Physics 2D Lecture Slides
Sept 30

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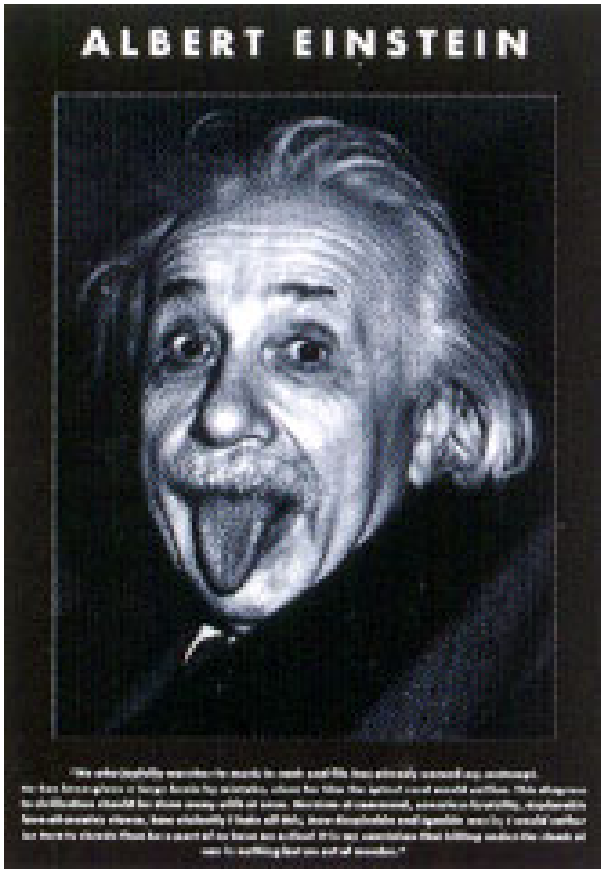
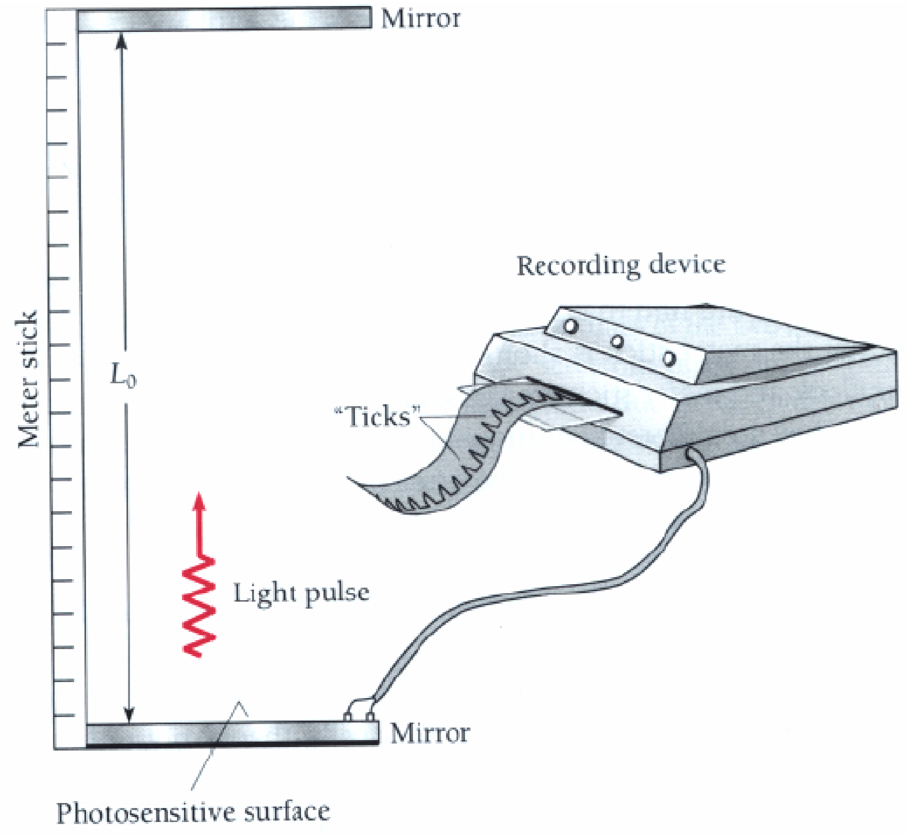
Einstein's Special Theory of Relativity



Einstein's Postulates of SR

- The laws of physics must be the same in all inertial reference frames
- The speed of light in vacuum has the same value ($c = 3.0 \times 10^8$ m/s) , in all inertial frames, regardless of the velocity of the observer or the velocity of the source emitting the light.

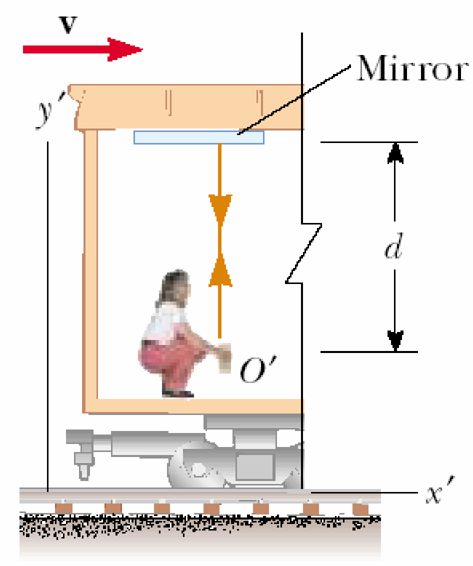
A Simple Clock Measuring a Time Interval

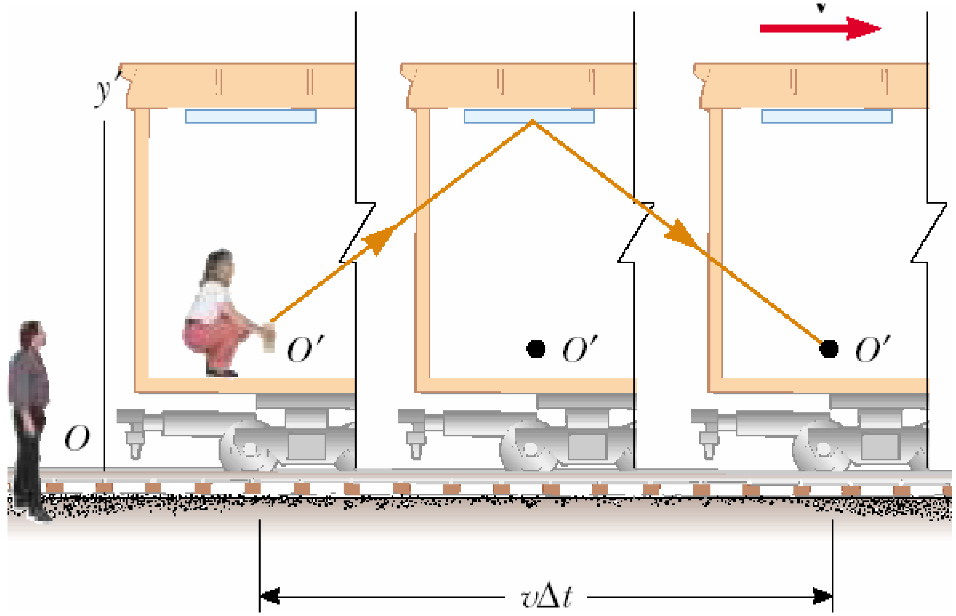


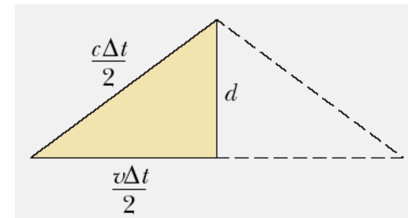
$$t = \int \Delta t$$

Time Dilation and "Proper" Time

Watching a time interval with a simple clock







Observer O' : $\Delta t' = \dfrac{2d}{c}$

Observer O : Apply Pythogoras Theorem

$$\left(\frac{c\Delta t}{2}\right)^2 = (d)^2 + \left(\frac{v\Delta t}{2}\right)^2, \text{ but } d = \left(\frac{c\Delta t'}{2}\right)$$

$$\therefore \quad c^2(\Delta t)^2 = c^2(\Delta t')^2 + v^2(\Delta t)^2$$

$$\Delta t = \frac{\Delta t'}{\sqrt{1-\left(\dfrac{v}{c}\right)^2}} = \gamma\,\Delta t'$$

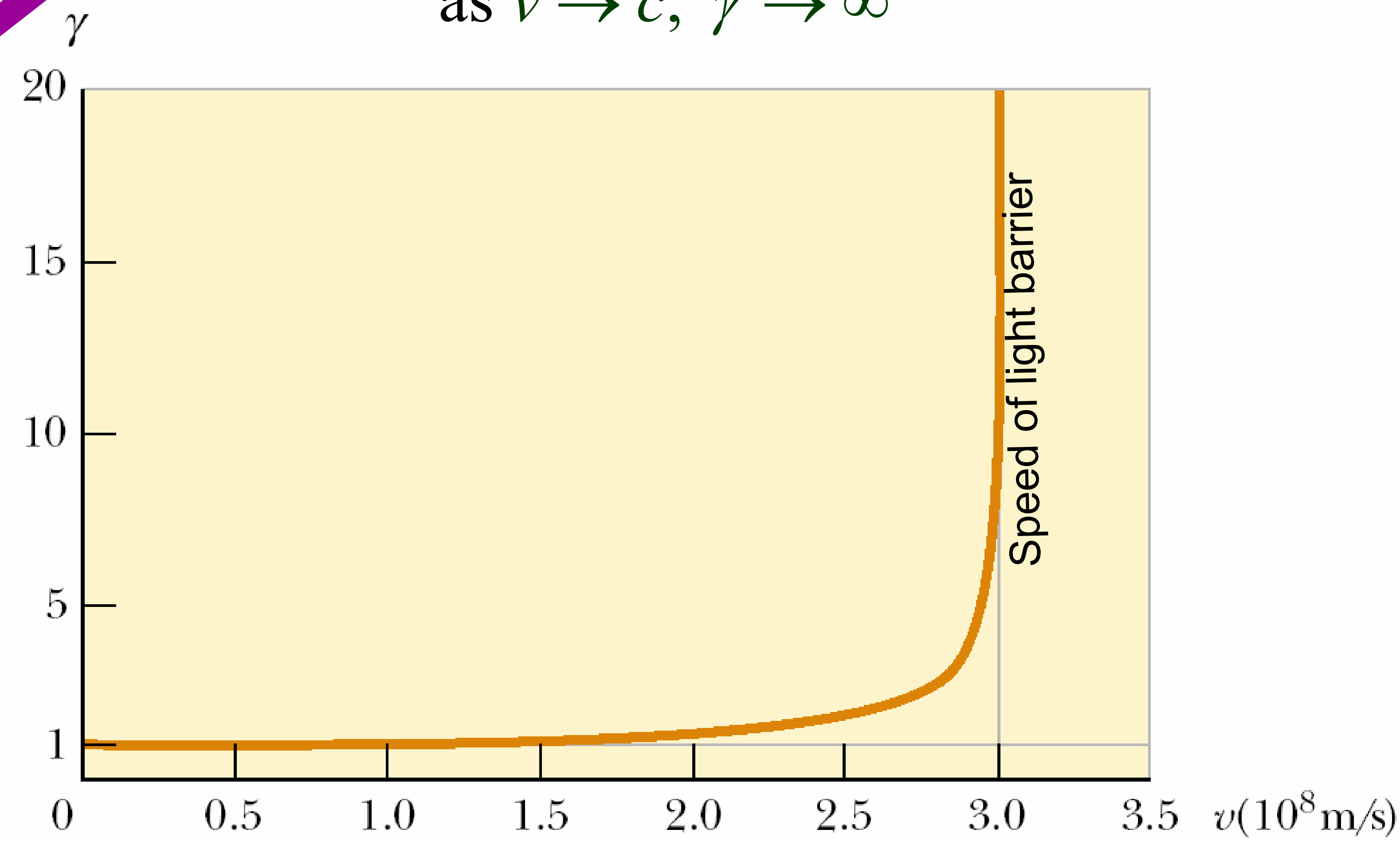
$$\Delta t > \Delta t'$$

The γ factor

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

as $v \to 0, \; \gamma \to 1$

as $v \to c, \; \gamma \to \infty$



Measuring Time: Period of a Pendulum

- Period of a pendulum is 3.0 s in the rest frame of the pendulum
- What is period of the pendulum as seen by an observer moving at v=0.95c

