Physics 2D Lecture Slides
Oct 8

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Definition (without proof) of Relativistic Momentum

$$\vec{p} = \frac{m\vec{u}}{\sqrt{1-(u/c)^2}} = \gamma m\vec{u}$$

With the new definition relativistic momentum is conserved in all frames of references : Do the exercise

New Concepts

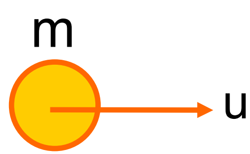
Rest mass = mass of object measured
In a frame of ref. where object is at rest

$$\gamma = \frac{1}{\sqrt{1-(u/c)^2}}$$

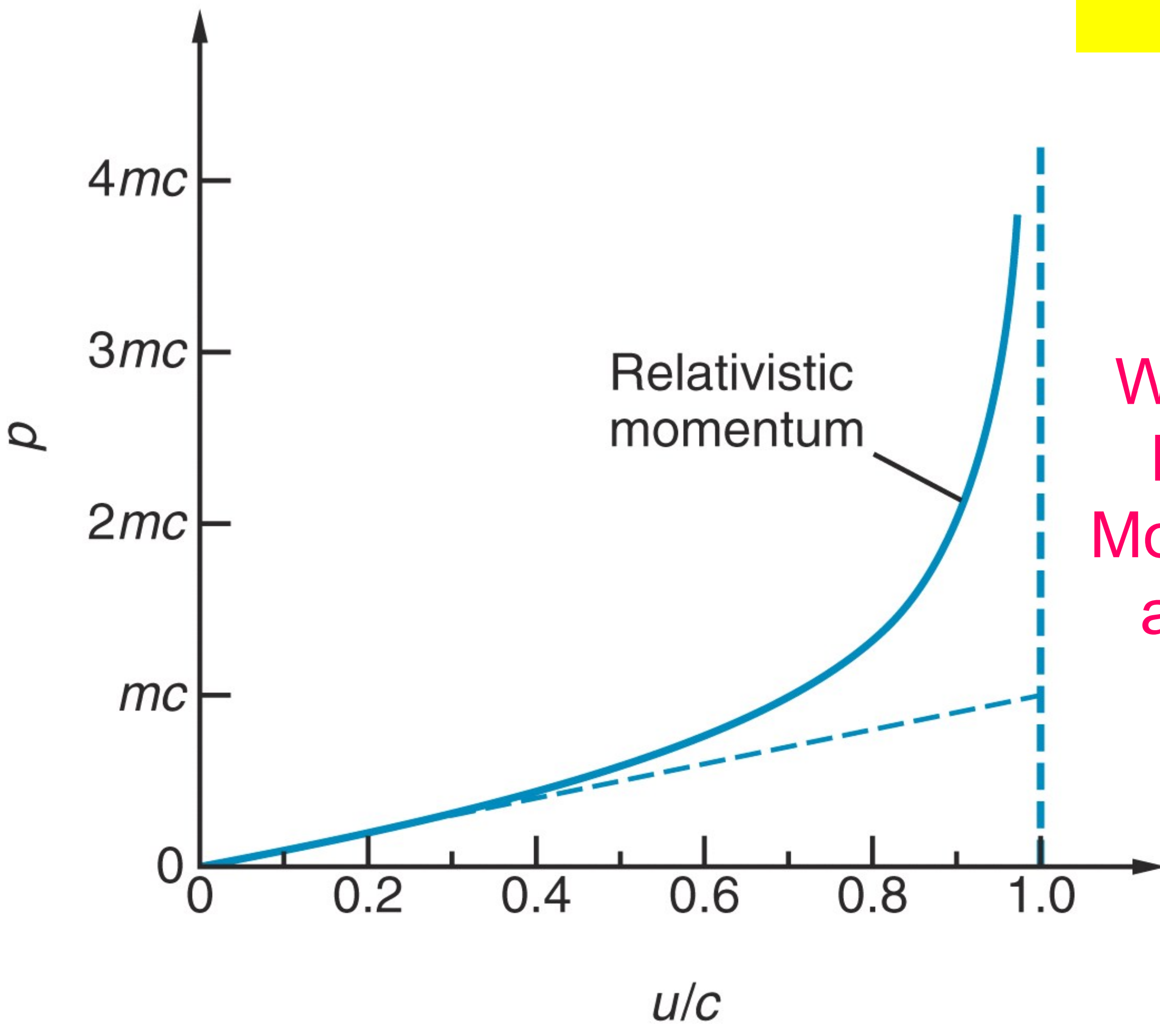
u is velocity of the object

NOT of a reference frame !

Nature of Relativistic Momentum



$$\vec{p} = \frac{m\vec{u}}{\sqrt{1-(u/c)^2}} = \gamma m\vec{u}$$



With the new definition of Relativistic momentum Momentum is conserved in all frames of references

Relativistic Force & Acceleration

$$\vec{p} = \frac{m\vec{u}}{\sqrt{1-(u/c)^2}} = \gamma m\vec{u}$$

Relativistic
Force
And
Acceleration

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}\left(\frac{m\vec{u}}{\sqrt{1-(u/c)^2}}\right) \quad use \quad \frac{d}{dt} = \frac{du}{dt}\frac{d}{du}$$

$$F = \left[\frac{m}{\sqrt{1-(u/c)^2}} + \frac{mu}{\left(1-(u/c)^2\right)^{3/2}} \times \left(\frac{-1}{2}\right)\left(\frac{-2u}{c^2}\right)\right]\frac{du}{dt}$$

$$F = \left[\frac{mc^2 - mu^2 + mu^2}{c^2\left(1-(u/c)^2\right)^{3/2}}\right]\frac{du}{dt}$$

$$F = \left[\frac{m}{\left(1-(u/c)^2\right)^{3/2}}\right]\frac{du}{dt} \quad \text{: Relativistic Force}$$

