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Modern Physics (2D)
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Quiz #6 (Nov 14)

Problem 1 : Matter Waves [10 pts]

A beam of electrons is incident on a slit of variable width. If it is possible to resolve a 1% difference in momentum, what slit width would be necessary to resolve the interference pattern of electrons if their kinetic energy is (a) 0.010 MeV (b) 1.0 MeV and (c) 100 MeV?

Problem 2: Out of Sight ! [10 pts]

A completely free electron in empty space is measured to have a location within a sphere of radius $R=1.0\times10^{-14}$ m, typical of an atomic nucleus. (a) Within what radius can you say with assurance that the electron will be found after 1.0s ? (b) Repeat the problem for an electron initially measured to lie within a sphere of radius $R=1.0\times10^{-10}$ m, the radius of an atom.

Hint: Should you use non-relativistic expressions for electron momentum and energy?

(1) We can resolve a 1% difference in momentum $\Rightarrow \frac{\Delta p}{p} = 0.01$, $\Delta p = (0.01)p$

the electrons have kinetic energy (a) 0.010 MeV

(b) 1.0 MeV

(c) 100 MeV

first, let's assume the uncertainty principle is minimally satisfied

$$\Rightarrow \Delta x \Delta p = \hbar/2$$

$$\Delta x = \frac{\hbar}{2\Delta p} = \frac{\hbar}{2(0.01)p}$$

and we take the uncertainty in its position to be the slit width

$$a = \Delta x$$

$$\Rightarrow a = \frac{\hbar}{2(0.01)p}$$

so we must solve for p in order to get the slit width, a.

We know the kinetic energy, so let's solve for γ

$$K = (\gamma - 1)mc^2 \quad \Rightarrow \quad \gamma = 1 + \frac{K}{mc^2}$$

$$\text{so} \quad \gamma = \begin{cases} \text{(a) } 1.02 \\ \text{(b) } 2.96 \\ \text{(c) } 197 \end{cases}$$

now solve for v,

$$\gamma = \frac{1}{\sqrt{1-(\frac{v}{c})^2}} \quad \Rightarrow \quad 1-(\frac{v}{c})^2 = \frac{1}{\gamma^2}$$

$$(\frac{v}{c})^2 = 1 - \frac{1}{\gamma^2}$$

$$\Rightarrow \quad v = c\sqrt{1-\frac{1}{\gamma^2}}$$

so $$v = \begin{cases} (a) & 5.91 \cdot 10^{7}\ m/s \\ (b) & 2.82 \cdot 10^{8}\ m/s \\ (c) & 3 \cdot 10^{8}\ m/s \end{cases}$$

and now we have for momentum

$$p = \gamma m v = \begin{cases} (a) & 5.49 \cdot 10^{-23}\ kg \cdot m/s \\ (b) & 7.6 \cdot 10^{-22}\ kg \cdot m/s \\ (c) & 5.38 \cdot 10^{-20}\ kg \cdot m/s \end{cases}$$

plug this into

$$a = \frac{\hbar}{2(0.01)p} = \begin{cases} (a) & 9.6 \cdot 10^{-11}\ m \\ (b) & 6.94 \cdot 10^{-12}\ m \\ (c) & 9.8 \cdot 10^{-14}\ m \end{cases}$$

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Another way to solve for momentum is by considering the relation

$$E = K + mc^2 = \sqrt{(pc)^2 + (mc^2)^2}$$

square both sides

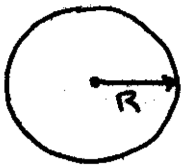
$$[K + mc^2]^2 = (pc)^2 + (mc^2)^2$$

so $$(pc)^2 = K^2 + 2K(mc^2)$$

$$(pc) = \sqrt{K^2 + 2K(mc^2)} = \begin{cases} 0.10\ MeV \\ 1.42\ MeV \\ 100.5\ MeV \end{cases}$$

which yields the same values as above after converting units.

(2) the electron is somewhere within a sphere of radius R, but we do not where exactly, so let's take its uncertainty in position to be



$$\Delta x = 2R$$ (note: if you took $\Delta x = R$, that's fine, we are only interested in an order of magnitude estimate)

by the uncertainty principle,

$$\Delta x \, \Delta p \geq \hbar/2$$

assume this is minimally satisfied, then

$$\Delta p = \frac{\hbar}{2\Delta x} = \frac{\hbar}{2(2R} = \frac{\hbar}{4R}$$

to get an answer in units eV/c, multiply & divide by c

$$\Delta p = \frac{\hbar}{4R}\frac{c}{c} = \left(\frac{\hbar c}{4R}\right)\frac{1}{c}$$

for $R = 1 \cdot 10^{-14}$ m

$$\Delta p = 4.9365 \text{ MeV}/c$$

so $\Delta p c > mc^2$, & we need to use relativity.

$$\Delta p = \gamma m v = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} m v \quad , \text{ or } \quad \frac{v}{c} = \frac{\Delta p c}{E} = \frac{\gamma m v c}{\gamma m c^2}$$

solving for the velocity we find

$$\left(\frac{v}{c}\right) = \frac{(\Delta p c)}{\sqrt{(mc^2)^2 + (\Delta p c)^2}} = 0.995$$

so after 1 s, the electron travels a distance

$$d = vt = (0.995\ c)(1\text{ s}) = 2.99 \cdot 10^8 \text{ m}$$

so the electron will be within a sphere of radius $R = 2.99 \cdot 10^8$ m

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(if your answer was close to this using the correct methods you get full credit)

(b) for $R = 1 \cdot 10^{-10}$ m

$$\Delta p = \left(\frac{\hbar c}{4R}\right)\frac{1}{c} = 4.9365 \cdot 10^{-4} \ \mathrm{MeV}/c$$

so $\Delta p c \ll mc^2$ and we can use non-relativistic physics

$$\Delta p = mv \quad \Rightarrow \quad v = \frac{\Delta p}{m} = \frac{\Delta p}{m}\left(\frac{c^2}{c^2}\right) = \left(\frac{\Delta p c}{mc^2}\right) c$$

so $v = 2.898 \cdot 10^5$ m/s

and $d = vt = 2.898 \cdot 10^5$ m

the electron lies within a radius of $2.898 \cdot 10^5$ m