Physics 2D Lecture Slides
Oct 7

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Lorentz Transformation Between Ref Frames

Lorentz Transformation	Inverse Lorentz Transformation
$x' = \gamma(x - vt)$	$x = \gamma(x' + vt)$
$y' = y$	$y = y'$
$z' = z$	$z = z'$
$t' = \gamma\left(t - \frac{vx}{c^2}\right)$	$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$

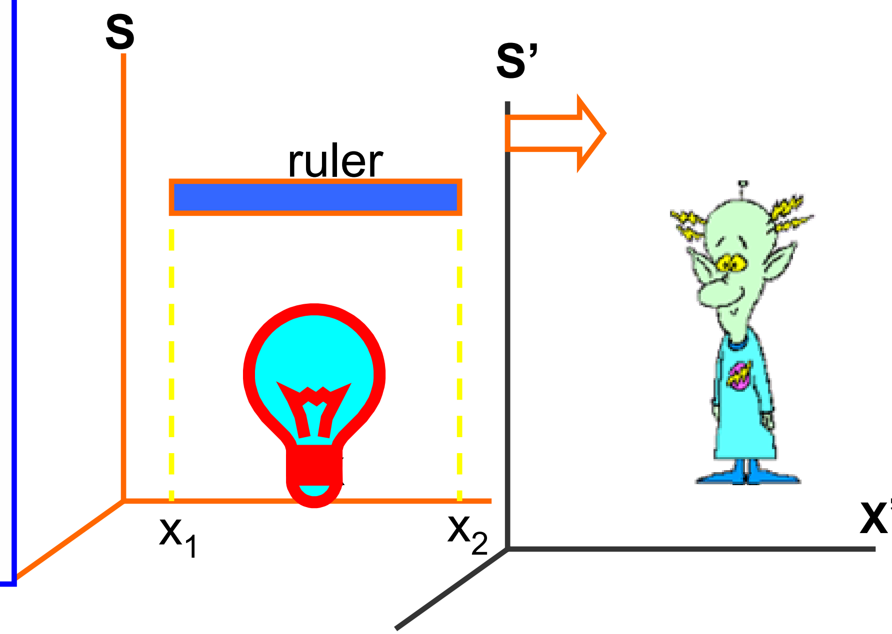
As v→0 , Galilean Transformation is recovered, as per requirement

Notice : SPACE and TIME Coordinates mixed up !!!

Lorentz Transform for Pair of Events

$$\left.\begin{aligned}\Delta x' &= \gamma(\Delta x - v\,\Delta t)\\ \Delta t' &= \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right)\end{aligned}\right\} S \longrightarrow S'$$

$$\left.\begin{aligned}\Delta x &= \gamma(\Delta x' + v\,\Delta t')\\ \Delta t &= \gamma\left(\Delta t' + \frac{v}{c^2}\Delta x'\right)\end{aligned}\right\} S' \longrightarrow S$$



Can understand Simultaneity, Length contraction & Time dilation formulae from this

Time dilation: Bulb in S frame turned on at t_1 & off at t_2 : What $\Delta t'$ did S' measure ?