Since Acceleration $\vec{a} = \frac{d\vec{u}}{dt}$,

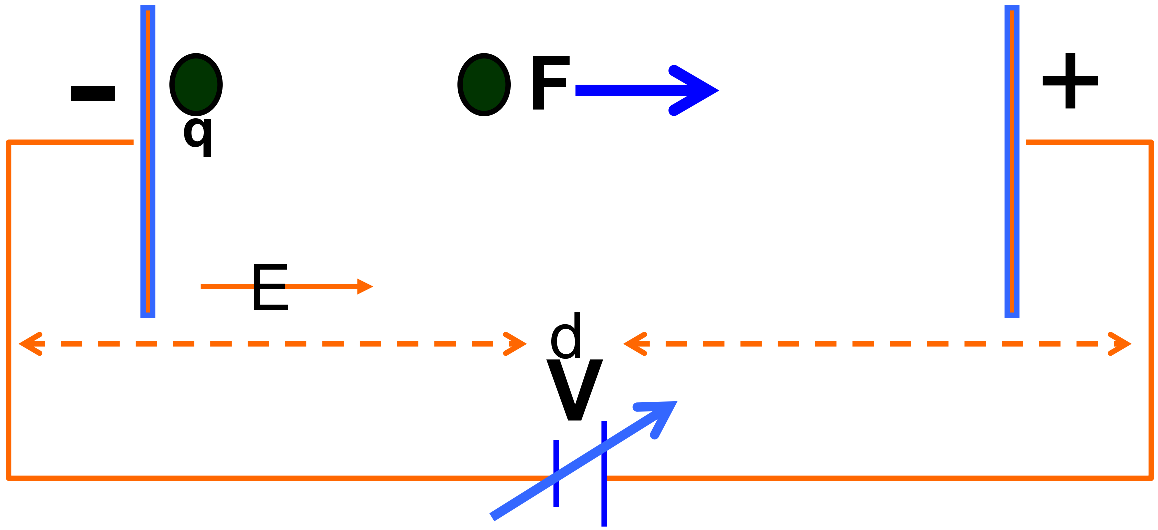
$$\Rightarrow \vec{a} = \frac{\vec{F}}{m}\left[1-(u/c)^2\right]^{3/2}$$

Reason why you cant quite get up to the speed of light no matter how hard you try!

Note: As $u/c \to 1$, $\vec{a} \to 0$!!!!

Its harder to accelerate when you get closer to speed of light

A Linear Particle Accelerator



Parallel Plates

$E = V/d$

$F = eE$

Charged particle q moves in straight line in a uniform electric field $\vec{E}$ with speed $\vec{u}$ accelarates under force $\vec{F}=q\vec{E}$

$$|\vec{a}| = \left|\frac{d\vec{u}}{dt}\right| = \left|\frac{\vec{F}}{m}\right|\left(1-\frac{u^2}{c^2}\right)^{3/2} = \left|\frac{q\vec{E}}{m}\right|\left(1-\frac{u^2}{c^2}\right)^{3/2}$$

larger the potential difference V across plates, larger the force on particle

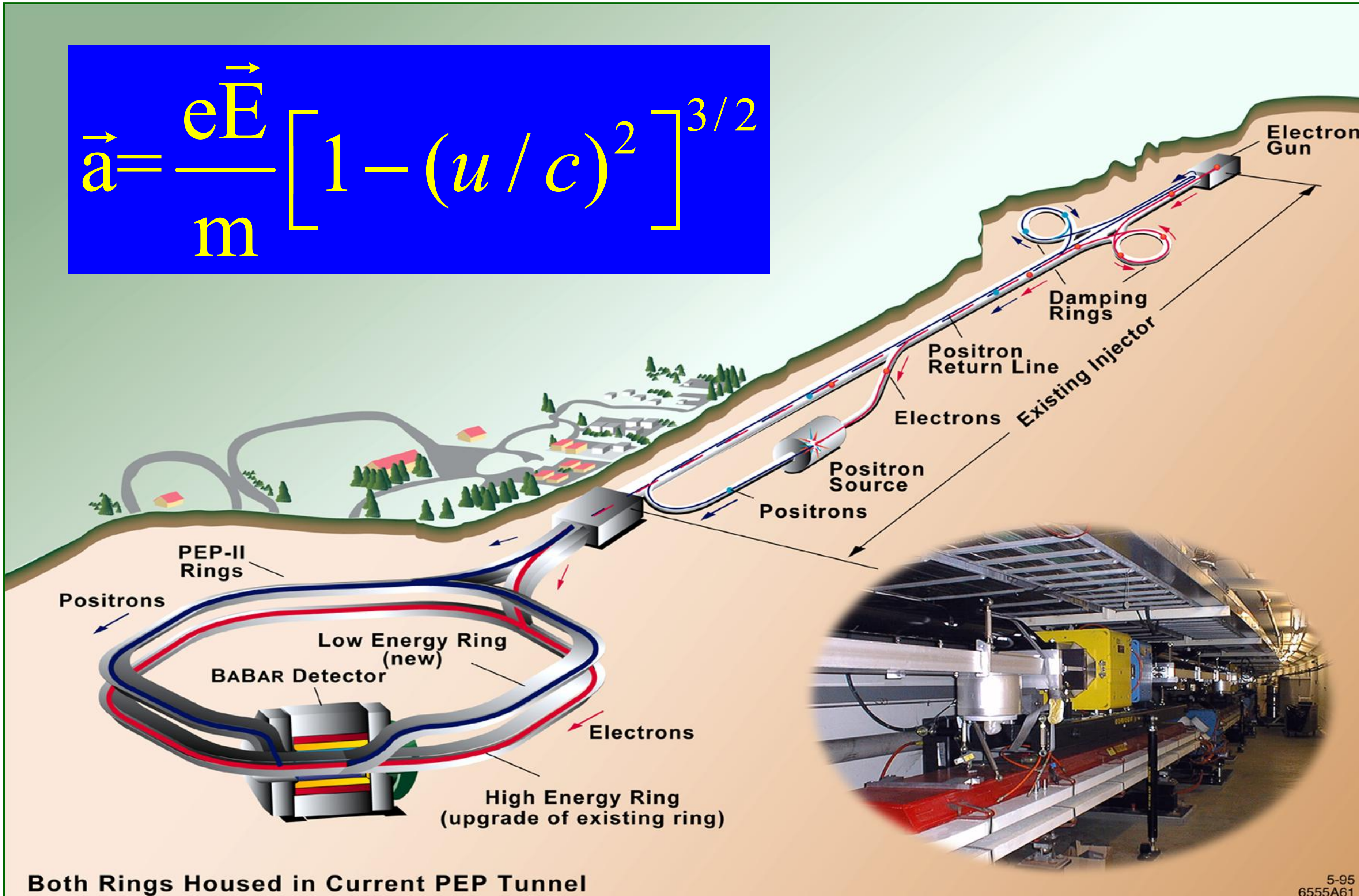
Under force, work is done on the particle, it gains Kinetic energy