Answer:

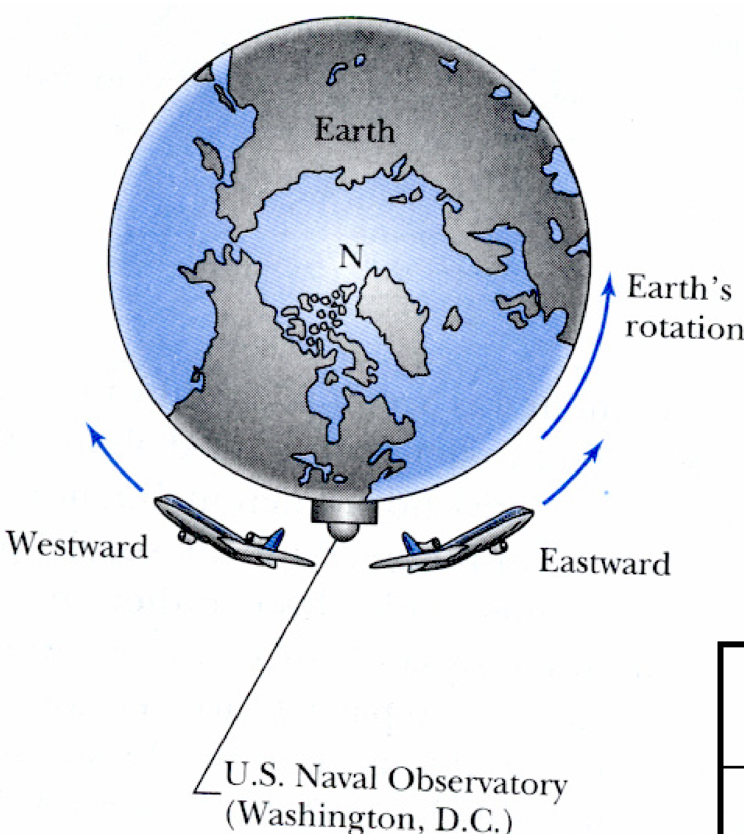
- Proper time T' = 3.0s
- Since motion is relative and time dilation does not distinguish between
  - relative motion →→ (V) from relative motion ← ← (-V)
- lets reformulate the problem like this (??)
  - A pendulum in a rocket is flying with velocity V =0.95c past a stationary observer
  - Moving clocks runs slower [w.r.t clock in observer's hand (rest)] by factor γ
  - ➔ Period T measured by observer = γ T'

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{1}{\sqrt{1-(0.95)^2}} = 3.2$$

$$\Rightarrow T = \gamma T' = 3.2 \times 3.0s = 9.6s$$

Moving pendulum slows down ➔ takes longer to complete a period

Round The World With An Atomic Clock

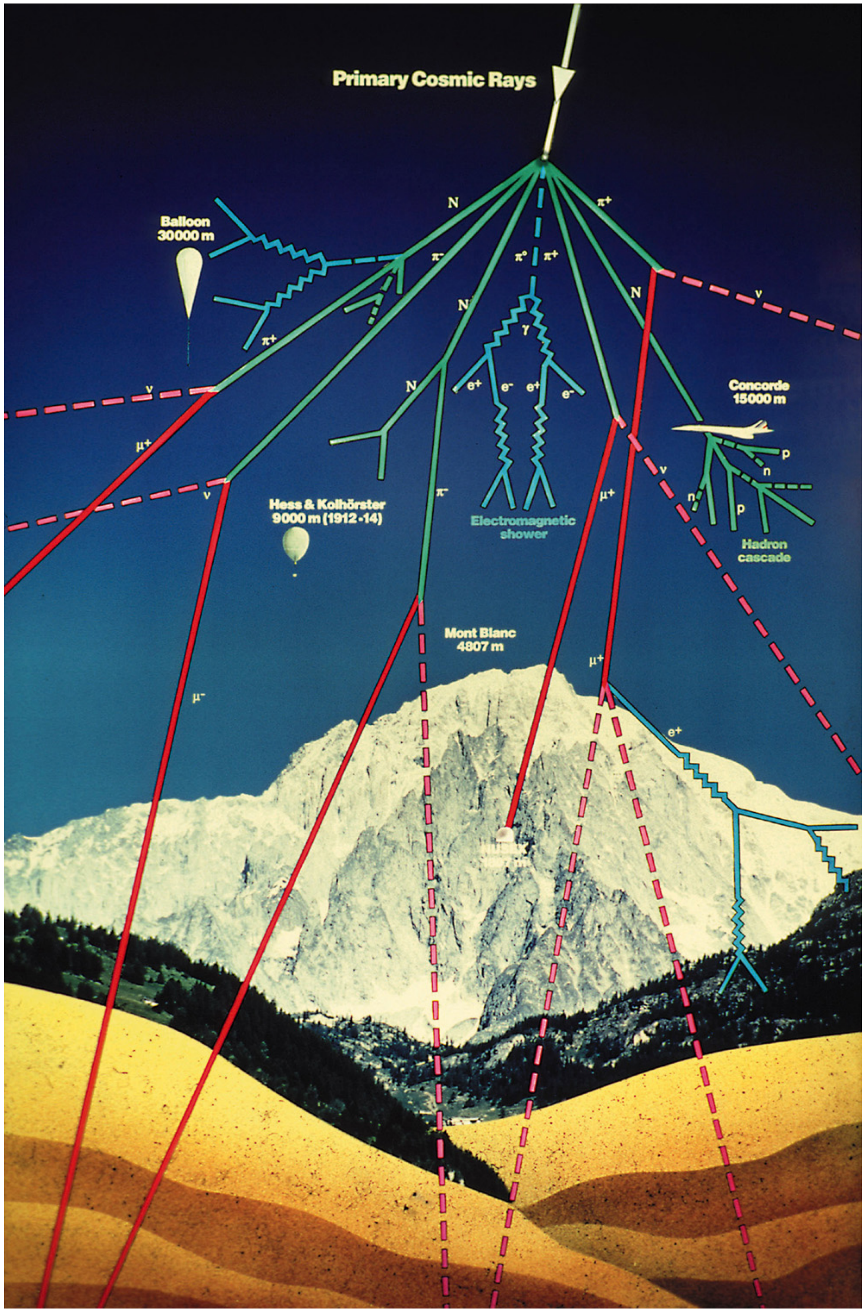


- Atomic Clock : certain atomic level transitions in Cesium atom
- Two planes take off from DC, travel east and west
  - Eastward trip took 41.2 hrs
  - Westward trip took 48.6
- Atomic clocks compared to similar ones kept in DC
- Need to account for Earth's rotation + GR etc

Travel	Predicted	Measured
Eastward	-40 ± 23 ns	-59 ± 10 ns
Westward	275 ± 21 ns	273 ± 7 ns

Flying clock ticked faster or slower than reference clock. Slow or fast is due to Earth's rotation

Cosmic Rays Bombarding the Earth



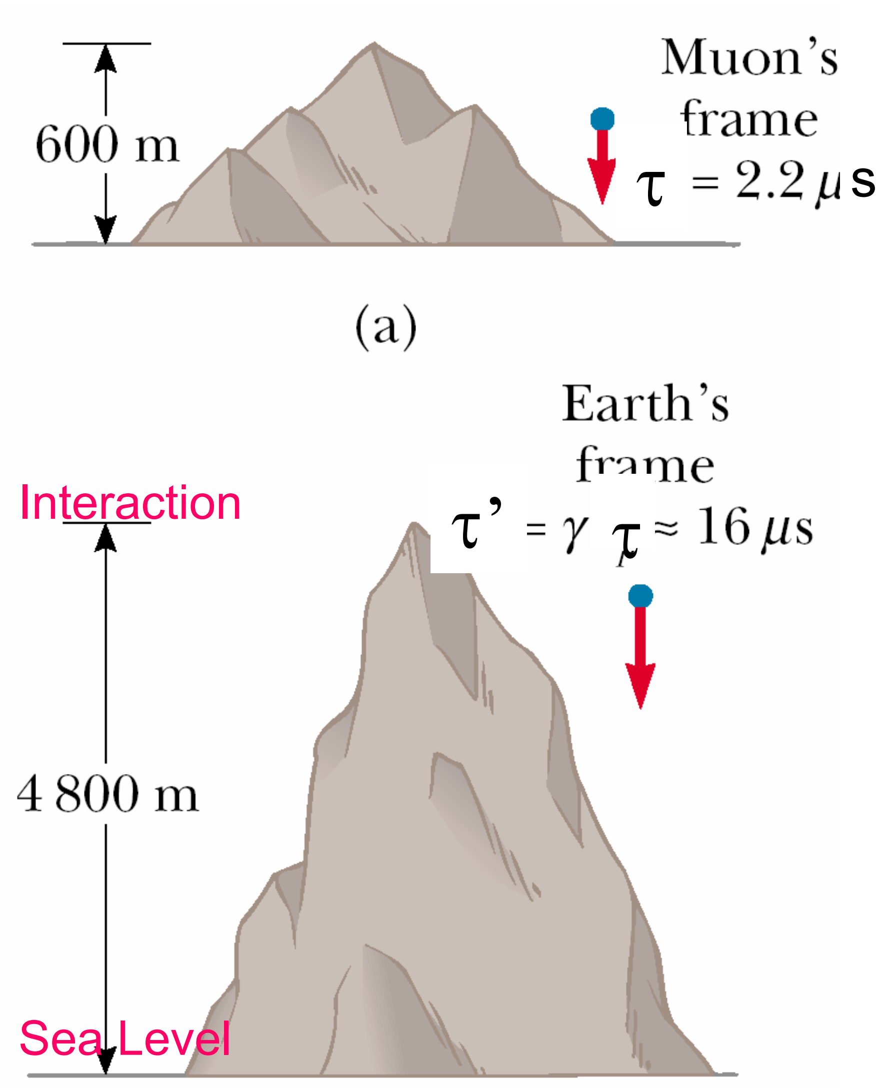
- Cosmic rays are messengers from space
- **Produced in violent collisions in the cosmos**
- **Typical Kinetic energy ~ 100 GeV**
- **Smash into Earth's outer atmosphere**
  - **4700 m from sea level**
- **Sometimes produce short lived Muons**

