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Modern Physics (2D)
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Quiz#3 (Oct 17 2003)

**Problem 1: Mystery Subatomic Particle [12 pts]**

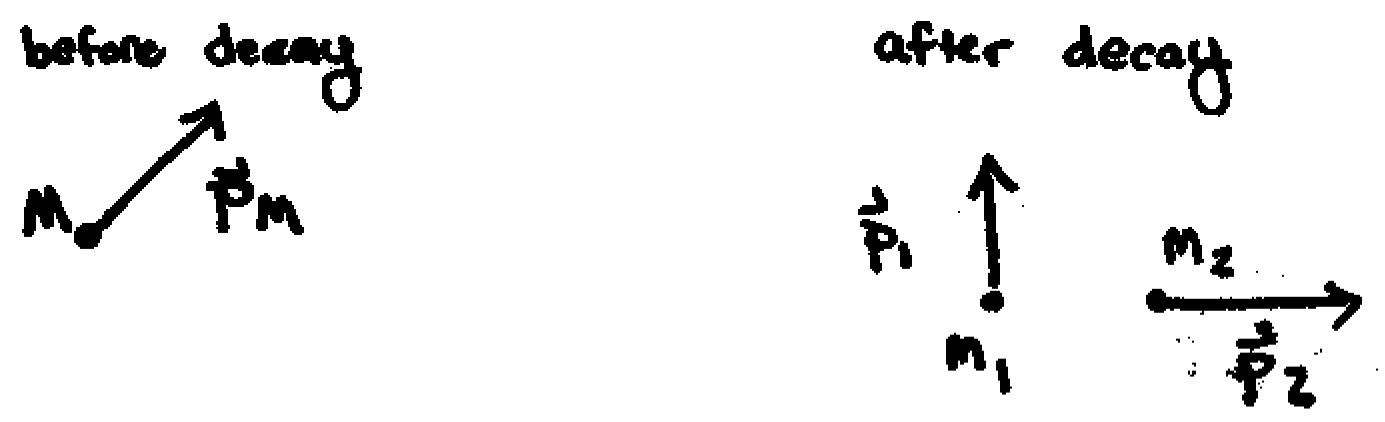
A particle of unknown mass M decays into two particles of known masses $m_1 = 0.5$ GeV/$c^2$ and $m_2 = 1.0$ GeV/$c^2$ whose momenta are measured to be $\mathbf{p}_1$=2 GeV/c along the positive y direction and $\mathbf{p}_2 = 1.5$ GeV/c along the positive x direction (1 GeV = $10^9$ eV). (a) Write down the expressions for conservation of energy and momenta before and after the decay (b) Find the unknown mass M and (b) the speed u with which it was traveling when it decayed. Pl. write down each step in your calculation clearly.

**Problem 2: The Need For Speed [8 pts]**

How much work has to be done on a proton to accelerate it (a) from rest to 0.01c (b) from 0.80c to 0.81c (c) 0.90c to 0.99c and finally (d) 0.99c to c? (e) Which particle has a higher kinetic energy: an electron moving with v=0.80c or a proton moving with v=0.80c ?

# QUIZ 3

(1)




(b) both energy and momentum are conserved

$$\vec{p}_M = \vec{p}_1 + \vec{p}_2 = 2\,\frac{GeV}{c} \uparrow + 1.5\,\frac{GeV}{c} \rightarrow$$

$p_{M_x} = 1.5\ GeV/c$

$p_{M_y} = 2\ GeV/c$

So $|\vec{p}_M| = \sqrt{\left(2\frac{GeV}{c}\right)^2 + \left(1.5\frac{GeV}{c}\right)^2} = 2.5\ \frac{GeV}{c}$

using this in the equation for energy conservation, we can solve for the mass $M$ of the decaying particle:

$$E_M = E_1 + E_2$$

$$\sqrt{(p_M c)^2 + (Mc^2)^2} = \sqrt{(p_1 c)^2 + (m_1 c^2)^2} + \sqrt{(p_2 c)^2 + (m_2 c^2)^2}$$

so

$$(Mc^2)^2 = \left[\sqrt{(p_1 c)^2 + (m_1 c^2)^2} + \sqrt{(p_2 c)^2 + (m_2 c^2)^2}\right]^2 - (p_M c)^2$$

So $M = 2.95\ GeV/c^2$

(c) using our result for $p_M$, we can solve for the speed of the decaying particle:

$$p_M = \gamma M u$$

$$\left(\frac{p_M}{M}\right)^2 = \frac{u^2}{1-\left(\frac{u}{c}\right)^2}$$

$$\Rightarrow \quad u^2\left[1 + \left(\frac{p_M}{Mc}\right)^2\right] = \left(\frac{p_M}{M}\right)^2$$

you can also use

$E_M = \gamma_M M c^2$

$p_M = \gamma_M M u$

so $\dfrac{p_M}{E_M} = \dfrac{u}{c^2}$

$\dfrac{u}{c} = \dfrac{p_M c}{E_M} = 0.65$

dividing both sides by $c^2$, we obtain

$$\frac{u^2}{c^2} = \frac{\left(\frac{P_M}{Mc}\right)^2}{\left[1 + \left(\frac{P_M}{Mc}\right)^2\right]} \Rightarrow u = 0.65\,c$$

(2) the work done on the proton is equal to the change in Kinetic Energy of the proton,

(a)
$$W = K_f - K_i = (\gamma_f - 1)mc^2 - (\gamma_i - 1)mc^2$$
$$= (\gamma_f - \gamma_i)mc^2$$
$$= \left(\frac{1}{\sqrt{1-\left(\frac{u_f}{c}\right)^2}} - \frac{1}{\sqrt{1-\left(\frac{u_i}{c}\right)^2}}\right)mc^2$$

$u_f = 0.01c$, $u_i = 0$

so $W = (938.3469 - 938.3)\text{ MeV} = 0.0469\text{ MeV}$

(b) $u_f = 0.81c$, $u_i = 0.80\,c$

$W = 36.2\text{ MeV}$

(c) $u_f = 0.99c$, $u_i = 0.90\,c$

$W = 4498.8\text{ MeV}$ (it takes more work as the particle velocity approaches the speed of light)

(d) $u_f = c$, $u_i = 0.99c$

$W = \infty$ (you cannot accelerate a massive particle to the speed of light, no matter how great the force is that you apply to it)

(e) $K_p > K_e$ for the same speed since $\gamma_p = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \gamma_e = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$

and $m_p > m_e \Rightarrow (\gamma_p - 1)m_pc^2 > (\gamma_e - 1)m_ec^2$

If 2 particles are travelling at the same speed, the more massive has greater K.E.