	Department of Physics University of California San Diego	Modern Physics (2D) Prof. V. Sharma Quiz # 7

Some Relevant Formulae, Constants and Identities

$\lambda = \frac{h}{p}$; $\Delta x.\Delta p \geq \frac{h}{4\pi}$; $\Delta E.\Delta t \geq \frac{h}{4\pi}$

Time Dep. S. Eq: $-\frac{\hbar^2}{2m}\frac{d^2\Psi(x,t)}{dx^2} + U(x)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$

Time Indep. S. Eq: $-\frac{\hbar^2}{2\mathrm{m}}\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\ \psi(x)$

Particle in box of length L: $\psi_n(0<x<L) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$ & $\mathrm{E_n} = \frac{\mathrm{n}^2\pi^2\hbar^2}{2mL^2}$

Planck's constant h = 6.626×10^{-34} J.s $=4.136\times10^{-15}$ eV.s

1 eV = 1.60×10^{-19} J

Electron mass = 9.1×10^{-31} Kg = 0.511 MeV/c^2

Energy in Hydrogen atom $\mathrm{E_n} = \frac{-ke^2}{2a_0}\left(\frac{1}{n^2}\right) = \left(\frac{-13.6\text{ eV}}{n^2}\right)$

$\sin\alpha\sin\beta = \frac{1}{2}\left(\cos(\alpha-\beta) - \cos(\alpha+\beta)\right)$

$\int \sin^2 x\, dx = \ = -\frac{1}{2}\cos x \sin x + \frac{1}{2}\mathrm{x} \ = \ \frac{1}{2}x - \frac{1}{4}\sin 2x$

Pl. write you answer in the Blue Book in indelible ink. Make sure your code number is prominently displayed on each page. Ask the proctor if you do not understand the question.

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Problem 1: Lazy "R" Us ! [12 pts]

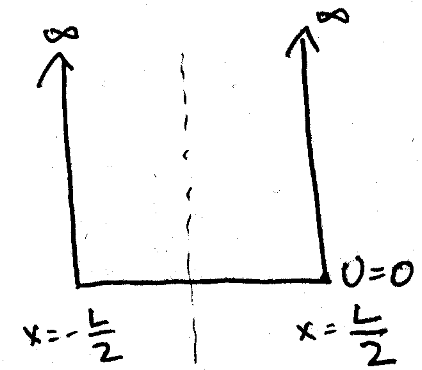
Consider a particle of mass m moving in a one-dimensional box with rigid walls (infinite potential) at $x = -L/2$ and $x = L/2$. (a) Draw this potential form (b) Find the normalization constant , the complete wavefunctions and probability densities for state $n = 1$, $n = 2$ & $n = 3$. (c) Neatly sketch the wavefunctions and probability densities in each case. Indicate the locations of the walls.

Problem 2: Hydrogen Atom Vs Particle In A Rigid Box [8 pts]

When a Hydrogen atom undergoes a transition from n=2 to n=1 level, a photon with λ=122nm is emitted. (a) If the atom is modeled as an electron trapped in a one-dimensional box, what is the width of the rigid box in order for the n=2 to n=1 transition to correspond to emission of a photon of this energy? (b) For a box with the width calculated in (a) , what is the ground state energy? (c) by how many eV is this energy different from the ground state energy of the Hydrogen atom?

Physics 2D Quiz 7

1a]



b] We know the answer for the well that goes from $x=0$ to $x=L$, so just translate the solutions!

$$\psi_n\left(-\tfrac{L}{2}<x<\tfrac{L}{2}\right)=\sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}\left(x+\frac{L}{2}\right)\right)$$

$$=\sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}+\frac{n\pi}{2}\right)$$

(Norm. constant is always $\sqrt{\frac{2}{L}}$).