- **Muon is electron like charged particle**
  - **~ 200 times heavier , same charge**
  - **Lifetime $\tau = 2.2\mu s = 2.2 \times 10^{-6}$ s**
  - **Produced with speed $v \equiv c$**
  - **Distance traveled in its lifetime**

$$d = c\tau = 650m$$

  - **Yet they seem to reach the surface!!**
    - **Why => Time Dilation**
    - **Must pay attention to frames of references involved**

Cosmic Rays Are Falling On Earth : Example of Time Dilation



- Two frames of references
  1. Riding on the Muon
  2. On surface of earth

– Muon Rider has "Proper Time"
  - Time measured by observer moving along with clock
  - $\Delta t' = \tau = 2.2\ \mu S$
  - $D' = v\,\Delta t' = 650m$

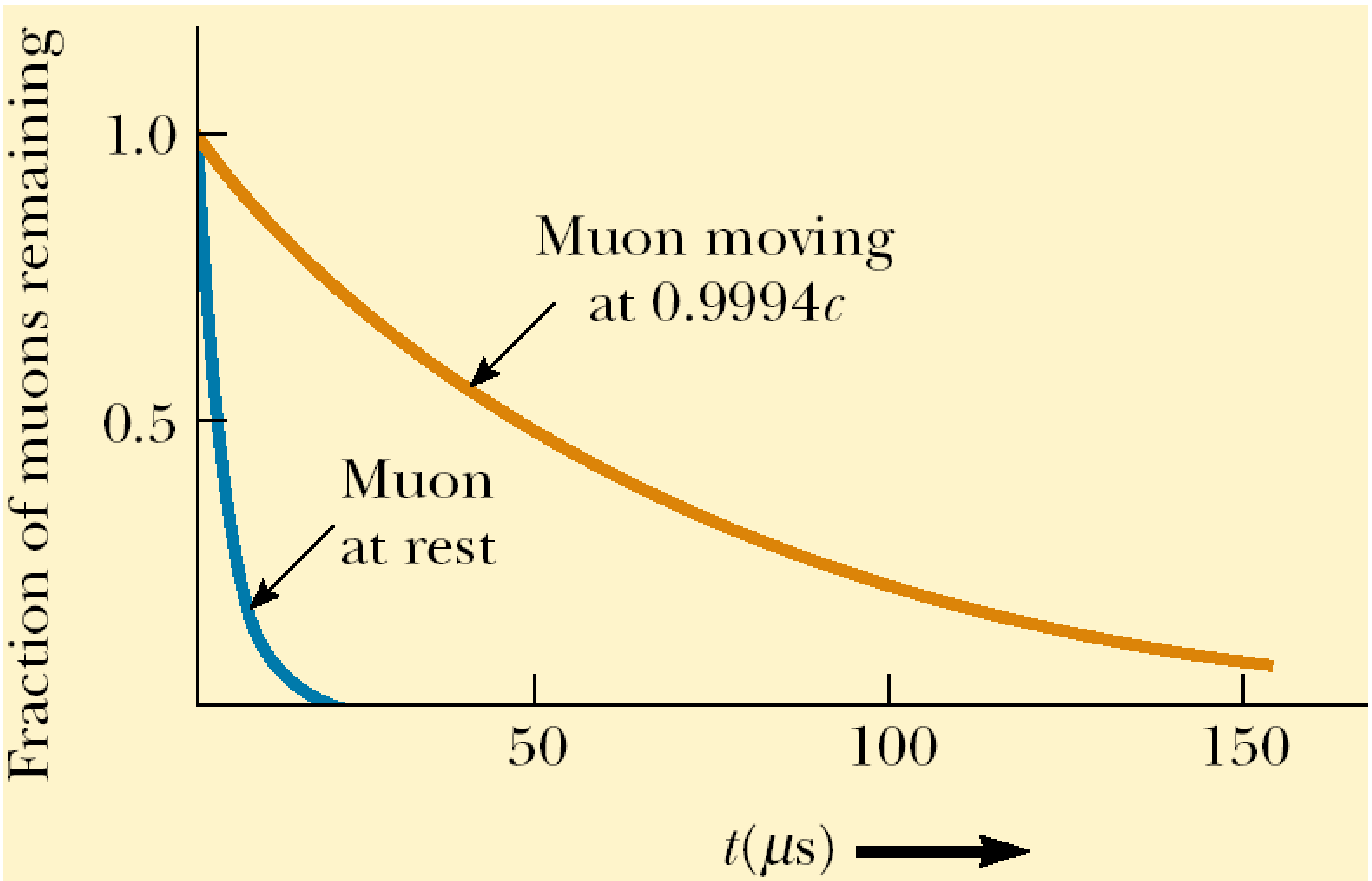
– Earthling watches a moving clock (muon's) run slower
  - $\Delta t' = \gamma\,\tau$
    - $v = 0.99c, \Rightarrow \gamma = 7.1$
    - $D = v\,\Delta t = 4700m$

Muon Decay Distance Distribution

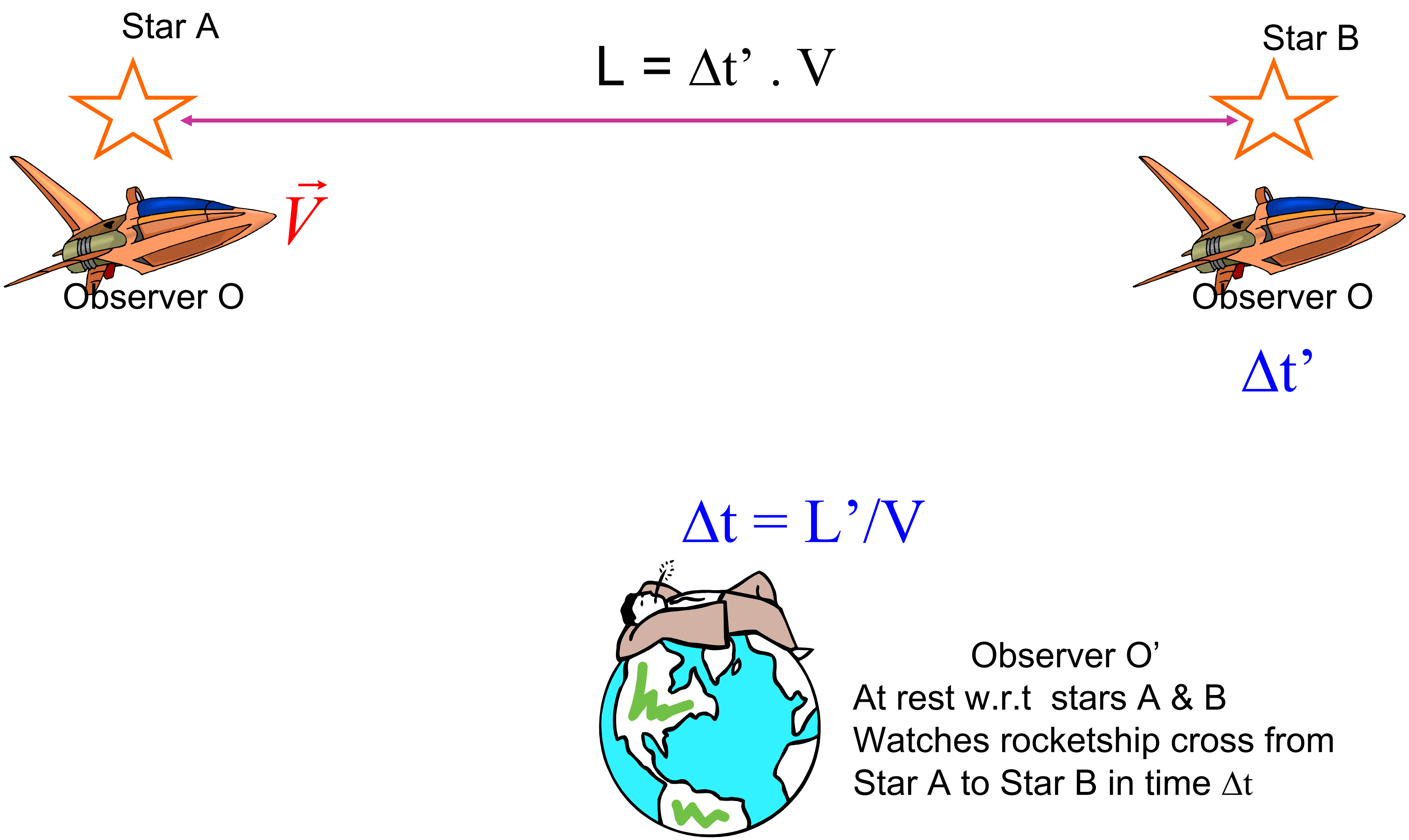
Relative to Observer on Earth Muons have a lifetime

$$t = \gamma\tau = 7.1\ \tau$$

Exponential Decay time Distribution : As in Radioactivity



Offsetting Penalty : Length Contraction



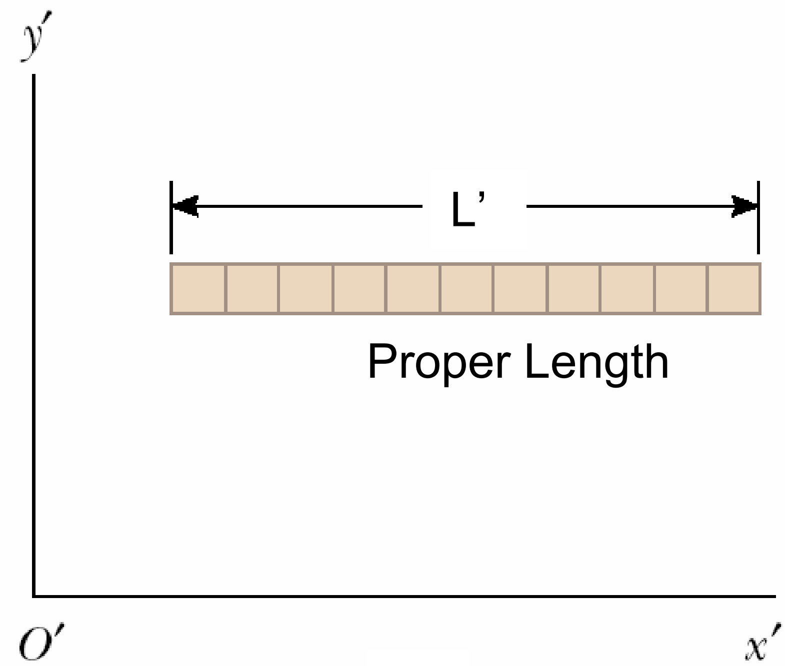
Rocketman Vs The Earthling

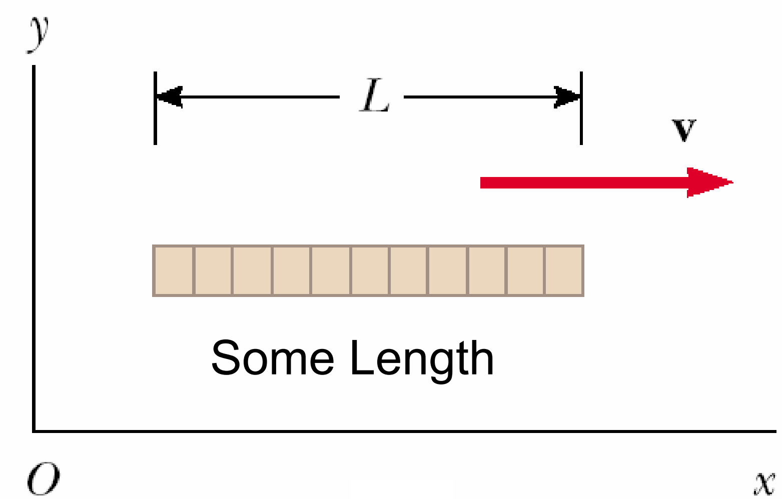
- Earth Observer saw rocketman take time $\Delta t = (L'/V)$
- Rocketman says he is at rest, Star B moving towards him with speed V from right passed him by in time $\Delta t'$, so
  - $L = \Delta t' . V$
  - But $\Delta t' = \Delta t / \gamma$ (time dilation)
  - $\Rightarrow L = V. (\Delta t / \gamma)$
    $= L'/\gamma$