New Unit of Energy

1 eV = 1.6×10^{-19} Joules
1 MeV = 1.6×10^{-13} Joules
1 GeV = 1.6x10-10 Joules

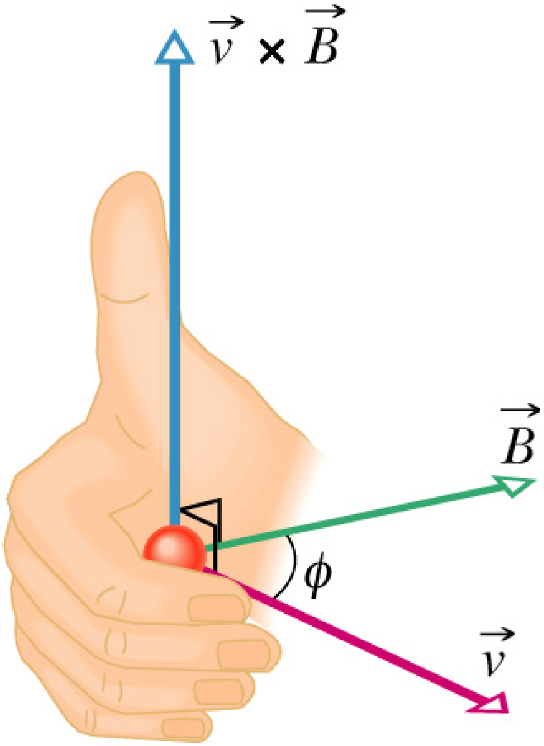
A Linear Particle Accelerator

$$\vec{a} = \frac{e\vec{E}}{m}\left[1-(u/c)^2\right]^{3/2}$$



PEP-II accelerator schematic and tunnel view

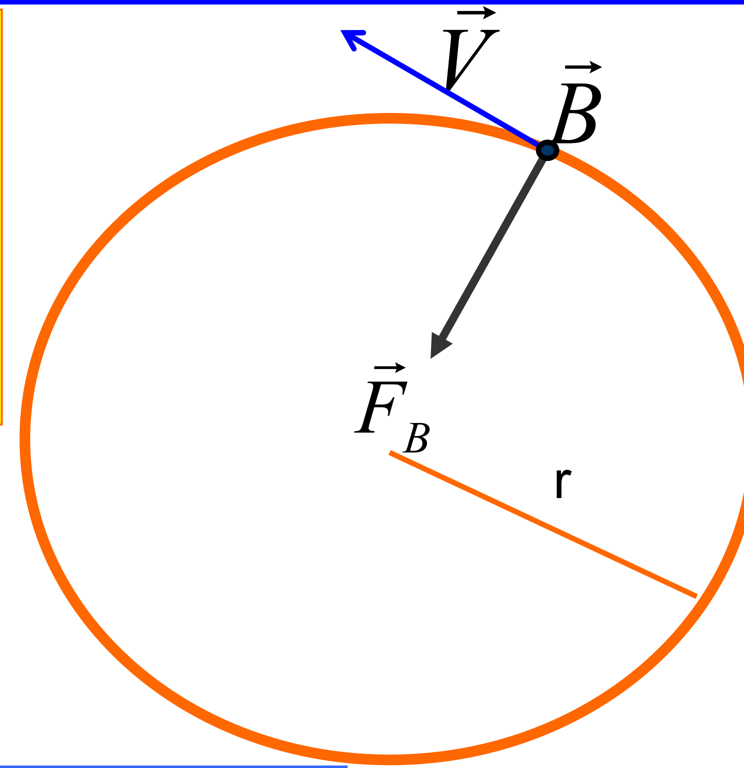
Magnetic Confinement & Circular Particle Accelerator



