

Some Useful Numbers, Equations and Identities

Speed of Light, $c = 2.998 \times 10^8 \text{m/s}$; $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$

$$x' = \gamma(x - vt); \quad t' = \gamma\left(t - \frac{xv}{c^2}\right)$$

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}; \quad u'_z = \frac{u_z}{\gamma\left(1 - \frac{u_z v}{c^2}\right)}$$

$$p = \frac{\mu}{\sqrt{1 - u^2/c^2}}; \quad E = \frac{mc^2}{\sqrt{1 - u^2/c^2}} = K + mc^2$$

$$E^2 = p^2 c^2 + m^2 c^4$$

$$f_{\text{obs}} = \frac{\sqrt{1 + v/c}}{\sqrt{1 - v/c}} f_{\text{source}}$$

For $x < 1$; $(1 \pm x)^n = 1 \pm nx + \frac{n(n-1)}{2}x^2 \pm \frac{n(n-1)(n-2)}{3}x^3 + ..$

Planck's Constant, $h = 6.626 \times 10^{-34} \text{J} \times \text{S} = 4.136 \times 10^{-15} \text{eV} \times \text{S}$

$1 \text{ eV} = 1.60 \times 10^{-19} \text{J}$; $1 \text{ MeV}/c = 5.344 \times 10^{-22} \text{Kg.m/s}$

Coulomb's Constant, $k = 8.99 \times 10^9 \text{N} \times \text{m}^2/\text{C}^2$

Gravitational Constant, $G = 6.67 \times 10^{-11} \text{N} \times \text{m}^2/\text{kg}^2$

Stefan – Boltzmann's Constant, $\sigma = 5.670399 \times 10^{-8} \text{W}/\text{m}^2 \times \text{K}^4$

Wien's Wavelength Displacement Constant = $2.898 \times 10^{-3} \text{m} \times \text{K}$

Boltzmann's Constant, $k = 1.381 \times 10^{-23} \text{J}/\text{K}$

Electron Mass = $9.11 \times 10^{-31} \text{Kg}$; Electron Charge = $1.602 \times 10^{-19} \text{C}$

Atomic Mass Unit $u = 1.6606 \times 10^{-27} \text{Kg}$ or $931.5 \text{ MeV}/c^2$

Proton Mass = $1.673 \times 10^{-27} \text{Kg}$ or $1.0073u$

Neutron Mass = $1.675 \times 10^{-27} \text{Kg}$ or $1.0087u$

Electron Rest Energy = $0.511 \text{ MeV}/c^2$; Proton Rest Energy = $938 \text{ MeV}/c^2$

$1 \text{ kW} \times \text{Hour} = 3.6 \times 10^6 \text{J}$

Intensity of Black Body Radiation $I = \sigma \times T^4$

Force on a charged particle in B field : $\vec{F} = q\vec{v} \times \vec{B}$

Centripetal Acc. = v^2/R where v and R are velocity and radius of orbit

Momentum of a charged particle in B field : $p = qBR$

Photoelectric Effect : $K_{\max} = hf - \phi$

Compton Wavelength λ_c for scattering off electron = 0.00243 nm

In Compton Scattering $\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)$

Construct. interfer. when path diff. between two adjacent rays is $d\sin\phi = n\lambda$

$$\text{TDSE : } -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x, t)}{\partial x^2} + U(x) \Psi(x, t) = i\hbar \frac{\partial \Psi(x, t)}{\partial t}$$

$$\text{TISE : } -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x)}{\partial x^2} + U(x) \psi(x) = E\psi(x)$$

Ground state energy E_1 for particle in a 1D rigid box = $\frac{\pi^2 \hbar^2}{2mL^2}$

Normalized wavefunction of particle in a 1D rigid box = $\sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L}x\right)$

$$[\hat{p}] = \frac{\hbar}{i} \frac{\partial}{\partial x}, \quad [p^2] = -\hbar^2 \frac{\partial^2}{\partial x^2}, \quad [\hat{E}] = i\hbar \frac{\partial}{\partial t}$$

Time dependent form of Wave Function: $\Psi(\vec{r}, t) = \psi(\vec{r})e^{-i\omega t}$; $\omega = E/\hbar$

Uncertainty on Observable Q : $\Delta Q = \sqrt{\langle Q^2 \rangle - \langle Q \rangle^2}$

Expectation Value $\langle Q \rangle = \int \int \Psi(x, t)^* [Q] \Psi(x, t) dx dt$

$$\langle f(r) \rangle = \int_0^\infty f(r) P(r) dr$$

$$\int u \cdot dv = uv - \int v \cdot du$$

$$\int \sin x \, dx = -\cos x$$

$$\int \sin^2 x \, dx = \frac{x}{2} - \frac{\sin 2x}{4}; \quad \int \cos^2 x \, dx = \frac{x}{2} + \frac{\sin 2x}{4}$$

$$\int x \sin^2 x \, dx = \frac{x^2}{4} - \frac{x \sin 2x}{4} - \frac{\cos 2x}{8}$$

$$\int x \cos^2 x \, dx = \frac{x^2}{2} + \frac{x \sin 2x}{4} + \frac{\cos 2x}{8}$$

$$\int x^2 \sin^2 x \, dx = \frac{x^3}{6} - \left(\frac{x^2}{4} - \frac{1}{8}\right) \sin 2x - \frac{x \cos 2x}{4}$$