two events occur at same place in S frame => $\Delta x = 0$

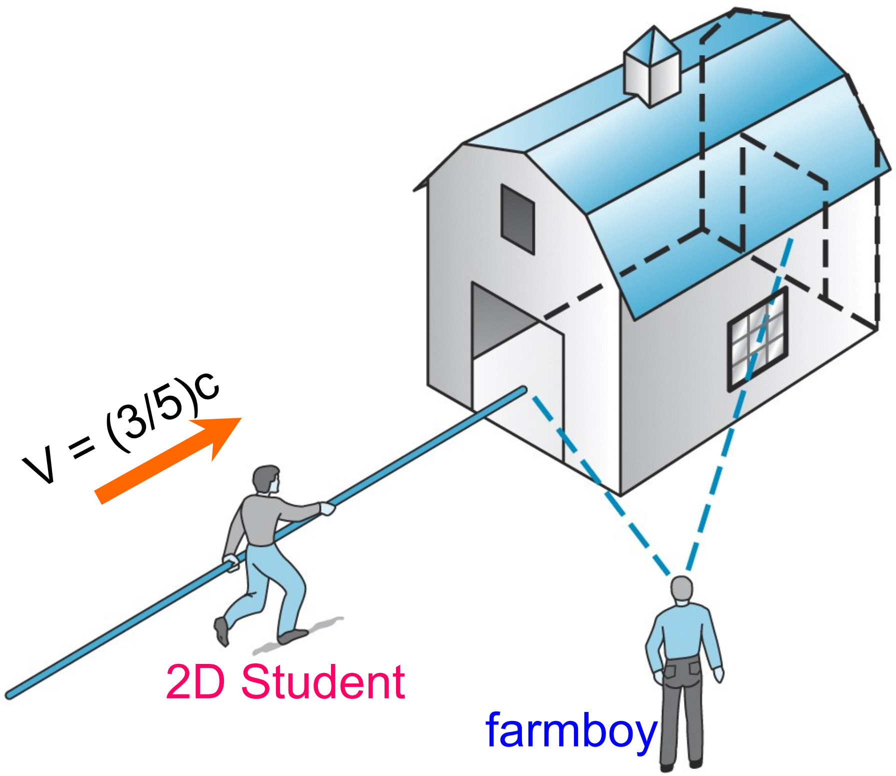
$$\Delta t' = \gamma\,\Delta t \quad (\Delta t = \text{proper time})$$

Length Contraction: Ruler measured in S between x_1 & x_2 : What $\Delta x'$ did S' measure ?

two ends measured at same time in S' frame => $\Delta t' = 0$

$$\Delta x = \gamma\,(\Delta x' + 0) \Rightarrow \Delta x' = \Delta x / \gamma \qquad (\Delta x = \text{proper length})$$

Fitting a 5m pole in a 4m barnhouse



Student with pole runs with v=(3/5)c

farmboy sees pole contraction factor

$$\sqrt{1-(3c/5c)^2} = 4/5$$

says pole just fits in the barn fully!

Student with pole runs with v=(3/5)c

Student sees barn contraction factor

$$\sqrt{1-(3c/5c)^2} = 4/5$$

says barn is only 3.2m long, too short

to contain entire 5m pole !

Farmboy says “You can do it”

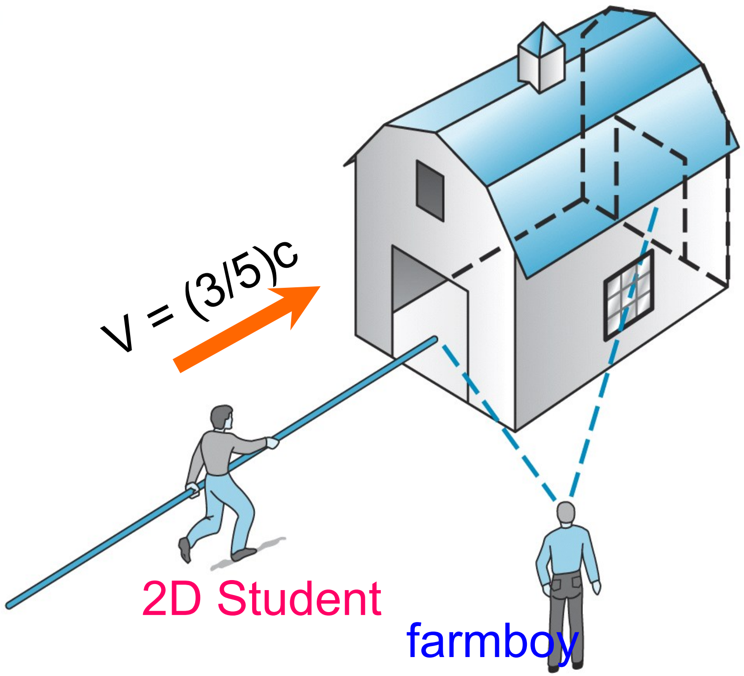
Student says “Dude, you are nuts”

Is there a contradiction ? Is Relativity wrong?