Classically

$$F = m\frac{v^2}{r}$$

$$qvB = m\frac{v^2}{r}$$

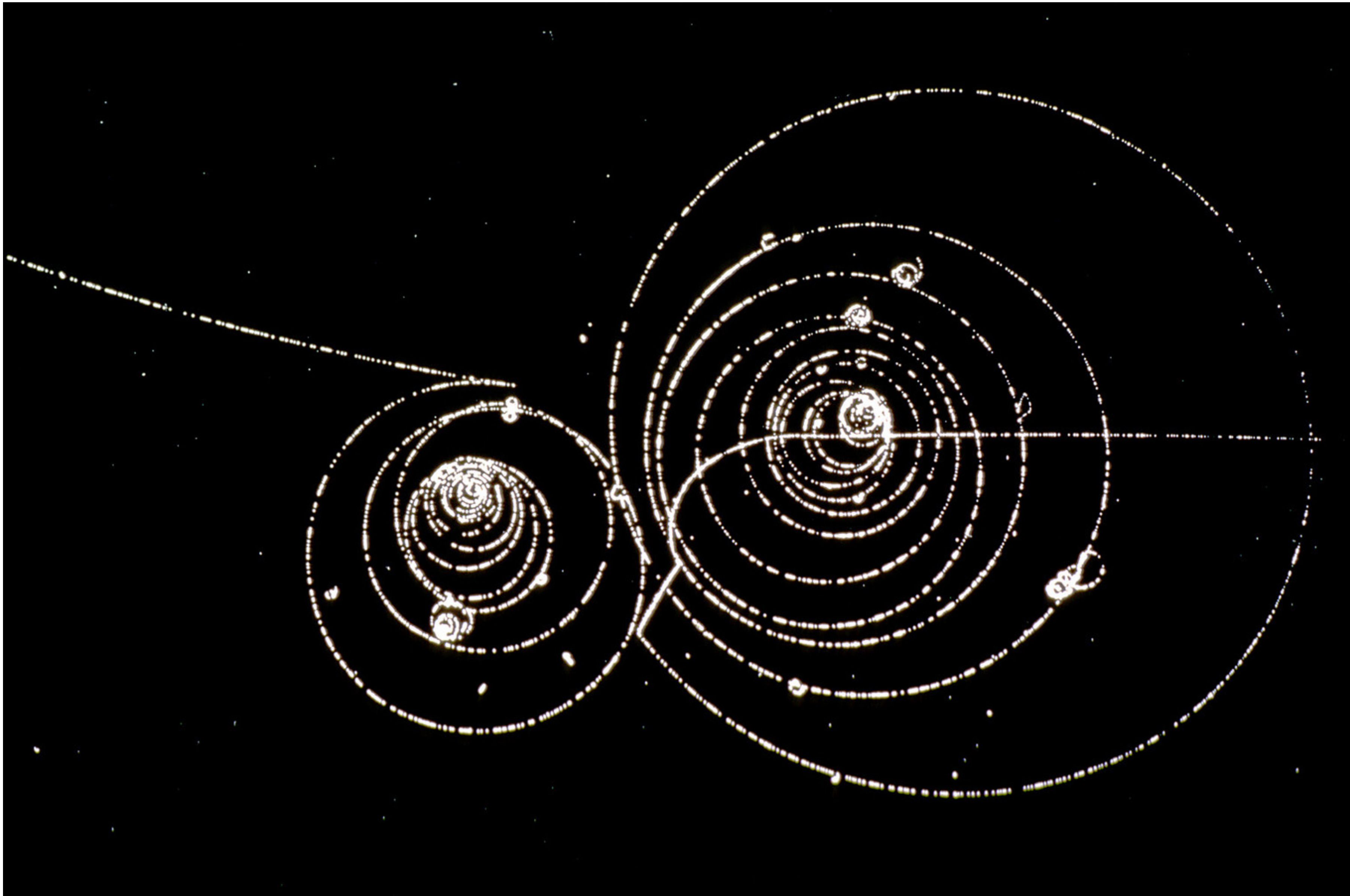


$$F = \frac{dp}{dt} = \frac{d(\gamma mu)}{dt} = \gamma m\frac{du}{dt} = quB$$

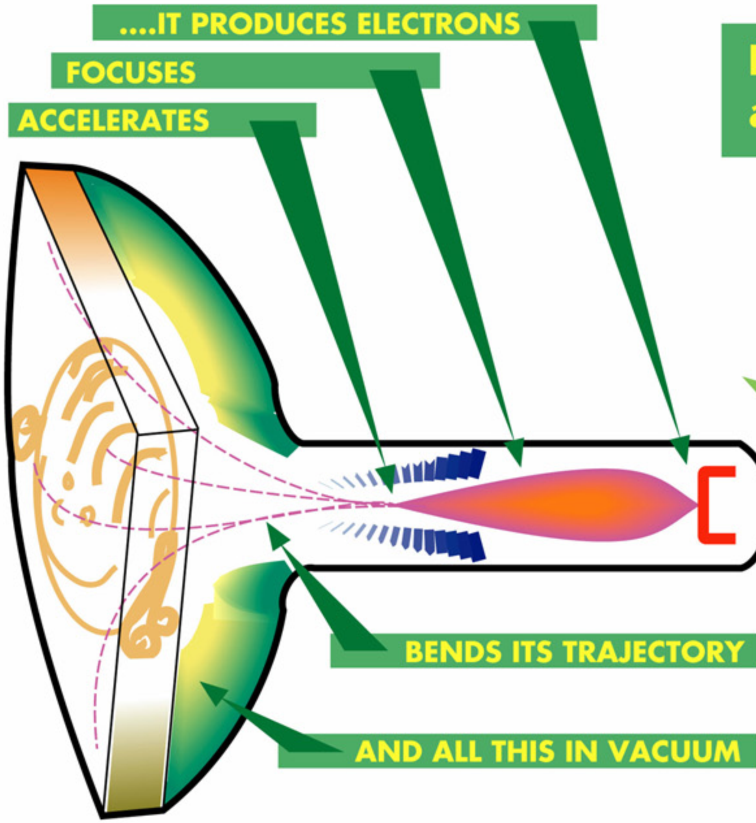
$$\frac{du}{dt} = \frac{u^2}{r} \text{ (Centripetal accelaration)}$$

$$\gamma m\frac{u^2}{r} = quB \quad \Rightarrow \gamma mu = qBr \Rightarrow p = qBr$$

Charged Form of Matter & Anti-Matter in a B Field

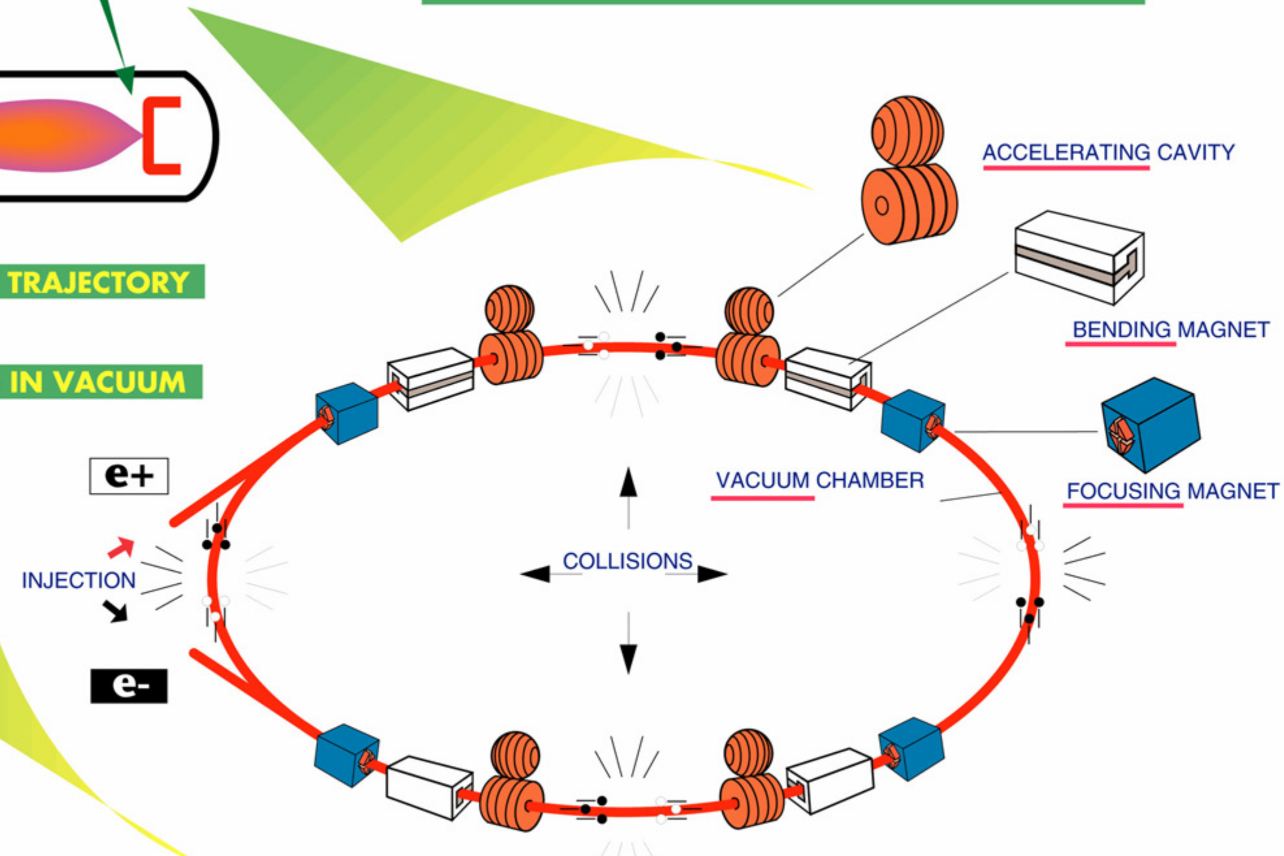


DID YOU KNOW YOUR TELEVISION SET IS AN ACCELERATOR ?