$$L = L'.\sqrt{1-\frac{V^2}{c^2}}$$

$$L \leq L'$$

Moving Rods Contract in direction Of relative motion

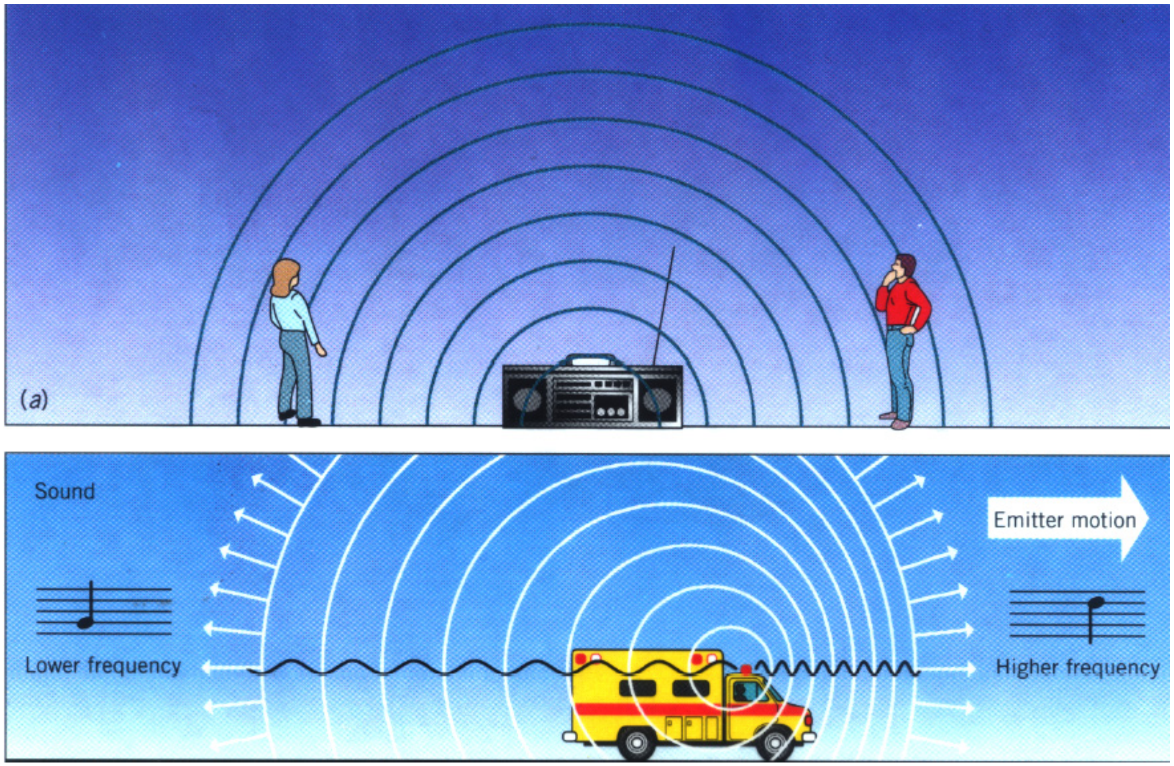




Immediate Consequences of Einstein's Postulates: Recap

- Events that are simultaneous for one Observer are not simultaneous for another Observer in relative motion
- Time Dilation : Clocks in motion relative to an Observer appear to slow down by factor γ
- Length Contraction : Lengths of Objects in motion appear to be contracted in the direction of motion by factor γ^{-1}
- New Definitions to keep track of the discussion :
  - Proper Time (who measures this ?)
  - Proper Length (who measures this ?)
  - Different clocks for different folks !

Doppler Effect In Sound : Reminder from 2A

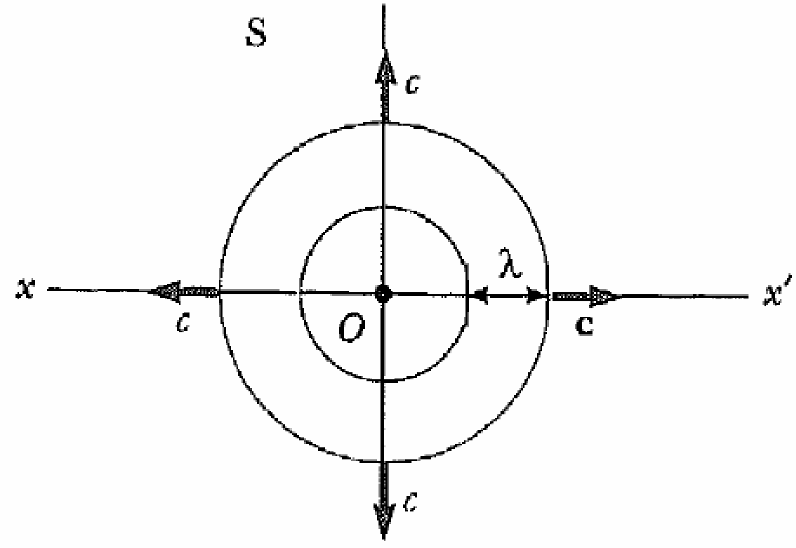


Observed Frequency of sound INCREASES if emitter moves towards the Observer
Observed Wavelength of sound DECREASES if emitter moves towards the Observer

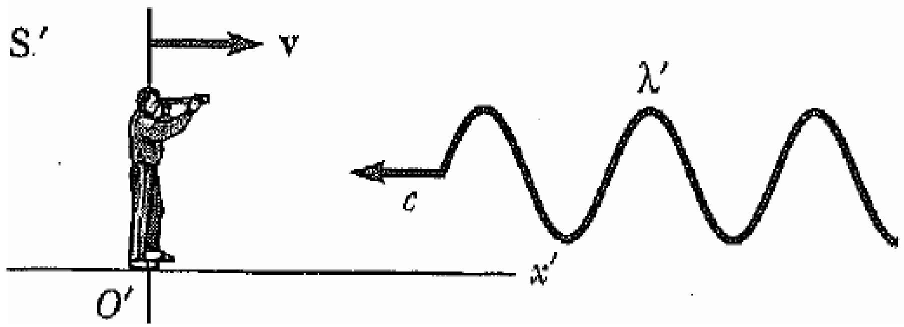
$$v = f\lambda$$

Time Dilation Example: Relativistic Doppler Shift

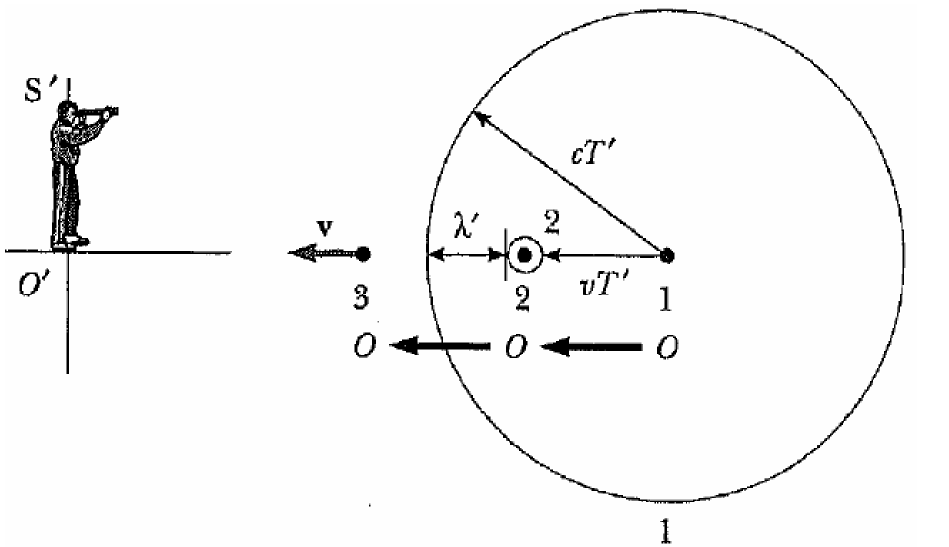
- Light : velocity c = f λ, f=1/T
- A source of light S at rest
- Observer S'approches S with velocity v
- S' measures f' or λ', c = f'λ'
- Expect f' > f since more wave crests are being crossed by Observer S'due to its approach direction than if it were at rest w.r.t source S



(a)



Relativistic Doppler Shift



Examine two successive wavefronts emitted by S at location 1 and 2

In S' frame, T' = time between two wavefronts

In time T', the Source moves by cT' w.r.t 1

Meanwhile Light Source moves a distance vT'