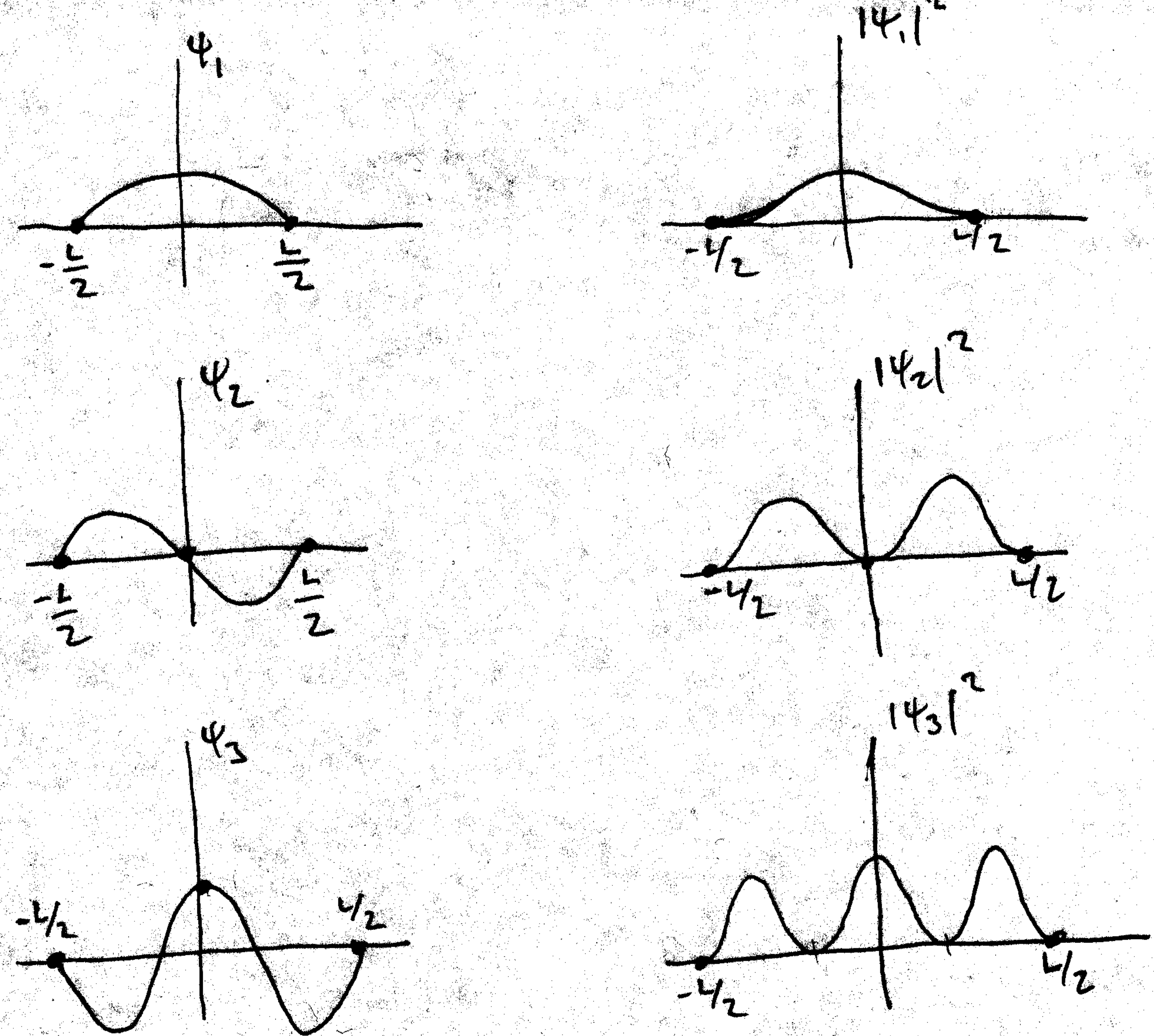
So for

$$\boxed{\begin{aligned}\psi_1&=\sqrt{\frac{2}{L}}\cos\left(\frac{\pi x}{L}\right) &&\Rightarrow |\psi_1|^2=\frac{2}{L}\cos^2\left(\frac{\pi x}{L}\right)\\ \psi_2&=\sqrt{\frac{2}{L}}\sin\left(\frac{2\pi x}{L}\right) &&\Rightarrow |\psi_2|^2=\frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)\\ \psi_3&=\sqrt{\frac{2}{L}}\cos\left(\frac{3\pi x}{L}\right) &&\Rightarrow |\psi_3|^2=\frac{2}{L}\cos^2\left(\frac{3\pi x}{L}\right)\end{aligned}}$$

b/c $\sin\left(x+\frac{\pi}{2}\right) = -\cos x$, $\sin(x+\pi)=-\sin x$.

$\sin\left(x+\frac{3\pi}{2}\right)=\cos x$, but we can drop the overall sign in the wavefunction.

c)



2a) $E_n = \frac{\hbar^2 \pi^2 n^2}{2mL^2} \Rightarrow E_2 - E_1 = (4-1)\frac{\hbar^2\pi^2}{2mL^2} = \frac{3\hbar^2\pi^2}{2mL^2}$

So $\frac{3\hbar^2\pi^2}{2mL^2} = \frac{hc}{\lambda} \Rightarrow L^2 = \frac{3\hbar^2\pi^2}{2mhc}\lambda$

$$\Rightarrow L = \left(\frac{3\hbar^2\pi^2}{2mhc}\lambda\right)^{1/2}$$

$$= \boxed{3.3\times 10^{-10}\ m}$$

b] $E_1 = \frac{\hbar^2 \pi^2}{2mL^2} = 5.5 \times 10^{-19} J = \boxed{3.4 eV}$

c] E_1 for H $= 13.6 eV$, so it's different by $\boxed{10.2 eV}$