Homework: You figure out who is right, if any and why.

Hint: Think in terms of observing three events

Fitting a 5m pole in a 4m barnhouse?



A: Arrival of right end of pole at left end of barn
B: Arrival of left end of pole at left end of barn
C: Arrival of right end of pole at right end of barn

Let S = Barn frame, S' = student frame

Answer : Simultaneity !

Event A : arrival of right end of pole at left end of barn: (t =0, t'=0) is reference

$L_0' $ = proper length of pole in S'

l_0 = length of barn in S frame $< L_0'$

In S: length of pole $L = L_0'\sqrt{1-(v/c)^2}$

The times in two frames are related:

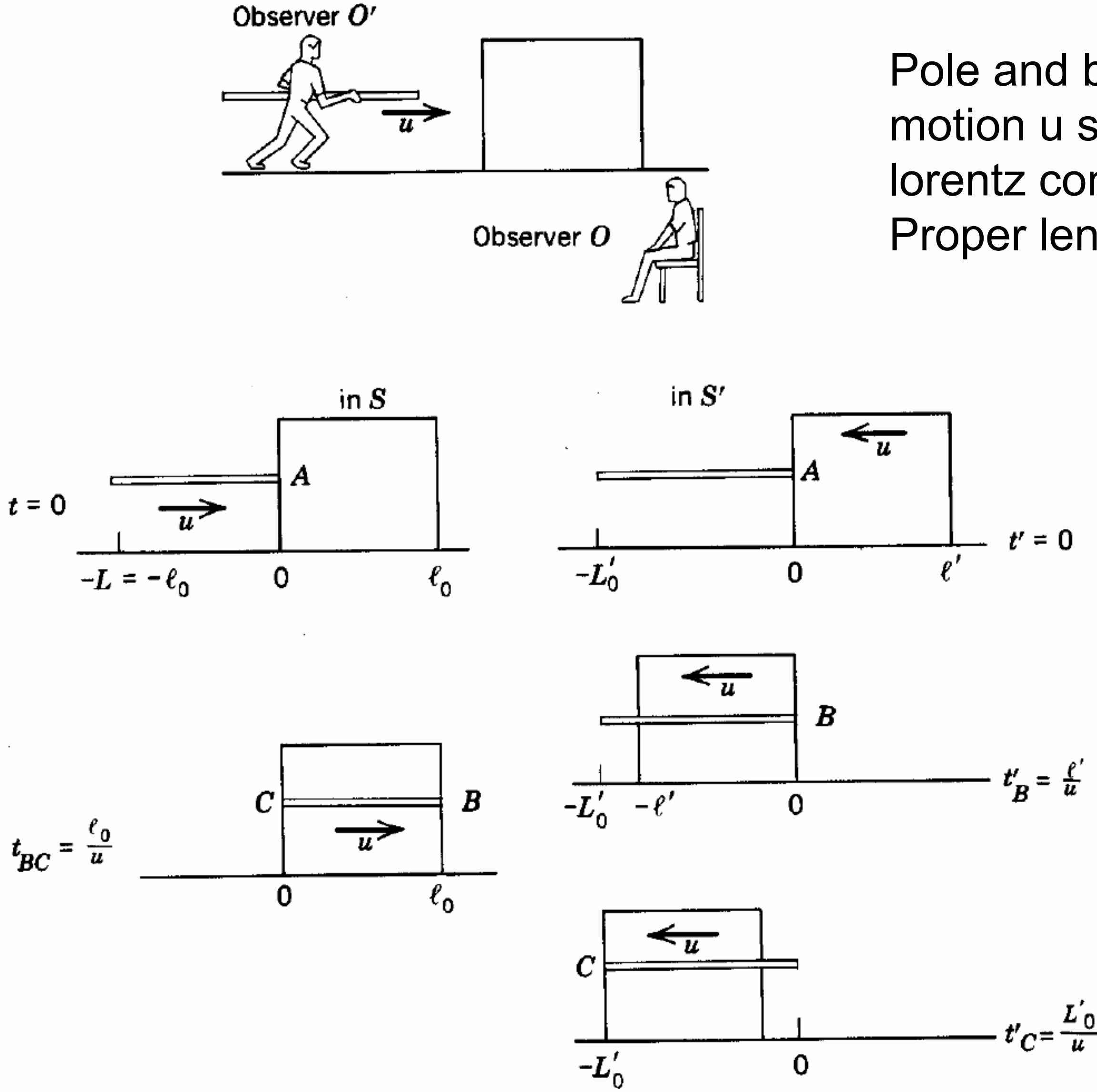
$$t_B' = \frac{l'}{v} = \frac{l_0}{v}\sqrt{1-(v/c)^2} = t_{BC}\sqrt{1-(v/c)^2}$$