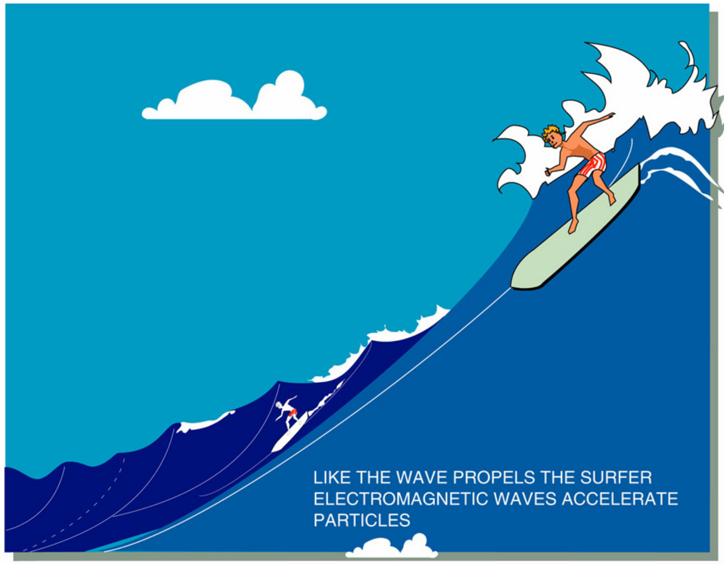
In your TV set, the electrons are accelerated to 20000 volts.

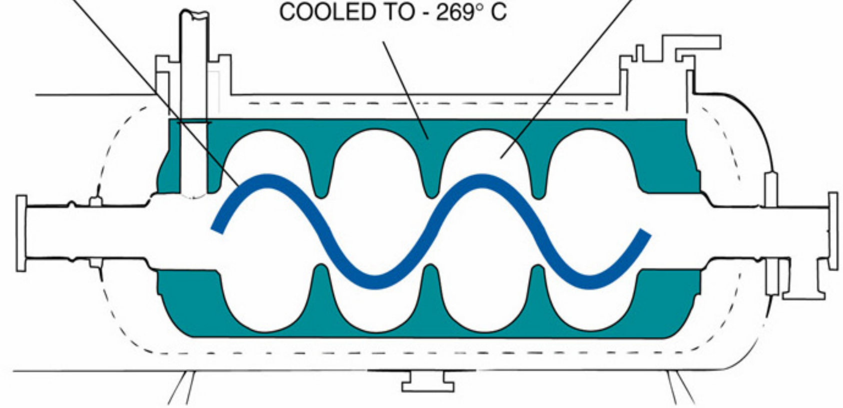
In LEP, they are accelerated to 100 000 000 000 volts.

