Agrees with HUP! (It's the minimum uncertainty).