$$t_C' = \frac{L_0'}{v} = \frac{l'}{v}\frac{1}{1-(v/c)^2} = \frac{t_{BC}}{\sqrt{1-(v/c)^2}}$$

$\Rightarrow$ Time gap in S' by which events B and C fail to be simultaneous

Farmboy sees two events as simultaneous
2D student can not agree
Fitting of the pole in barn is relative !

Farmboy Vs 2D Student

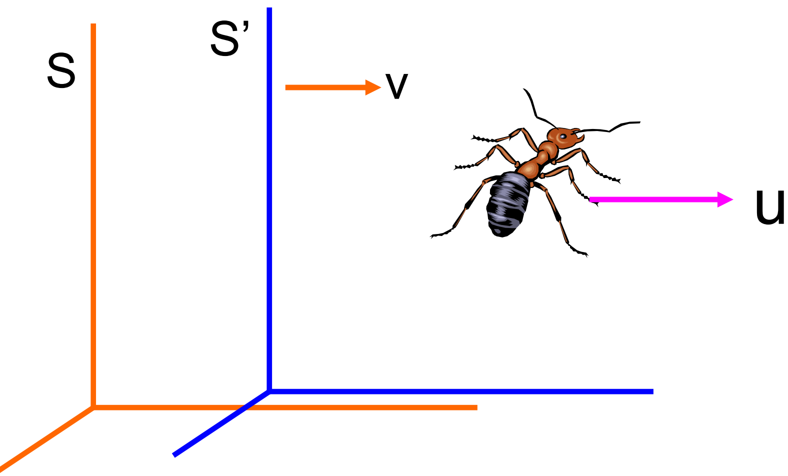


Pole and barn are in relative motion u such that lorentz contracted length of pole = Proper length of barn

