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Modern Physics (2D)
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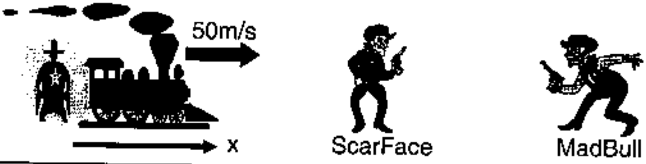
Quiz#2 (Oct 10 2003)

Problem 1: Wild Wild West ! [10 pts]



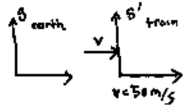
In the old west, a sheriff riding on a train traveling 50m/s sees a shootout between two bandits standing on the earth 50m apart parallel to the train. The sheriff's instruments indicate that in his reference frame the two men fired simultaneously. (a) Which of the two bandits, the first one the train passes from the left (ScarFace) or the second one (MadBull) sould be arrested for firing the first shot? (b) how much earlier did he fire? (c) who was struck first?
Hint: Deduce your answers from the Lorentz transformation rules.

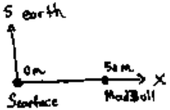
Problem 2: Fast, Very Very Fast ! [10 pts]

Some "Cosmic ray" protons coming from deep space have an astounding kinetic energy K of 3.0×10^{20} eV. This is enough energy to "warm" your brain cells by a quite a few degrees. Using proton mass of 938.0 MeV, (a) calculate the Lorentz factor γ and (b) the speed u of this cosmic messenger with respect to the ground based detector?
Hint: Correct answer is not $u = c$!

QUIZ 2

(1)





in the earth frame, the positions of the bandits are

$$x_S = 0\,m \quad , \; x_M = 50\,m$$

(for simplicity we place Scarface at the origin and MadBull 50 m away to the right)

in the frame S', the bandits are observed to fire at the same time

$$\Rightarrow \; t'_S = t'_M$$

let us find the time lapse between gun blasts as measured in the earth frame $\boxed{S}$. Use Lorentz transformation

$$t'_S = \gamma\left(t_S - v\frac{x_S}{c^2}\right)$$

$$t'_M = \gamma\left(t_M - v\frac{x_M}{c^2}\right)$$

$$\Delta t' = t'_S - t'_M = \gamma\left(t_S - v\frac{x_S}{c^2}\right) - \gamma\left(t_M - v\frac{x_M}{c^2}\right)$$

$$\Delta t' = \gamma\,\Delta t - \frac{v\gamma}{c^2}\Delta x$$

but $\Delta t' = 0$

$$\Rightarrow \; \Delta t = v\frac{\Delta x}{c^2} = \frac{v}{c^2}(x_S - x_M)$$

$$t_S - t_M = -2.77 \cdot 10^{-14}\,s$$

the negative sign tells us that $t_S < t_M$, so Scarface fires his gun first.

(b) from the calculation we see that Scarface fires $2.77 \cdot 10^{-14}$ s earlier than MadBull.

(c) In the earth frame from (b) it is clear that MadBull will be struck first. (This is sufficient for full credit)

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NOT NECESSARY (but you may interested)

does the sheriff in the train see that MadBull is struck first?

We know MadBull is struck first in the earth frame, so let's call t_{M_f} the time in the earth frame at which MadBull is hit, and t_{S_f} the time at which Scarface is hit in this frame. Clearly $t_{M_f} < t_{S_f}$ (MadBull is struck first in earth frame). Now let's Lorentz transform to the train,

$v = 50$ m/s

$$t'_{M_f} = \gamma\left(t_{M_f} - \frac{v x_M}{c^2}\right) = \gamma\left(t_{M_f} - \frac{(50\text{m})v}{c^2}\right)$$

$$t'_{S_f} = \gamma\left(t_{S_f} - \frac{v x_S}{c^2}\right) = \gamma(t_{S_f}) \qquad x_S = 0$$

using $t_{M_f} < t_{S_f}$ we obtain $t'_{M_f} < t'_{S_f}$

So the Sheriff will see MadBull struck first as well

(2) $K = (\gamma - 1) m c^2$

(a) solve for γ

$$\frac{K}{mc^2} = \gamma - 1$$

$$\gamma = 1 + \frac{K}{mc^2} = 0.32 \cdot 10^{12} \quad \text{or} \quad 3.2 \cdot 10^{11}$$

(b) $$\gamma = \frac{1}{\sqrt{1 - \left(\frac{u}{c}\right)^2}} \Rightarrow 1 - \left(\frac{u}{c}\right)^2 = \frac{1}{\gamma^2}$$

$$\left(\frac{u}{c}\right)^2 = 1 - \frac{1}{\gamma^2}$$

$$\left(\frac{u}{c}\right) = \sqrt{1 - \frac{1}{\gamma^2}}$$

your calculator will show $\frac{u}{c} = 1$, but it's actually $\frac{u}{c} \sim 0.99999\ldots$ since a proton can never go the speed of light

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{(1-\beta)}\sqrt{1+\beta}}$$

now $\beta = \frac{u}{c} \sim 1$ so $1 + \beta \sim 2$

$$\Rightarrow \gamma \sim \frac{1}{\sqrt{2}\sqrt{1-\beta}}$$

solving for β, we obtain

$$1-\beta = \frac{1}{2\gamma^2} \approx 5 \cdot 10^{-24}$$

so we would have to go to the 24TH decimal point to see the difference between u and c.

$$\frac{u}{c} = 0{,}999\ 999\ 999\ 999\ 999\ 999\ 999\ 995$$

24TH position