



Department of Physics
University of California San Diego

Modern Physics (2D)
Prof. V. Sharma
Quiz # 6 (Feb 21 2003)

Some Relevant Formulae, Constants and Identities

$$E = \gamma mc^2; K = \gamma mc^2 - mc^2; p = \gamma mu$$

$$\lambda = \frac{h}{p}$$

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

$$\Delta E \cdot \Delta t \geq \frac{h}{4\pi}$$

$$\text{Bragg Scattering : } n\lambda = 2d\sin\theta$$

$$\text{Compton Scatter: } \Delta\lambda = \left(\frac{h}{m_e c} \right) (1 - \cos\theta) = (0.0243 \text{ \AA}) (1 - \cos\theta)$$

$$\text{Planck's Constant } h = 6.626 \times 10^{-34} \text{ J}\cdot\text{s} = 4.136 \times 10^{-15} \text{ eV}\cdot\text{s}$$

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$$

$$\text{Electron Mass} = 9.1 \times 10^{-31} \text{ Kg} = 0.511 \text{ MeV}/c^2$$

$$\text{Neutron Mass} = 939.6 \text{ MeV}/c^2 = 1.675 \times 10^{-27} \text{ Kg}$$

$$\text{Proton Mass} = 938.3 \text{ MeV}/c^2 = 1.673 \times 10^{-27} \text{ Kg}$$

$$\text{Speed of Light in Vacuum } c = 2.998 \times 10^8 \text{ m/s}$$

Pl. write you answer in the Blue Book in indelible ink. Make sure your code number is prominently displayed on each page.

If you have trouble understanding the question, pl. ask the proctor



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Problem 1 : The More You Know Now, The Less You Know Later [10 pts] :

You made a measurement that establishes the position of a proton with an accuracy of $\pm 1.0 \times 10^{-11}$ m. (a) calculate the uncertainty in the proton's momentum measurement as a result of your measurement (b) calculate the uncertainty in the proton's velocity. (c) do you need to use the non-relativistic or the relativistic form of the expression for momentum, and why? (d) Calculate the uncertainty in the proton's position 1.0 s later, How does it compare with the size of a nucleus (10 fm)?

Problem 2: Planck Guessed Right! [10 pts]:

The frequency of oscillation of a harmonic oscillator of mass m and

spring constant k is $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$. The

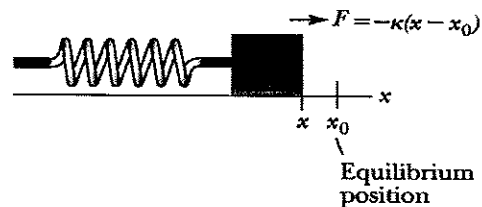
total energy of the oscillator

$$E = \frac{p^2}{2m} + \frac{kx^2}{2} \text{ where } p \text{ is its momentum}$$

when its displacement from the equilibrium is x . In Classical physics, the minimum energy of the oscillator is $E=0$, but things are different in the quantum world. Using the Uncertainty relations

(a) write an expression for an estimate of the total energy E in terms of x only (b) What is the value of x^2 which minimizes the energy E ? (c) Now calculate the Energy E and show that $E_{\min} \cong$

$$\frac{1}{2} hf .$$



Physics 2D Quiz 6

a) $\Delta x = 10^{-11} \text{ m}$.

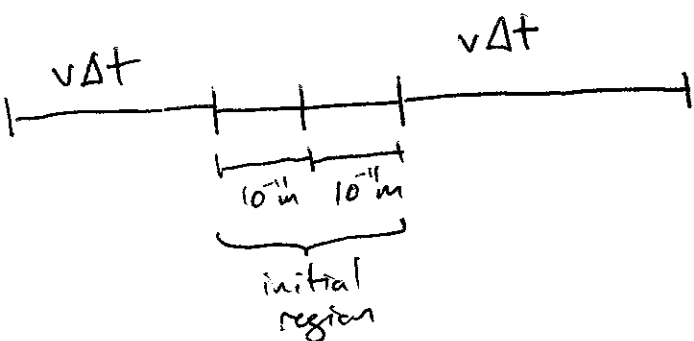
$$\text{So } \Delta p = \frac{h}{2\Delta x} = \boxed{5.27 \times 10^{-24} \frac{\text{kg}\cdot\text{m}}{\text{s}}} = 9.9 \times 10^{-3} \frac{\text{MeV}}{c}$$

b) This is a small momentum, so can use $p = mv$.

$$\Delta v = \frac{\Delta p}{m} = \boxed{3150 \text{ m/s}}$$

c) Non-rel. is OK. That's a pretty small velocity we just found (compared to the speed of light).
Or could check that $KE \ll \text{rest energy}$.

d) Worst case scenario: The proton travels with $v = 3150 \text{ m/s}$ in the same direction the whole time, starting at the edge of its initial region:



$$\text{So the new } \Delta x = 10^{-11} \text{ m} + v\Delta t$$

$$v\Delta t = 3150 \text{ m, so}$$

$$\boxed{\Delta x = 3150 \text{ m}}$$

That's much bigger than the size of a nucleus!

2a) Using $p = \langle p \rangle + \Delta p \approx \Delta p$ (since $\langle p \rangle = 0$)

$$\Delta p = \Delta p = \frac{h}{2(\Delta x)}$$

Now, saying $x \approx \Delta x$ yields

$$E = \frac{p^2}{2m} + \frac{1}{2} k x^2 = \boxed{\frac{h^2}{8m x^2} + \frac{1}{2} k x^2}$$

b) $\frac{\partial E}{\partial x^2} = -\frac{h^2}{8m(x^2)^2} + \frac{1}{2}k = 0$

$$\Rightarrow (x^2)^2 = \frac{h^2}{8m k} \cdot 2 = \frac{h^2}{4mk}$$

$$\text{So } \boxed{x^2 = \frac{h}{2\sqrt{mk}}}$$

(Could also do $\frac{\partial E}{\partial x}$, and solve for x^2)

c) Plugging in gives

$$E = \frac{h^2 \cdot 2\sqrt{mk}}{8m \cdot h} + \frac{1}{2} \frac{h k}{2\sqrt{mk}} = h \sqrt{\frac{k}{m}} \cdot \frac{1}{2} = \boxed{\frac{1}{2} h F} \quad \underline{\text{Done!}}$$