In rest frame of pole, Event B precedes C

Lorentz Velocity Transformation Rule

S and S' are measuring ant's speed u along x, y, z axes



In S' frame, $u_{x'} = \dfrac{x_2' - x_1'}{t_2' - t_1'} = \dfrac{dx'}{dt'}$

$dx' = \gamma(dx - v\,dt)$, $dt' = \gamma\left(dt - \dfrac{v}{c^2}dx\right)$

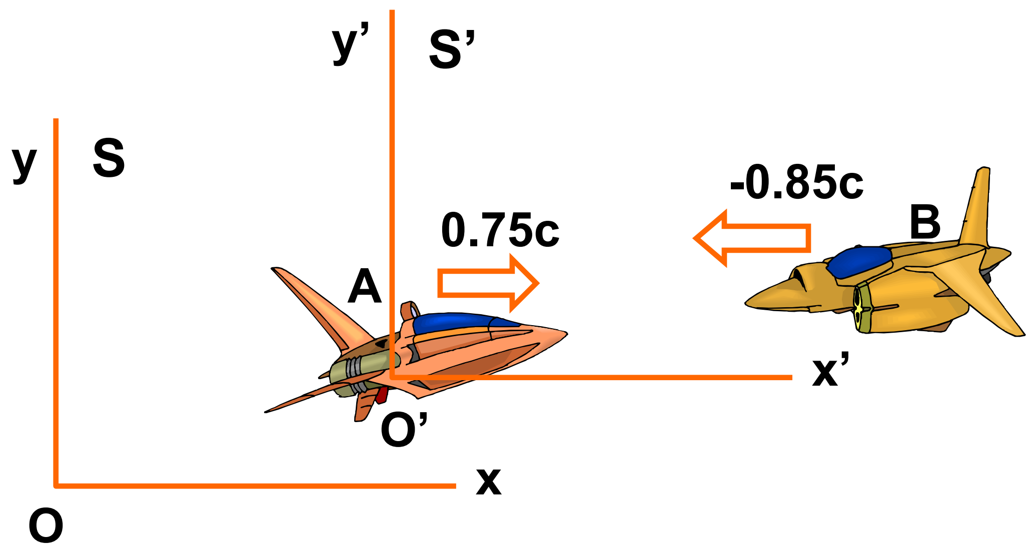
$u_{x'} = \dfrac{dx - v\,dt}{dt - \dfrac{v}{c^2}dx}$, divide by dt'

$u_{x'} = \dfrac{u_x - v}{1 - \dfrac{v u_x}{c^2}}$

For v << c, $u_{x'} = u_x - v$

(Galilean Trans. Restored)

Does Lorentz Transform "work" ?



Two rockets travel in opposite directions

An observer on earth (S) measures speeds = 0.75c And 0.85c for A & B respectively

What does A measure as B's speed?

Place an imaginary S' frame on Rocket A $\Rightarrow$ v = 0.75c relative to Earth Observer S

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{-0.850c - 0.750c}{1 - \frac{(-0.850c)(0.750c)}{c^2}} = -0.977c$$

Consistent with Special Theory of Relativity

Velocity Transformation Perpendicular to S-S' motion

$$dy' = dy, \quad dt' = \gamma(dt - \frac{v}{c^2}dx)$$

$$u_y' = \frac{dy'}{dt'} = \frac{dy}{\gamma(dt - \frac{v}{c^2}dx)}$$

divide by dt on RHS

$$u_y' = \frac{u_y}{\gamma(1 - \frac{v}{c^2}u_x)}$$

There is a change in velocity in the direction $\perp$ to S-S' motion !

Similarly Z component of Ant's velocity transforms as

$$u_z' = \frac{u_z}{\gamma(1 - \frac{v}{c^2}u_x)}$$

Inverse Lorentz Velocity Transformation

Inverse Velocity Transform:

