Department of Physics
University of California San Diego

Modern Physics (2D)
Prof. V. Sharma
Quiz # 2 (Jan 24 2003)

Some Relevant Formulae, Constants and Identities

$\gamma = \left[1-(u/c)^2\right]^{-1/2}$

$p = \gamma m u$

$K = \gamma mc^2 - mc^2$

$E = KE + mc^2 = \gamma mc^2$

Centripetal Acc.= $\frac{u^2}{r}$

$p = \gamma m u = qBR$

$E^2 = (pc)^2 + (mc^2)^2$

1 eV = 1.60×10^{-19} J

Electron Mass = 8.2×10^{-14} J = 0.511 MeV

Speed of Light in Vaccum c = 2.998×10^8m/s

Electron Charge = 1.602×10^{-19} C

Atomic mass unit u = 1.6605×10^{-27}kg = 931.49 MeV/c^2

Pl. write you answer in the Blue Book in indelible ink. Make sure your code number is prominently displayed on each page.

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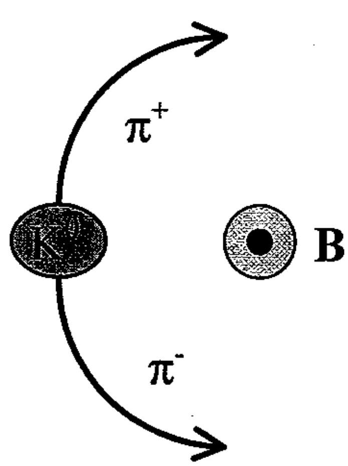
Problem 1: **<u>Fundamentals Of A Circular Accelerator</u>** [10 pts]
A particle of charge q and mass m moves in a circular orbit with a fixed speed v in the presence of a constant magnetic field B. Show that the frequency of its orbital motion is $f = \frac{qB}{2\pi m}\left(\sqrt{1-(v/c)^2}\right)$.
(Hint: The period is the amount of time it takes for the particle to go around the circle once, and the frequency is the reciprocal of the period).

Problem 2 **<u>: A Particle From The Early Universe</u>** [10 pts]
The K^0 meson is an uncharged particle that decays into two charged pions according to $K^0 \rightarrow \pi^+ + \pi^-$. The pions have opposite charge but the same mass, m_π=140 MeV/c^2. The magnitude of the charge of either pion is the same as the magnitude of the charge of an electron. Suppose a K^0 meson at rest decays into two pions in a detector in which a magnetic field B = 2T is present. The pions move in a plane perpendicular to the B field. If the radius of curvature of the pions is 34.4cm, find (a) the momenta of the pions (in units of MeV/c) (b) speed of the pions (c) the mass of the K^0 meson in units of MeV/c^2



Physics 2D Quiz 2 Solns

1] $\vec{F} = q\vec{v}\times\vec{B}$, so $F = qvB$ here $(\vec{v}\perp\vec{B})$

Also, $\vec{F} = \frac{\partial \vec{p}}{\partial t} = \frac{\partial}{\partial t}(\gamma m\vec{v}) = \gamma m\frac{\partial \vec{v}}{\partial t}$ (since $v=$const, $\frac{\partial \gamma}{\partial t}=0$)

so $F = \gamma m\frac{v^2}{r}$, where r is the radius of the circle.

Period = Time for one cycle = $\frac{2\pi r}{v}$, so $f = \frac{v}{2\pi r}$

Solve for r using $qvB = \frac{\gamma m v^2}{r} \Rightarrow r = \frac{\gamma m v}{qB}$

Thus $f = \frac{v}{2\pi r} = \frac{v}{2\pi}\frac{qB}{\gamma m v} = \boxed{\frac{qB}{2\pi m}\sqrt{1-\left(\frac{v^2}{c^2}\right)}}$

2] a] $p = qBr = (1.6\times10^{-19})(2)(0.344)\,\frac{kg\cdot m}{s}$

$= 1.1\times10^{-19}\,\frac{kg\cdot m}{s}$.

To convert, use $1\,\frac{MeV}{c} = \frac{(1.6\times10^{-19})\times10^{6}}{3\times10^{8}}\,\frac{kg\cdot m}{s} = 5.33\times10^{-22}\,\frac{kg\cdot m}{s}$

So $\boxed{p = 206.25\,\frac{MeV}{c}}$

b] $p = \gamma m v$, so $p^2 = \frac{m^2 v^2}{1 - v^2/c^2}$

$$\Rightarrow p^2 - \frac{v^2 p^2}{c^2} = m^2 v^2$$

$$\Rightarrow v^2\left(m^2 + \frac{p^2}{c^2}\right) = p^2 \Rightarrow v = \frac{p}{\left(m^2 + \frac{p^2}{c^2}\right)^{1/2}}$$

Use $m = 140\ \mathrm{MeV}/c^2$ and $p = 206.25\ \frac{\mathrm{MeV}}{c}$ to get

$$\boxed{v = 0.83c}$$

c] Energy cons: $E_K = E_{\pi^+} + E_{\pi^-}$

$E_K = M_K c^2$ and $E_{\pi^+} = E_{\pi^-} = \sqrt{p^2c^2 + m_\pi^2 c^4}$

So $M_K c^2 = 2\sqrt{p^2c^2 + m_\pi^2 c^4}$

Use p, m_π from above to get $\boxed{M_K = 498.6\ \frac{\mathrm{MeV}}{c^2}}$