Distance between successive wavefront

$$\lambda' = cT' - vT'$$

$\lambda' = cT' - vT'$, use $f = c/\lambda$

$$f' = \frac{c}{(c-v)T'}, \quad T' = \frac{T}{\sqrt{1-(v/c)^2}}$$

Substituting for T', use f=1/T

$$\Rightarrow f' = \frac{\sqrt{1-(v/c)^2}}{1-(v/c)}$$

$$\Rightarrow f' = \frac{\sqrt{1+(v/c)}}{\sqrt{1-(v/c)}}\, f$$

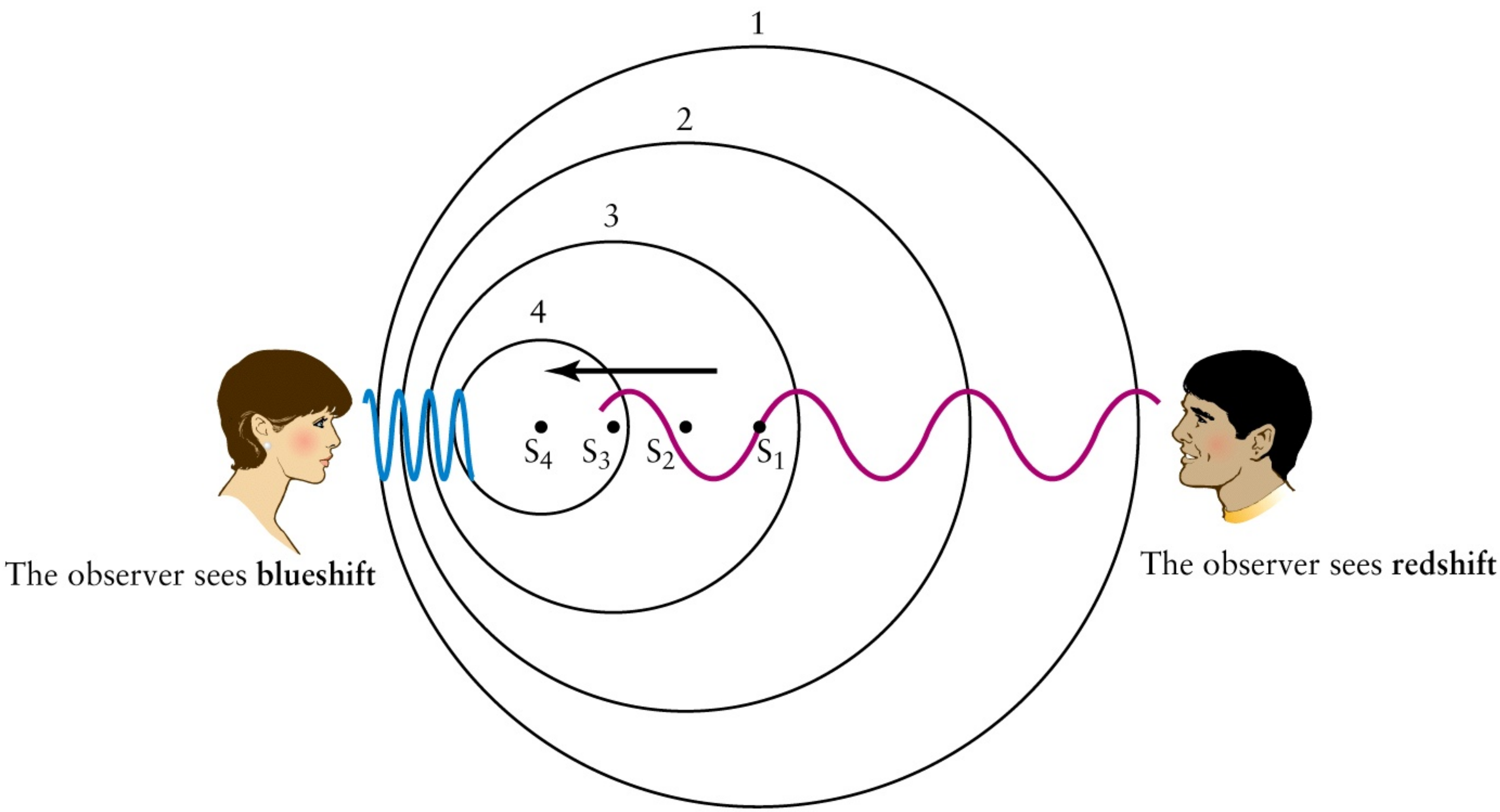
better remembered as:

$$f_{obs} = \frac{\sqrt{1+(v/c)}}{\sqrt{1-(v/c)}}\, f_{source}$$

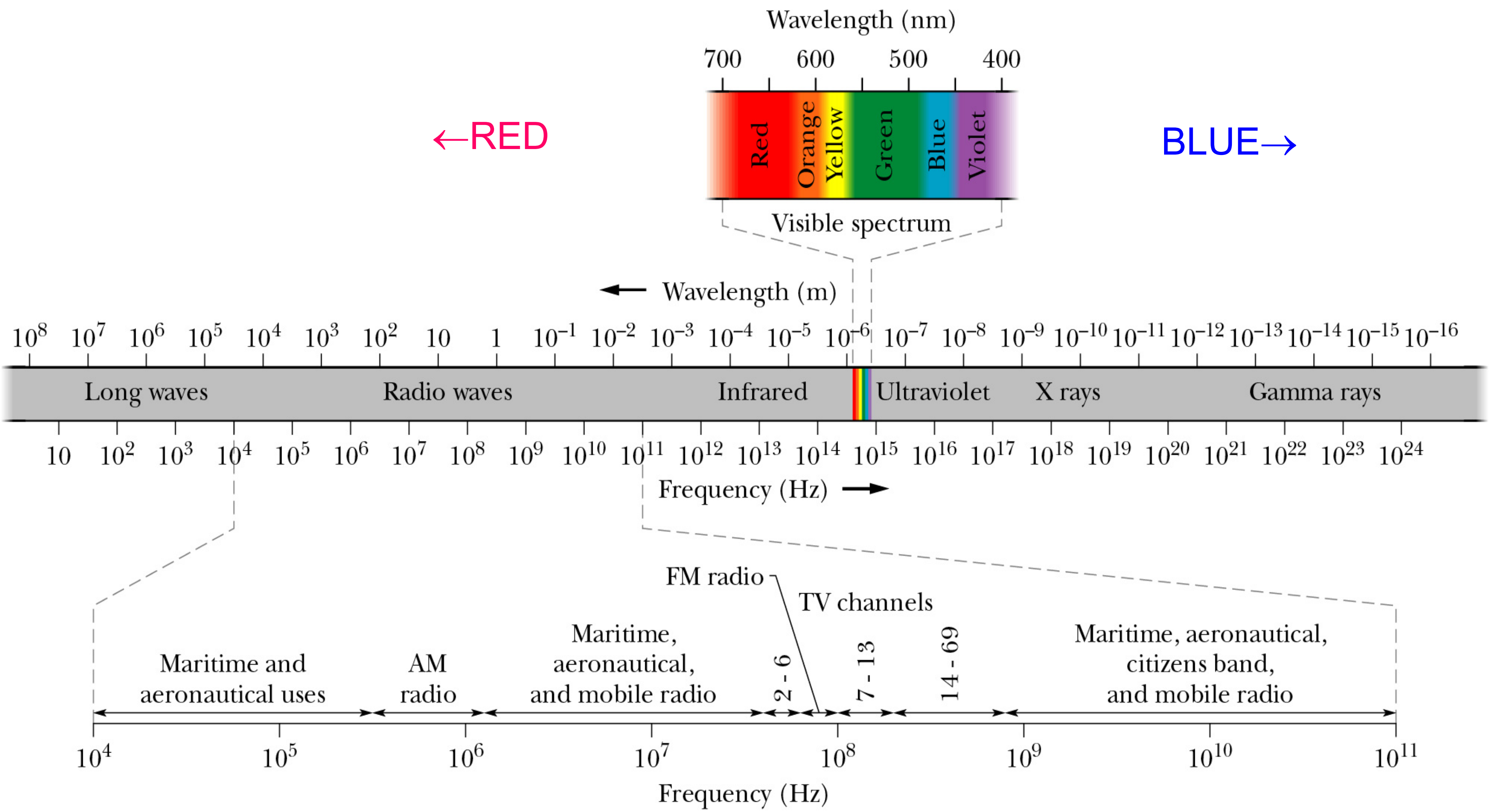
f_{obs} = Freq measured by observer approching light source

Relativistic Doppler Shift

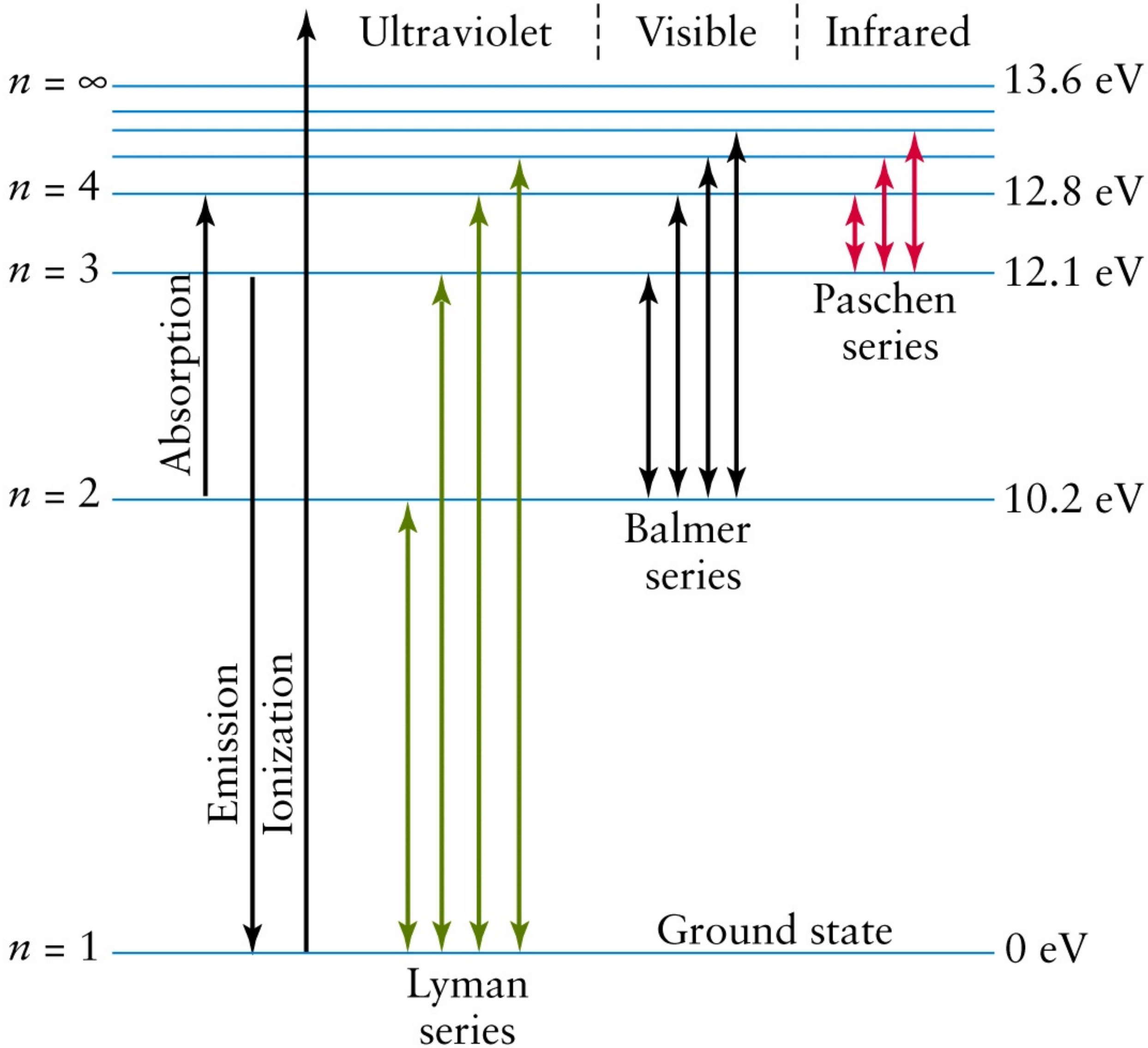
$$f_{obs} = \frac{\sqrt{1+(v/c)}}{\sqrt{1-(v/c)}} f_{source}$$