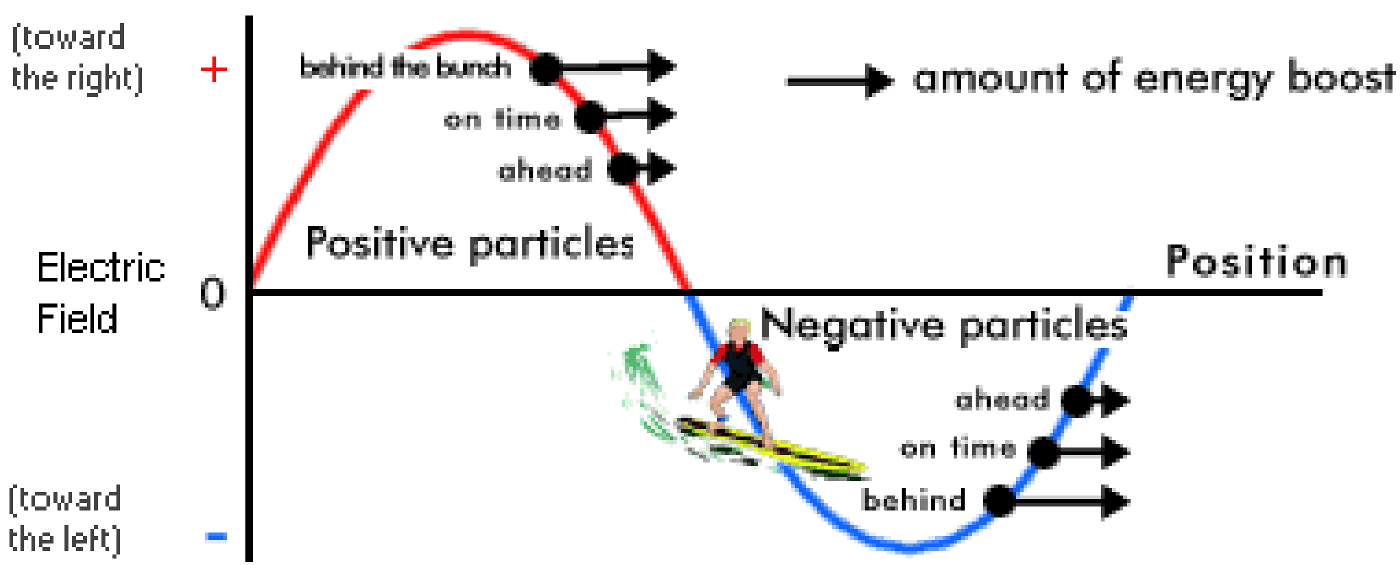


CERN AC - Z34 va - V13/3/98

Accelerating Electrons Thru RF Cavities







Circular Particle Accelerator: LEP @ CERN, Geneve

Magnets Keep Circular Orbit of Particles

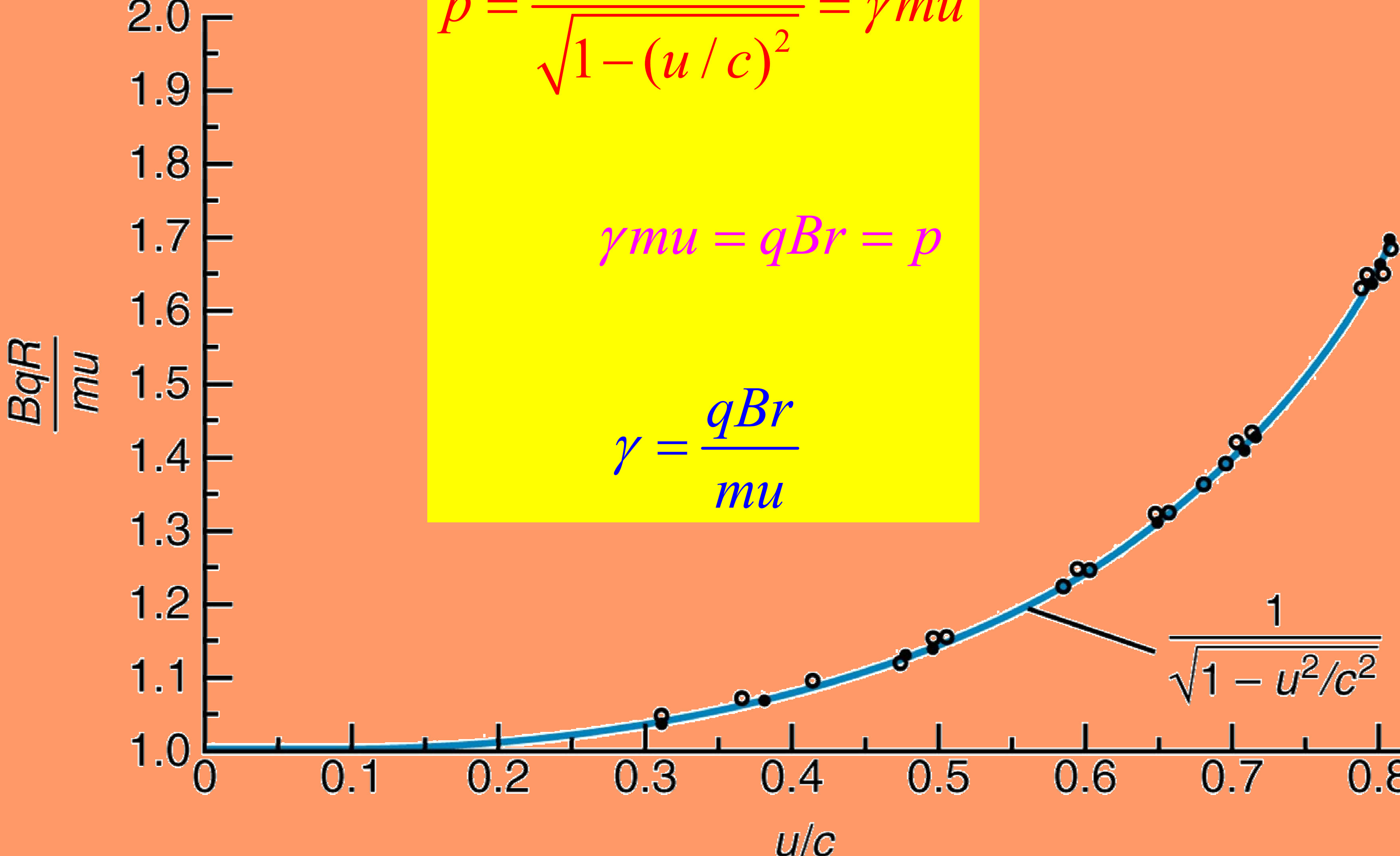
Inside A Circular Particle Accelerator @ CERN

Test of Relativistic Momentum In Circular Accelerator

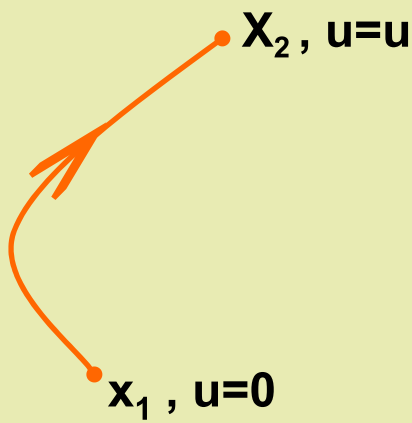
$$\vec{p} = \frac{m\vec{u}}{\sqrt{1-(u/c)^2}} = \gamma m\vec{u}$$

$$\gamma m u = qBr = p$$

$$\gamma = \frac{qBr}{mu}$$



Relativistic Work Done & Change in Energy



$$W = \int_{x_1}^{x_2} \vec{F}.d\vec{x} = \int_{x_1}^{x_2} \frac{d\vec{p}}{dt}.d\vec{x}$$

$$p = \frac{mu}{\sqrt{1-\frac{u^2}{c^2}}} \quad \therefore \quad \frac{d\vec{p}}{dt} = \frac{m\frac{du}{dt}}{\left[1-\frac{u^2}{c^2}\right]^{3/2}},$$ substitute in W,

$$\therefore W = \int_0^u \frac{m\frac{du}{dt}udt}{\left[1-\frac{u^2}{c^2}\right]^{3/2}}$$ (change in var x → u)

$$W = \int_0^u \frac{mudu}{\left[1-\frac{u^2}{c^2}\right]^{3/2}} = \frac{mc^2}{\left[1-\frac{u^2}{c^2}\right]^{1/2}} - mc^2 = \gamma mc^2 - mc^2$$

Work done is change in Kinetic energy K

$K = \gamma mc^2 - mc^2$ or

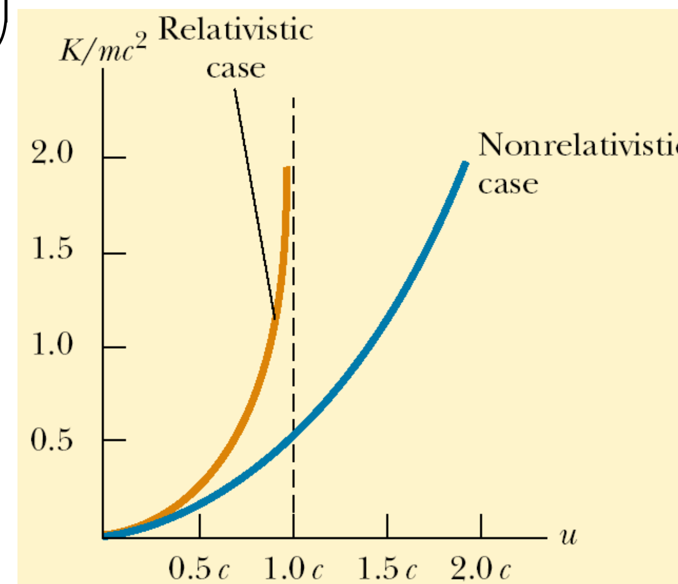
Total Energy $E = \gamma mc^2 = K + mc^2$

But Professor… Why Can’s ANYTHING go faster than light ?

$$K = \frac{mc^2}{\left[1-\frac{u^2}{c^2}\right]^{1/2}} - mc^2 \Rightarrow \left(K+mc^2\right)^2 = \left(\frac{mc^2}{\left[1-\frac{u^2}{c^2}\right]^{1/2}}\right)^2$$