$$\sin(\theta_1) \sin(\theta_2) = (1/2)[\cos(\theta_1 - \theta_2) - \cos(\theta_1 + \theta_2)]$$

$$\cos(\theta_1) \cos(\theta_2) = (1/2)[\cos(\theta_1 - \theta_2) + \cos(\theta_1 + \theta_2)]$$

$$\sin(\theta_1) \cos(\theta_2) = (1/2)[\sin(\theta_1 - \theta_2) + \sin(\theta_1 + \theta_2)]$$

$$\int_{-1}^{+1} e^{-ax} \, dx = \frac{1}{a}[e^a - e^{-a}]$$

$$\int_{-1}^{+1} x e^{-ax} \, dx = \frac{1}{a^2}[e^a - e^{-a} - a(e^a + e^{-a})]$$

$$\int_0^{+\infty} e^{-ax^2} \, dx = \frac{1}{2} \sqrt{\frac{\pi}{a}}, \quad a > 0$$

$$\int_0^{+\infty} x^2 e^{-ax^2} \, dx = \frac{1}{4a} \sqrt{\frac{\pi}{a}}, \quad a > 0$$

$$\int_0^{+\infty} x^{(2n)} e^{-ax^2} \, dx = \frac{1 \cdot 3 \cdot \dots \cdot (2n-1)}{2^{(n+1)}} \sqrt{\frac{\pi}{a^{2n+1}}}, \quad a > 0$$

$$\int_0^{+\infty} x^{(2n+1)} e^{-ax^2} \, dx = \frac{n!}{2a^{(n+1)}}, \quad a > 0$$

$$\int_0^{+\infty} x^n e^{-x} \, dx = n!$$

$$\int_0^{+\infty} x^n e^{-x/\alpha} \, dx = n! \alpha^{n+1}$$

$$\int_1^{+\infty} e^{-ax} dx = \frac{e^{-a}}{a}$$

$$\int_1^{+\infty} x e^{-ax} dx = \frac{e^{-a}}{a^2}(1+a)$$

$$\int_0^b x^2 e^{-x} dx = 2 - (2 + 2b + b^2)e^{-b}$$

$$\int_0^b x^3 e^{-x} dx = 6 - (6 + 6b + 3b^2 + b^3)e^{-b}$$

$$\int_0^b x^4 e^{-x} dx = 24 - (24 + 24b + 12b^2 + 4b^3 + b^4)e^{-b}$$

$$\int_0^b x^2 e^{-x} dx = 2 - (2 + 2b + b^2)e^{-b}$$

The ground state wavefunction for 1D oscillator under $U(x) = \frac{1}{2}m\omega^2 x^2$ has the form:

$$\psi_0(x) \propto e^{-\frac{m\omega x^2}{2\hbar}}$$

The wavefunction for Oscillator's first excited state:

$$\psi_1(x) \propto \sqrt{\frac{m\omega}{\hbar}} x e^{-\frac{m\omega x^2}{2\hbar}}$$

Next excited state:

$$\psi_2(x) \propto \left(1 - \frac{2m\omega x^2}{\hbar}\right) e^{-\frac{m\omega x^2}{2\hbar}}$$

The energy of the nth Oscillator state $E_n = (n + \frac{1}{2})\hbar\omega$.

Volume element in Sph. polar coordinates is $dV = r^2 dr \sin\theta d\theta d\phi$

Information about Hydrogenic atom with Z protons in the nucleus:

$$\text{Reduced Mass } \mu = \frac{M_1 \cdot M_2}{M_1 + M_2}$$

$$R_{1,0}(r) = \left(\frac{Z}{a_0}\right)^{3/2} 2e^{-\frac{Zr}{a_0}}$$

$$R_{2,0}(r) = \left(\frac{Z}{2a_0}\right)^{3/2} \left(2 - \frac{Zr}{a_0}\right)e^{-\frac{Zr}{2a_0}}$$

$$R_{2,1}(r) = \left(\frac{Z}{2a_0}\right)^{3/2} \frac{Zr}{\sqrt{3}a_0} e^{-\frac{Zr}{2a_0}}$$

$$\text{Radial Prob. Density } P(r) = r^2 |R(r)|^2$$

$$Y_0^0(\theta, \phi) = \Theta(\theta)\Phi(\phi) = \frac{1}{2\sqrt{\pi}}$$

$$Y_1^0(\theta, \phi) = \frac{1}{2}\sqrt{\frac{3}{\pi}} \cos \theta \text{ and } Y_1^{\pm 1}(\theta, \phi) = \mp \frac{1}{2}\sqrt{\frac{3}{2\pi}} \sin \theta e^{\pm i\phi}$$

$$\text{Orbit radius } r_n = \frac{n^2 \hbar^2}{Zm_e k e^2}$$

$$\text{bohr radius } a_0 = \frac{\hbar^2}{m_e k e^2} = 0.0529 \text{ nm}$$

$$\text{Energy } E_n = \frac{-kZ^2 e^2}{2a_0} \frac{1}{n^2} \text{ for } n = 1, 2, 3, 4, \dots$$

$$\text{Rydberg Constant } R = 1.0973732 \times 10^7 \text{ m}^{-1}$$
