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Modern Physics
(2D)
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Quiz # 5 (Nov 7)

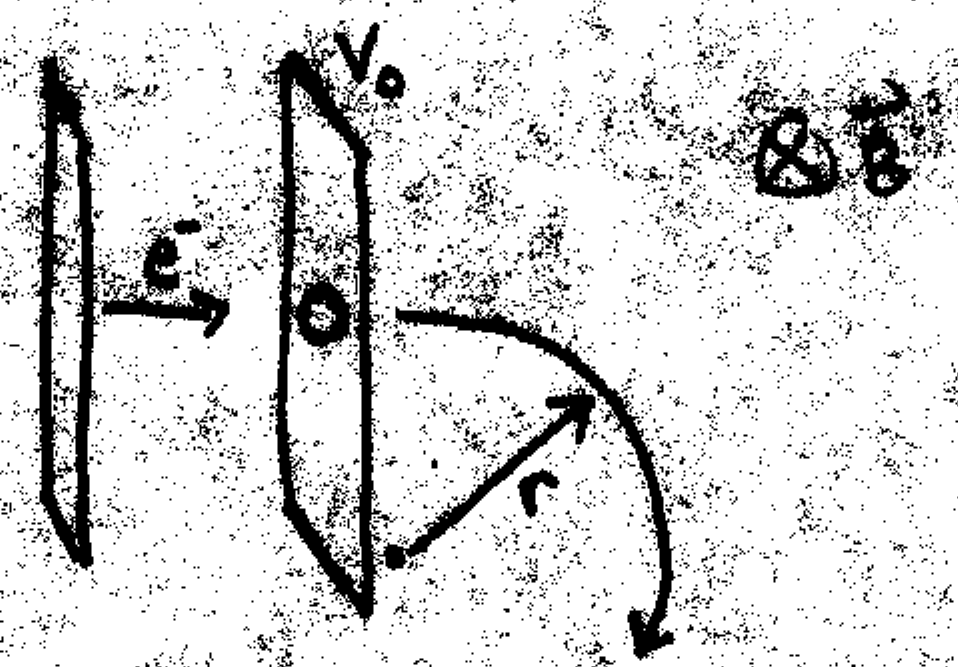
Problem 1 : A Similar But Different Experiment [10 pts]

In a different version of J.J. Thomson's famous measurement of electron's e/m , non-relativistic electrons are accelerated (from rest) thru a measured potential difference V_0. They are then passed into a region of known magnetic field B and the radius R of their circular orbit is measured. (a) Draw a schematic diagram of the setup (b) Write an expression for the kinetic energy K of the electron (in terms of V_0) as they enter the region of B field. (c) Write an expression for the orbit radius R in terms of the charge q , mass m and the speed u of the electron and the external B field. (d) Eliminate speed u from results of (b) and (c) and derive an expression for e/m in terms of the experimentally measurable quantities V_0, B and R.

Problem 2: A Funky kind of Atom [10 pts]

Positronium is a Hydrogen like atom consisting of a positron (a positively charged antimatter form of electron) and an electron revolving around each other. Using the ideas developed in the Bohr model, write expressions for (a) the total energy of the "atom" (b) Expression for Newton' second law and equality of forces (c) Expression for kinetic energy K in terms of the Coulomb constant ***k*** , electron charge ***e*** and the orbit radius ***r*** (c) The allowed radii (relative to the center of mass of the two particles. (d) The allowed energy levels of the system. (e) Calculate the ground state energy for this system in the units of ***eV***. Is the ***Positronium*** atom more tightly bound than the Hydrogen atom?

(1) (a)



(b) the electrons are accelerated through a potential difference V_0 from rest, so

$$K = eV_0$$

(c) a centripetal force provided by the magnetic field puts the electron into uniform circular motion, so

$$F_{centripetal} = F_B$$

$$m\frac{v^2}{r} = evB$$

$$\Rightarrow \boxed{r = \frac{mv}{eB}}$$

(d) from (b) $K = \frac{1}{2}mv^2 = eV_0$

$$\Rightarrow v = \sqrt{\frac{2eV_0}{m}}$$

put into equation for radius of orbit

$$r = \frac{m}{eB}\sqrt{\frac{2eV_0}{m}}$$

square both sides

$$r^2 = \left(\frac{m}{e}\right)^2 \frac{1}{B^2}\left(\frac{2eV_0}{m}\right) = \left(\frac{m}{e}\right)\frac{2V_0}{B^2}$$

$$\Rightarrow \boxed{\frac{e}{m} = \frac{2V_0}{B^2 r^2}}$$

(2) (a) $E = \frac{1}{2}\mu v^2 - \frac{ke^2}{r}$

we use reduced mass $\mu = \frac{m_e m_e}{m_e + m_e} = \frac{m_e}{2}$

(b) centripetal force = coulomb force

$$\frac{\mu v^2}{r} = \frac{Ke^2}{r^2}$$

(c) from (b)

$$\mu v^2 = \frac{Ke^2}{r}$$

$$\Rightarrow \boxed{\frac{1}{2}\mu v^2 = \frac{1}{2}\frac{Ke^2}{r}}$$

in Bohr model, we have angular momentum quantization

$$\mu v r = n\hbar$$

$$r = \frac{n\hbar}{\mu v}$$

from (b) $\frac{\mu v^2}{r} = \frac{ke^2}{r^2}$

so $\mu v^2 r = ke^2$

and thus $(\mu v r)\, v = ke^2$

use angular momentum conservation $\mu v r = n\hbar$

$$(n\hbar)\, v = ke^2$$

$$\Rightarrow v = \frac{ke^2}{n\hbar}$$

plug this into the expression for the radius

$$r = \frac{n\hbar}{\mu v} = \frac{n\hbar}{\mu}\left(\frac{n\hbar}{ke^2}\right) = \frac{n^2\hbar^2}{\mu ke^2}$$

$$\boxed{r = \frac{n^2\hbar^2}{\mu ke^2}}$$

(d) from (a)

$$E = \frac{1}{2}\mu v^2 - \frac{ke^2}{r}$$

from (c)

$$\frac{1}{2}\mu v^2 = \frac{1}{2}\frac{ke^2}{r}$$

so

$$E = -\frac{1}{2}\frac{ke^2}{r} = -\frac{1}{2}\frac{\mu k^2 e^4}{n^2\hbar^2}$$

$$\boxed{E = -\frac{1}{2}\left(\frac{\mu k^2 e^4}{\hbar^2}\right)\frac{1}{n^2}}$$

(e) $\mu = \frac{m_e}{2}$ $\Rightarrow$ $E = -\left(\frac{13.6\,\text{eV}}{2}\right)\frac{1}{n^2} = \frac{-6.8\,\text{eV}}{n^2}$

positronium is not bound as tightly as Hydrogen ($-6.8 > -13.6\,\text{eV}$ so it takes less energy to dissociate positronium constituents)