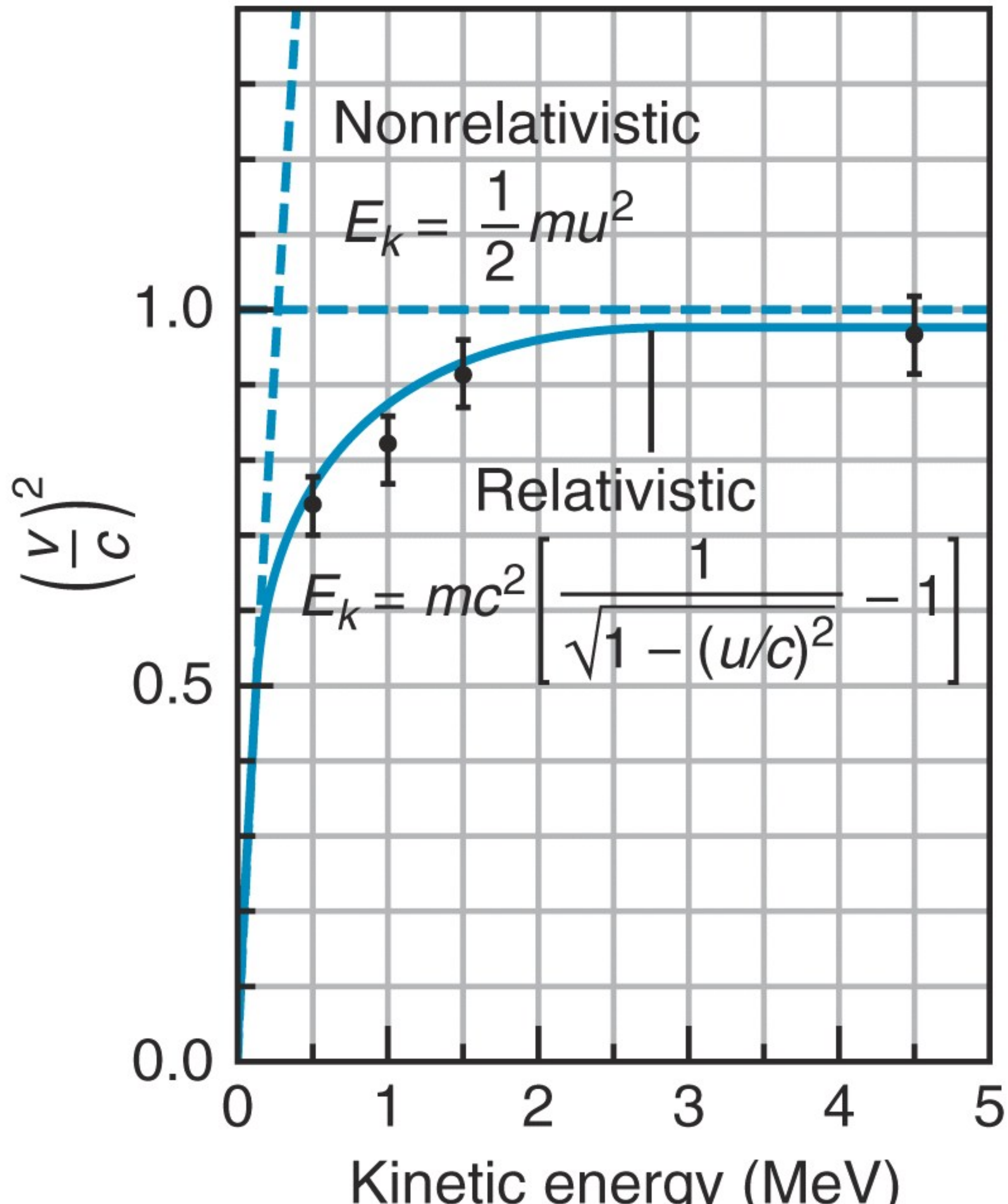
$$\Rightarrow \left[1-\frac{u^2}{c^2}\right] = m^2c^4\left[K+mc^2\right]^{-2}$$

$$\Rightarrow u = c\sqrt{1-\left(\frac{K}{mc^2}+1\right)^{-2}}$$ (Parabolic in u Vs $\frac{K}{mc^2}$)



Non-relativistic case: $K = \frac{1}{2}mu^2 \Rightarrow u = \sqrt{\frac{2K}{m}}$

Relativistic Energy



A Digression: How to Handle Large/Small Numbers

- Example: consider very energetic particle with very large Energy E

$$\gamma = \frac{E}{mc^2} = \frac{mc^2 + K}{mc^2} = 1 + \frac{K}{mc^2}$$

- Lets Say $\gamma = 3\text{x}10^{11}$, Now calculate u from → $\frac{u}{c} = \left[1 - \frac{1}{\gamma^2}\right]^{1/2}$
- Try this on your el-cheapo calculator, you will get u/c =1, u=c due to limited precision.
- In fact u ≅c but not exactly!, try to get this analytically

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{(1-\beta)(1+\beta)}}$$

Since $\beta = \frac{\text{u}}{\text{c}} \cong 1, \quad 1+\beta = 2$

$$\gamma \approx \frac{1}{\sqrt{2}\sqrt{1-\beta}}$$

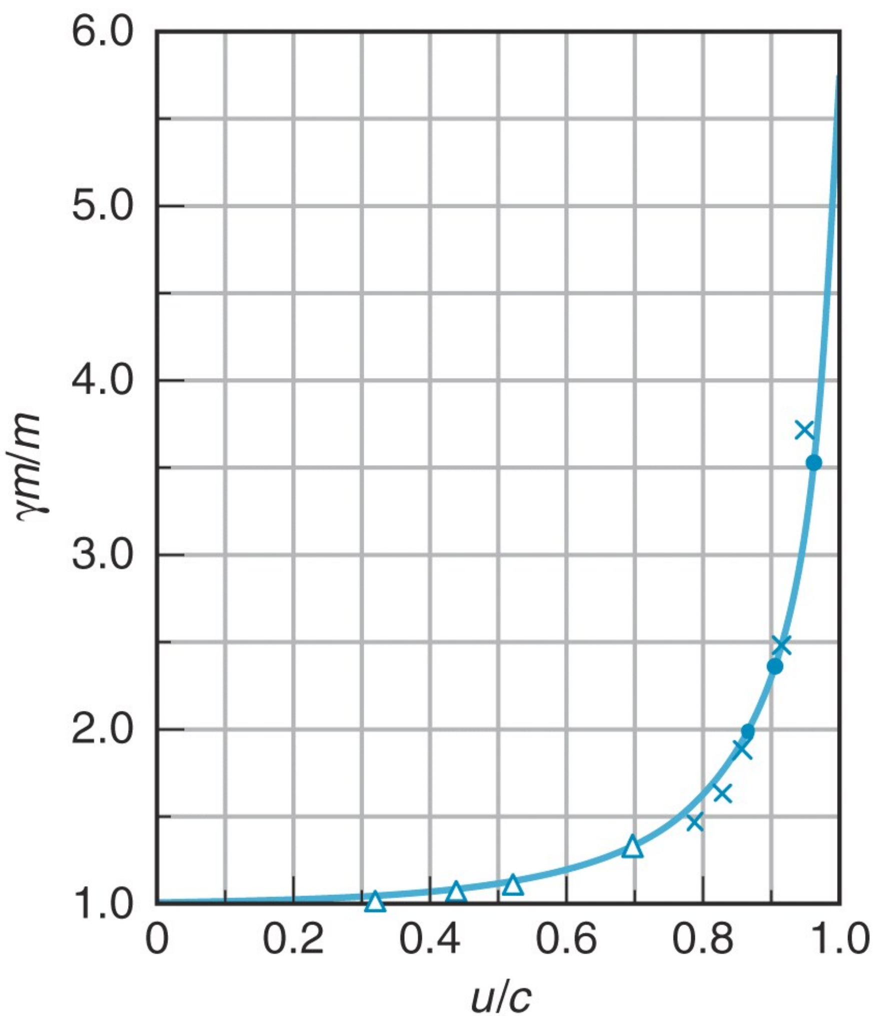
$$\Rightarrow 1-\beta = \frac{1}{2\gamma^2} = 5\times10^{-24}, \quad u = \beta c$$