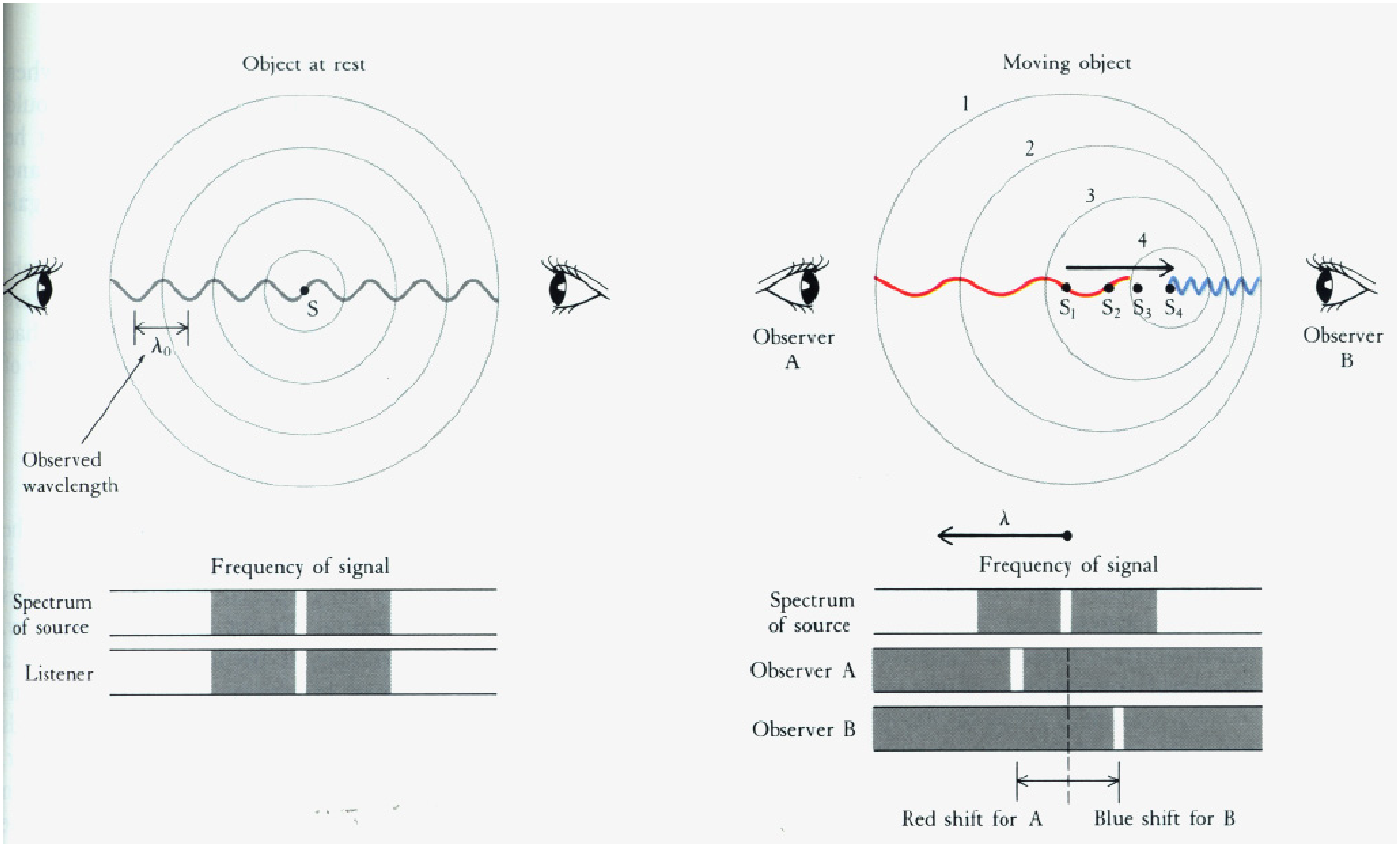
Doppler Shift & Electromagnetic Spectrum



Fingerprint of Elements: Emission & Absorption Spectra



Spectral Lines and Perception of Moving Objects



Doppler Shift in Spectral Lines and Motion of Stellar Objects

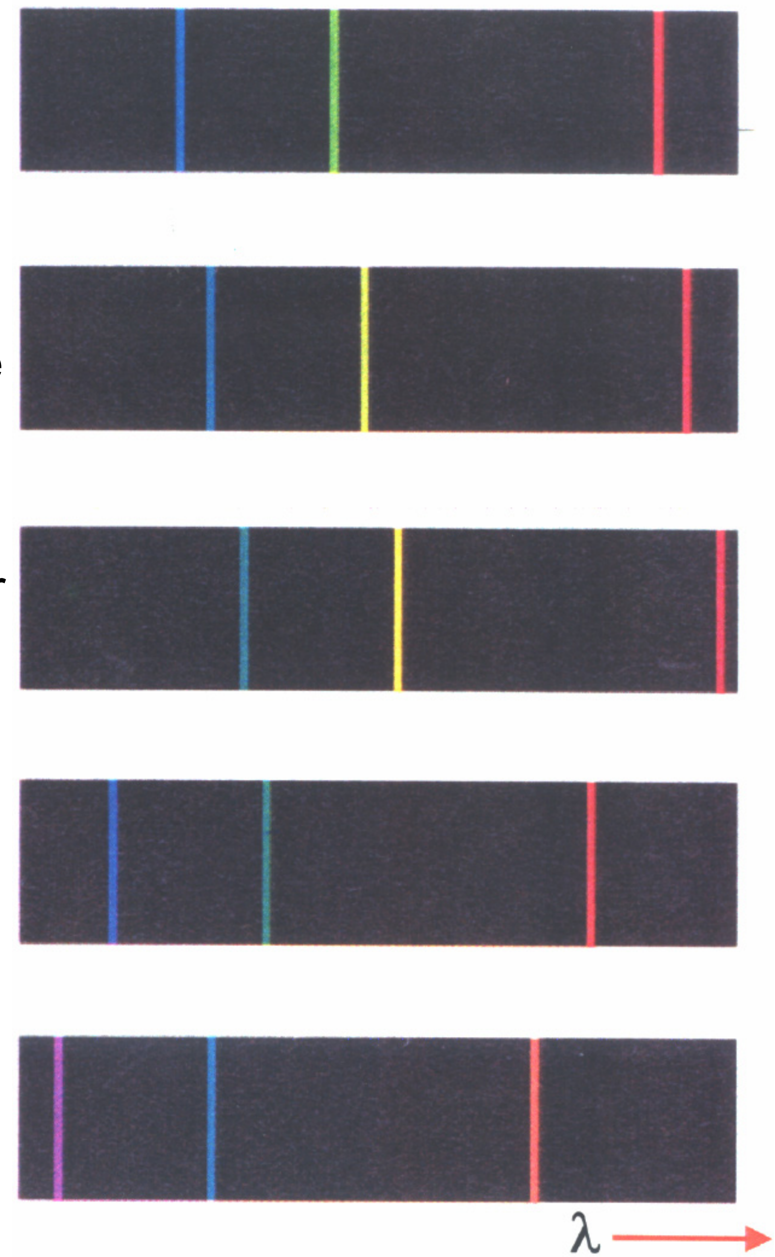
Laboratory Spectrum, lines at rest wavelengths

Lines Redshifted, Object moving away from me

Larger Redshift, object moving away even faster

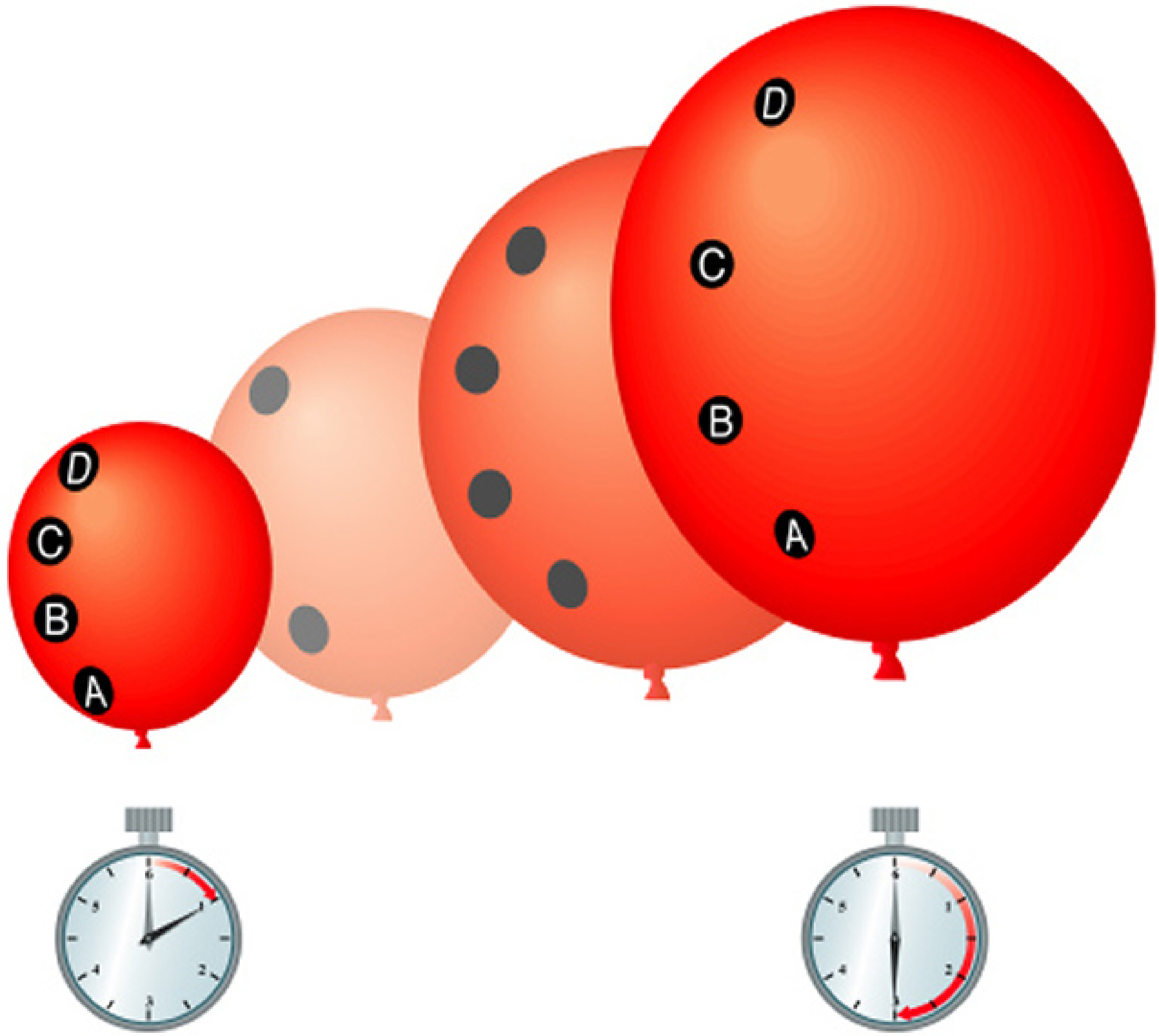
Lines blueshifted, Object moving towards me