$$\Rightarrow u = 0.999\ 999\ 999\ 999\ 999\ 999\ 999\ 995\text{c !!}$$

Such particles are routinely produced in violent cosmic collisions

In Quizzes, you are Expected to perform Such simple approximations

When Electron Goes Fast it Gets “Fat”



$$E = \underbrace{\gamma m}c^2$$

As $\frac{v}{c} \to 1, \quad \gamma \to \infty$

Apparent Mass approaches ∞

Relativistic Kinetic Energy & Newtonian Physics

Relativistic KE $= \gamma mc^2 - mc^2$

When $u << c$, $\left[1 - \frac{u^2}{c^2}\right]^{-\frac{1}{2}} \cong 1 + \frac{1}{2}\frac{u^2}{c^2} + \ldots$smaller terms

so $K \cong mc^2[1 + \frac{1}{2}\frac{u^2}{c^2}] - mc^2 = \frac{1}{2}mu^2$ (classical form recovered)

Total Energy of a Particle

$$E = \gamma mc^2 = KE + mc^2$$

For a particle at rest, u = 0

$\Rightarrow$ Total Energy $E = mc^2$

Relationship between P and E

$$\boxed{E = \gamma m c^2} \Rightarrow E^2 = \gamma^2 m^2 c^4$$

$$\boxed{p = \gamma m u} \Rightarrow \boxed{p^2 c^2 = \gamma^2 m^2 u^2 c^2}$$

$$\Rightarrow E^2 - p^2 c^2 = \gamma^2 m^2 c^4 - \gamma^2 m^2 u^2 c^2 = \gamma^2 m^2 c^2 (c^2 - u^2)$$

$$= \frac{m^2 c^2}{1 - \frac{u^2}{c^2}} (c^2 - u^2) = \frac{m^2 c^4}{c^2 - u^2} (c^2 - u^2) = m^2 c^4$$

$$\boxed{E^2 = p^2 c^2 + (mc^2)^2}$$ ....important relation

For particles with zero rest mass like photon (EM waves)

$$\boxed{E = pc \text{ or } p = \frac{E}{c}}$$ (light has momentum!)

Relativistic Invariance : $\boxed{E^2 - p^2 c^2 = m^2 c^4}$: In all Ref Frames

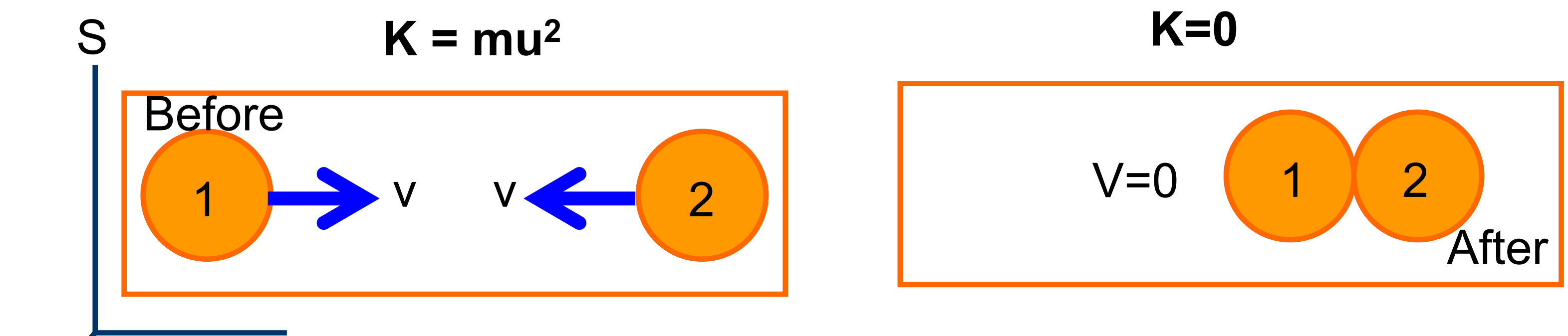
Rest Mass is a "finger print" of the particle

Mass Can "Morph" into Energy & Vice Verca

- Unlike in Newtonian mechanics
- In relativistic physics : Mass and Energy are the same thing
- New word/concept : Mass-Energy , just like spacetime
- It is the mass-energy that is always conserved in every reaction : Before & After a reaction has happened
- Like squeezing a balloon :
  - **If you squeeze mass, it becomes (kinetic) energy & vice verca !**
    - **CONVERSION FACTOR = C^2**
    - **This exchange rate never changes !**

Mass is Energy, Energy is Mass : Mass-Energy Conservation

Examine Kinetic energy Before and After Inelastic Collision: Conserved?



Mass-Energy Conservation: sum of mass-energy of a system of particles before interaction must equal sum of mass-energy after interaction

$$E_{before} = E_{after}$$

$$\frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}} + \frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}} = Mc^2 \Rightarrow M = \frac{2m}{\sqrt{1-\frac{u^2}{c^2}}} > 2m$$

Kinetic energy has been transformed into mass increase

Kinetic energy is not lost, its transformed into more mass in final state

$$\Delta M = M - 2m = \frac{2K}{c^2} = \frac{2}{c^2}\left(\frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}} - mc^2\right)$$