Larger blueshift, object approaching me faster

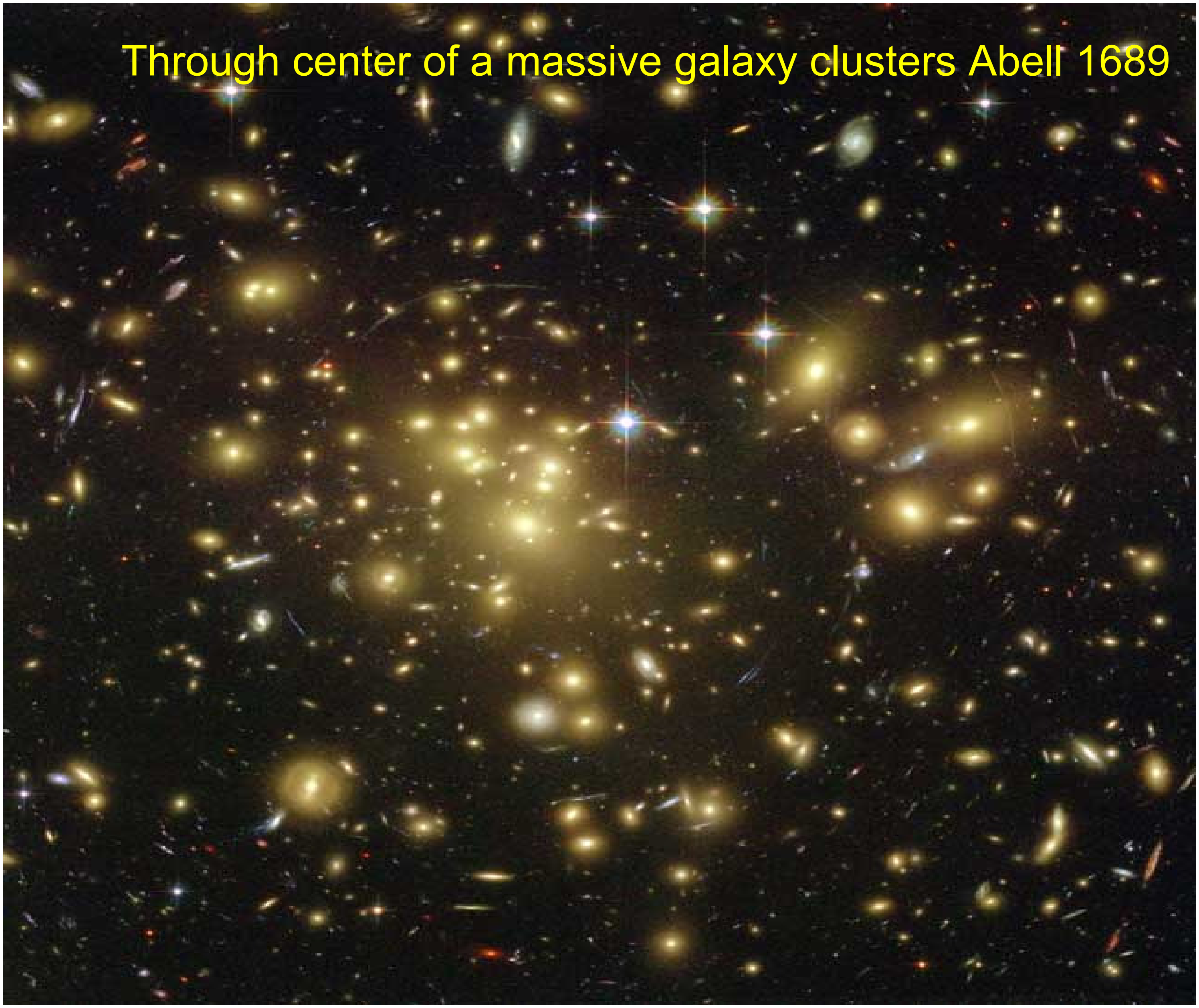


Cosmological Redshift & Discovery of the Expanding Universe:

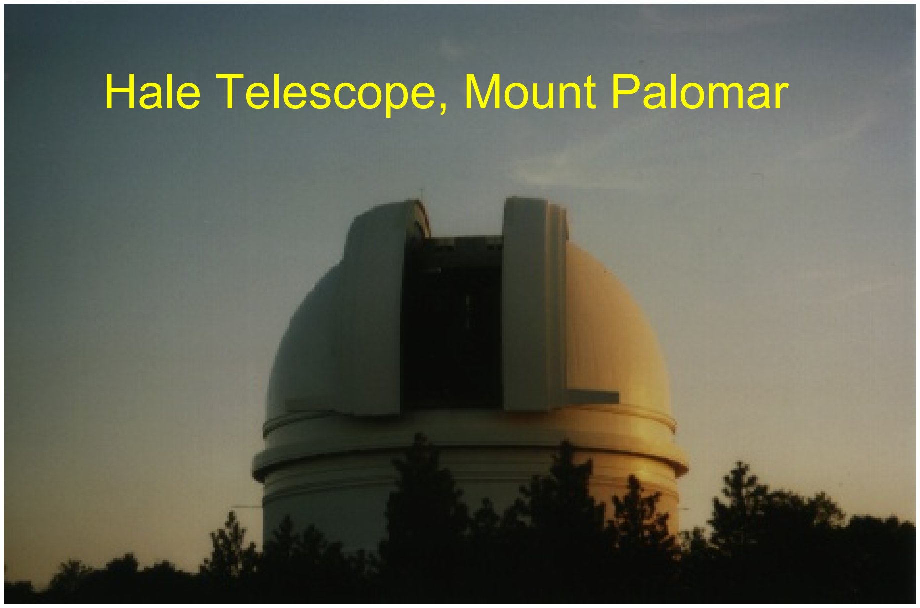
[Space itself is Expanding]

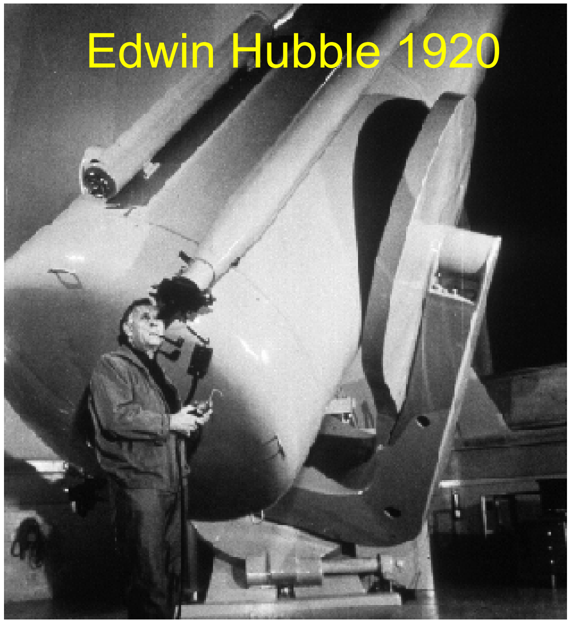


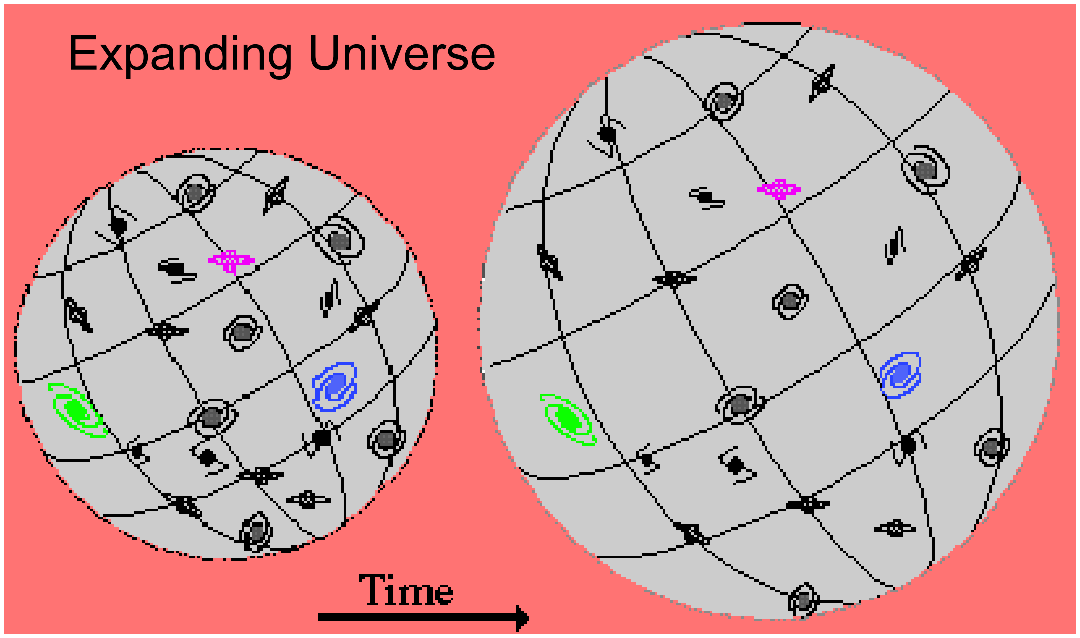
Seeing Distant Galaxies Thru Hubble Telescope



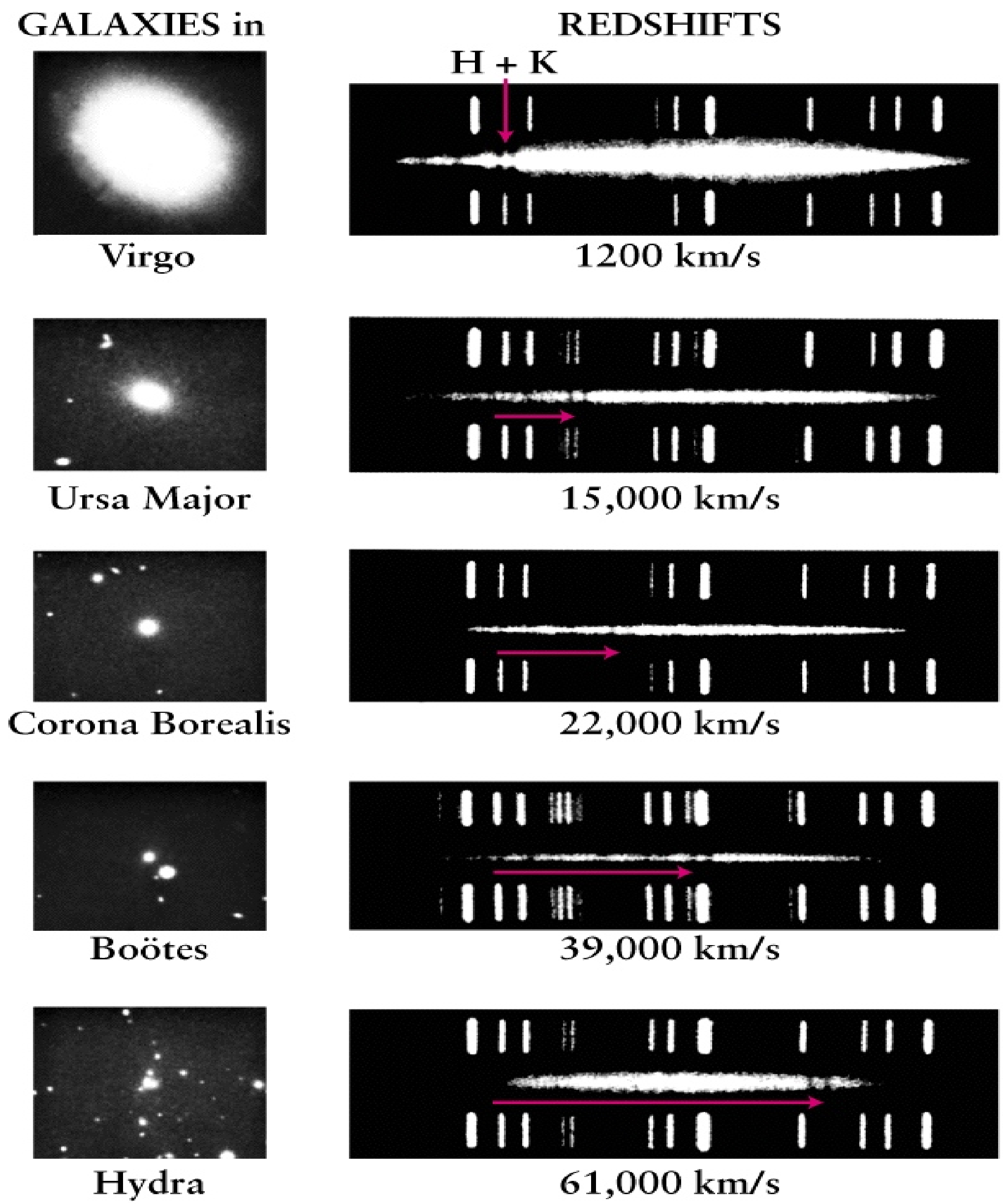
Expanding Universe, Edwin Hubble & Mount Palomar





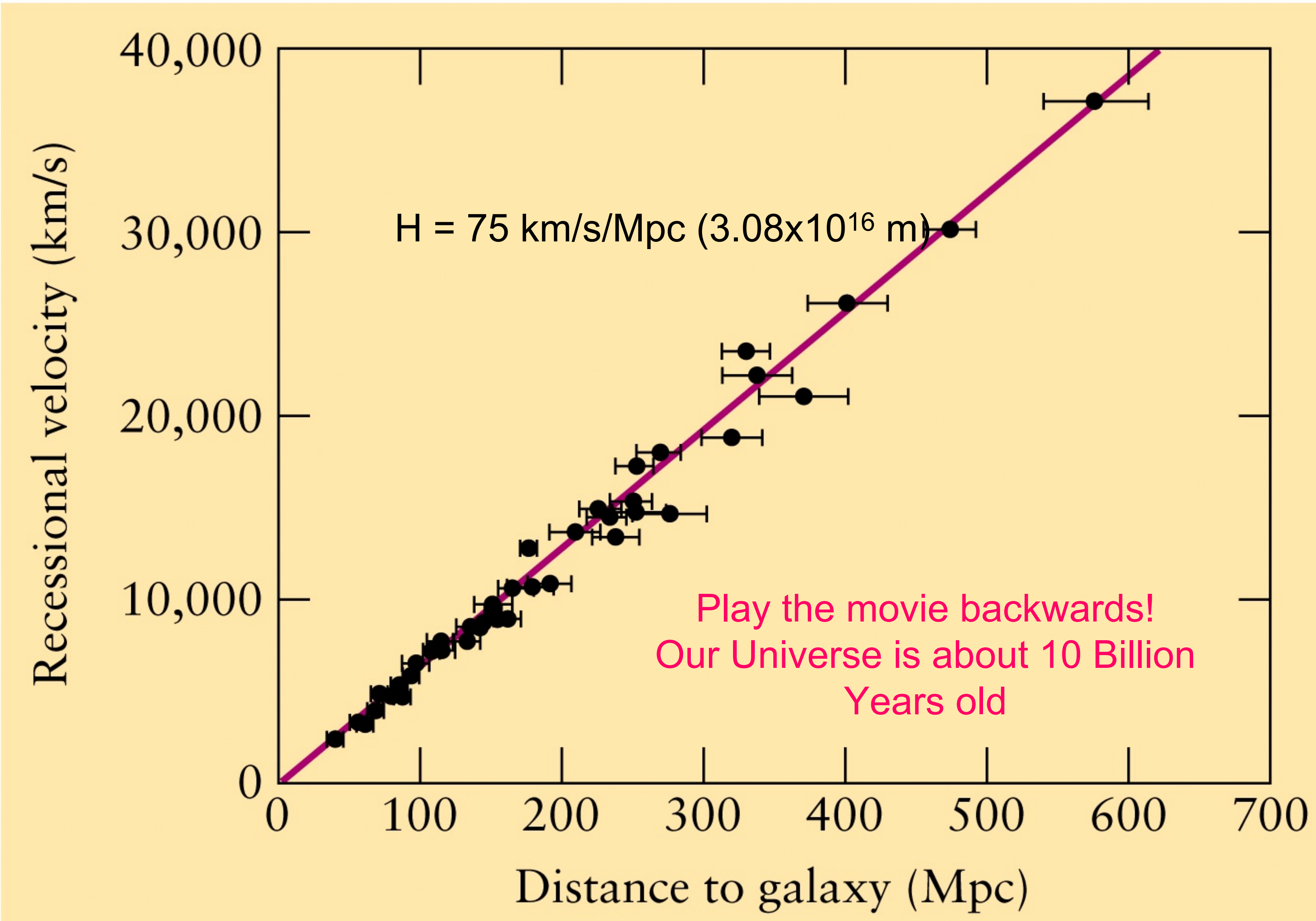


Galaxies at different locations in our Universe travel at different velocities



Hubble's Measurement of Recessional Velocity of Galaxies

V = H d : Farther things are faster they go



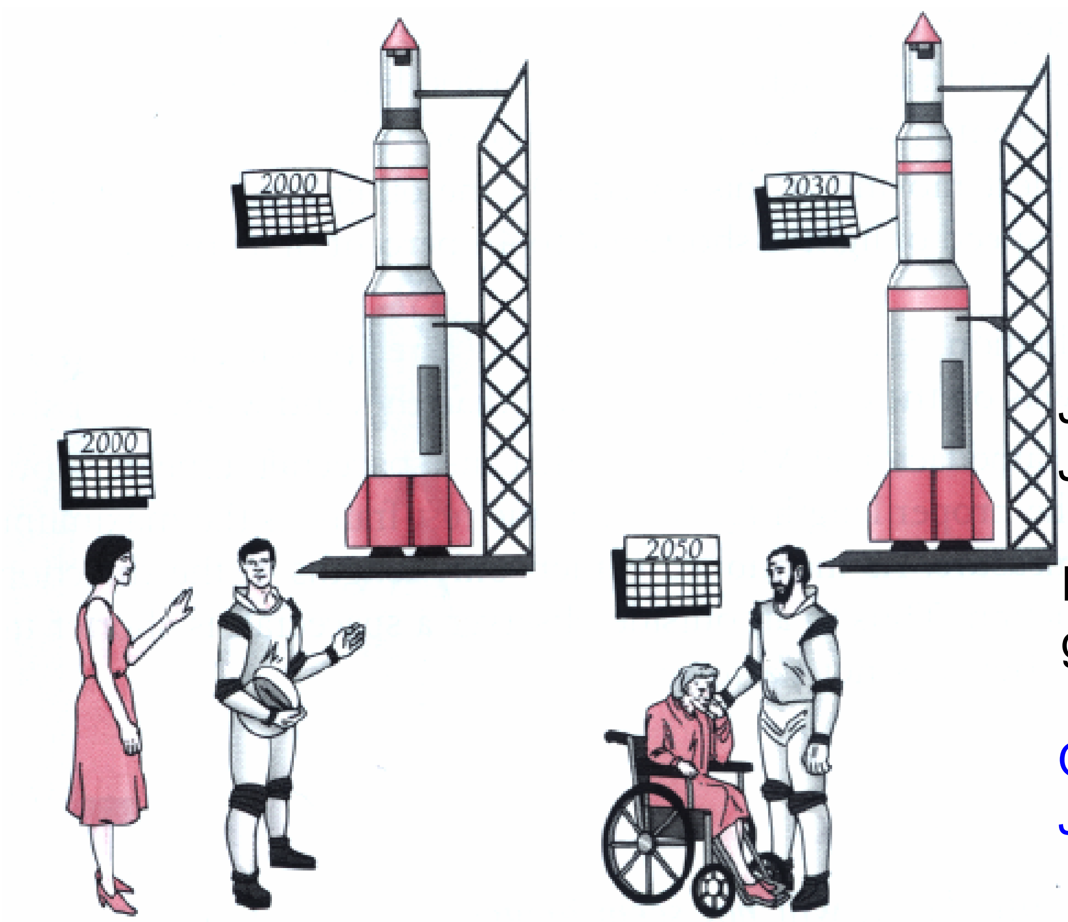
Now for Something Totally Different : Paradox !

A paradox is an apparently self-contradictory statement, the underlying meaning of which is revealed only by careful scrutiny. The purpose of a paradox is to arrest attention and provoke fresh thought

``A paradox is not a conflict within reality. It is a conflict between reality and your feeling of what reality should be like." - Richard Feynman

Construct a few paradoxes in Relativity & analyze them

Jack and Jill's Excellent Adventure: Twin Paradox



Jill sees Jack's heart slow down

Factor : $\sqrt{1-(v/c)^2}$

$$= \sqrt{1-(0.8c/c)^2} = 0.6$$

For every 5 beats of her heart
She sees Jack's beat only 3 !

Jack has only 3 thoughts for 5 that Jill has ! Every things slows!

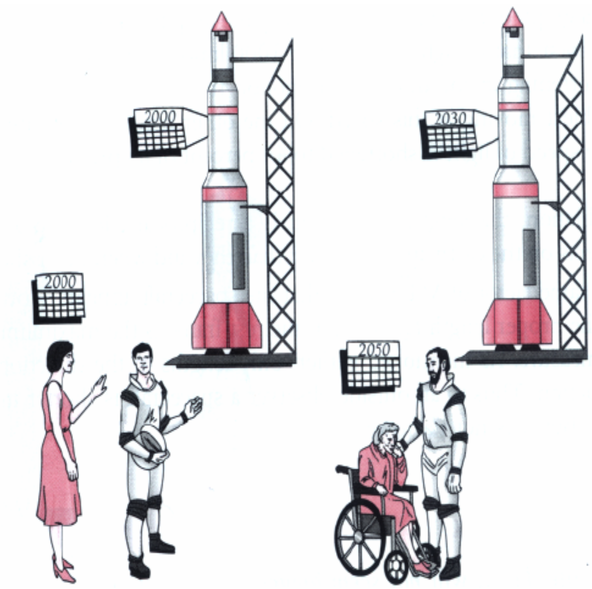
Finally Jack returns after 50 yrs gone by according to Jill's calendar

Only 30 years have gone by Jack
Jack is 50 years old, Jane is 70 !

Jack & Jill are 20 yr old twins, with same heartbeat
Jack takes off with V = 0.8c to a star 20 light years away
Jill stays behind, watches Jack by telescope

Is there a paradox here ??

Twin Paradox ?



- Paradox : Turn argument around, motion is relative
- Jack claims he at rest, Jill is moving v=0.8c
- Should not Jill be 50 years old when 70 year old Jack returns from space Odyssey?
- No ! …because Jack is not traveling in a inertial frame of reference
  - TO GET BACK TO EARTH HE HAS TO TURN AROUND => decelerate/accelerate
- But Jill always remained in Inertial frame
- Time dilation formula applies to Jill's observation of Jack but not to Jack's observation of Jill

Non-symmetric aging verified with atomic clocks taken on airplane trip around world and compared with identical clock left behind. Observer who departs from an inertial system will always find its clock slow compared with clocks that stayed in the system