$$u_x = \frac{u_{x'} + v}{1 + \frac{v u_{x'}}{c^2}}$$

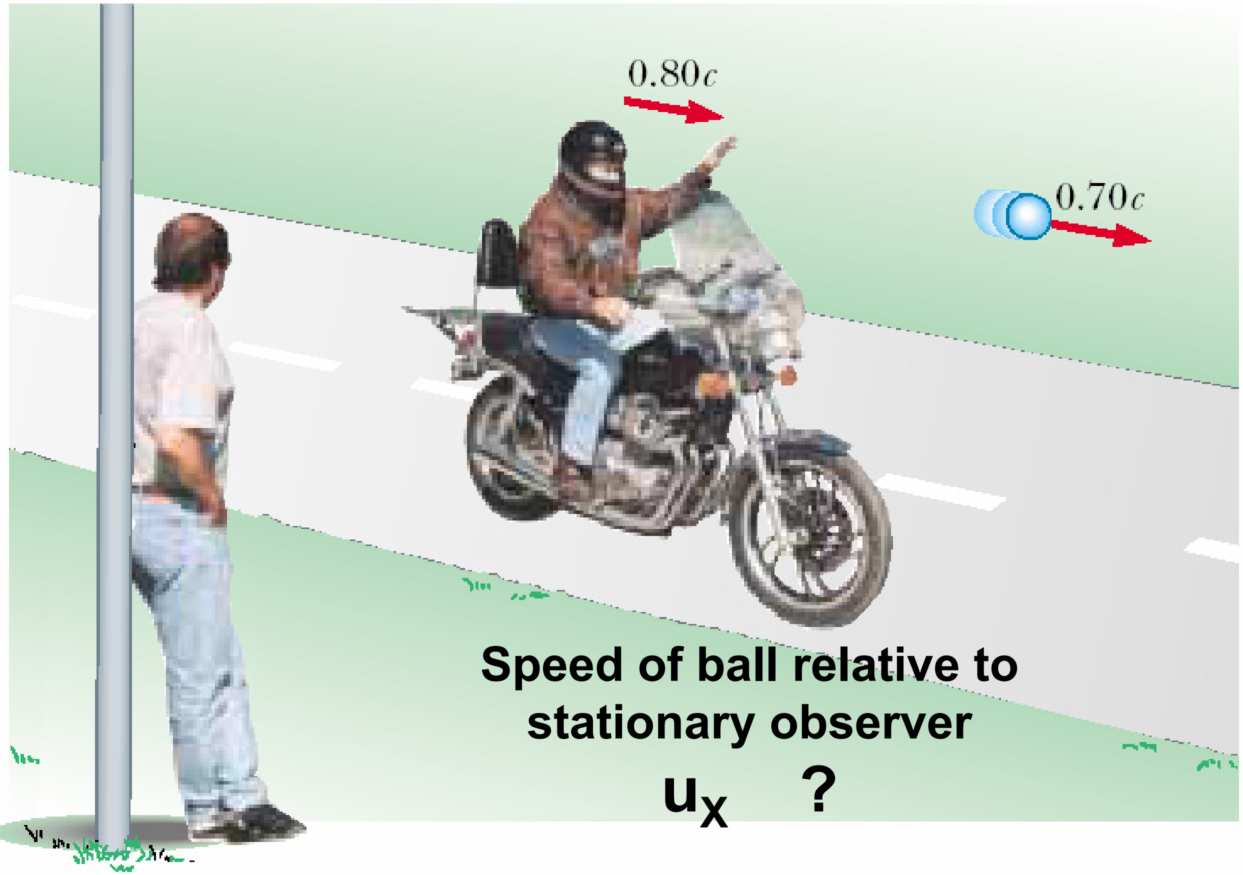
$$u_y = \frac{u_y'}{\gamma(1 + \frac{v}{c^2} u_x')}$$

$$u_z = \frac{u_z'}{\gamma(1 + \frac{v}{c^2} u_x')}$$

As usual, replace

$v \Rightarrow -v$

Example of Inverse velocity Transform



Biker moves with speed = 0.8c past stationary observer

Throws a ball forward with speed = 0.7c

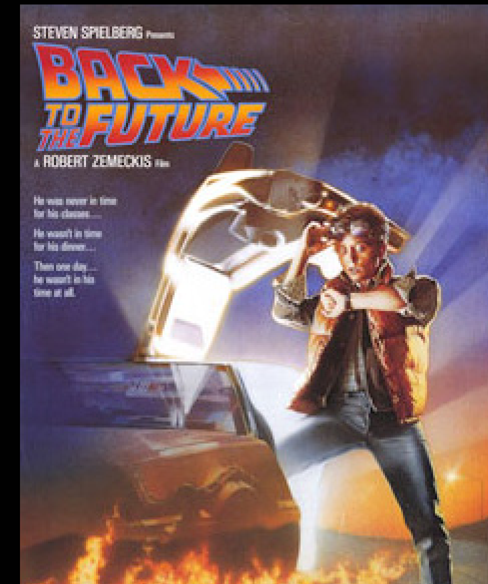
What does stationary observer see as velocity of ball ?

Place S' frame on biker
Biker sees ball speed

$u_{x'} = 0.7c$

$$u_x = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}} = \frac{0.70c + 0.80c}{1 + \frac{(0.70c)(0.80c)}{c^2}} = 0.96c$$

Hollywood Yarns !

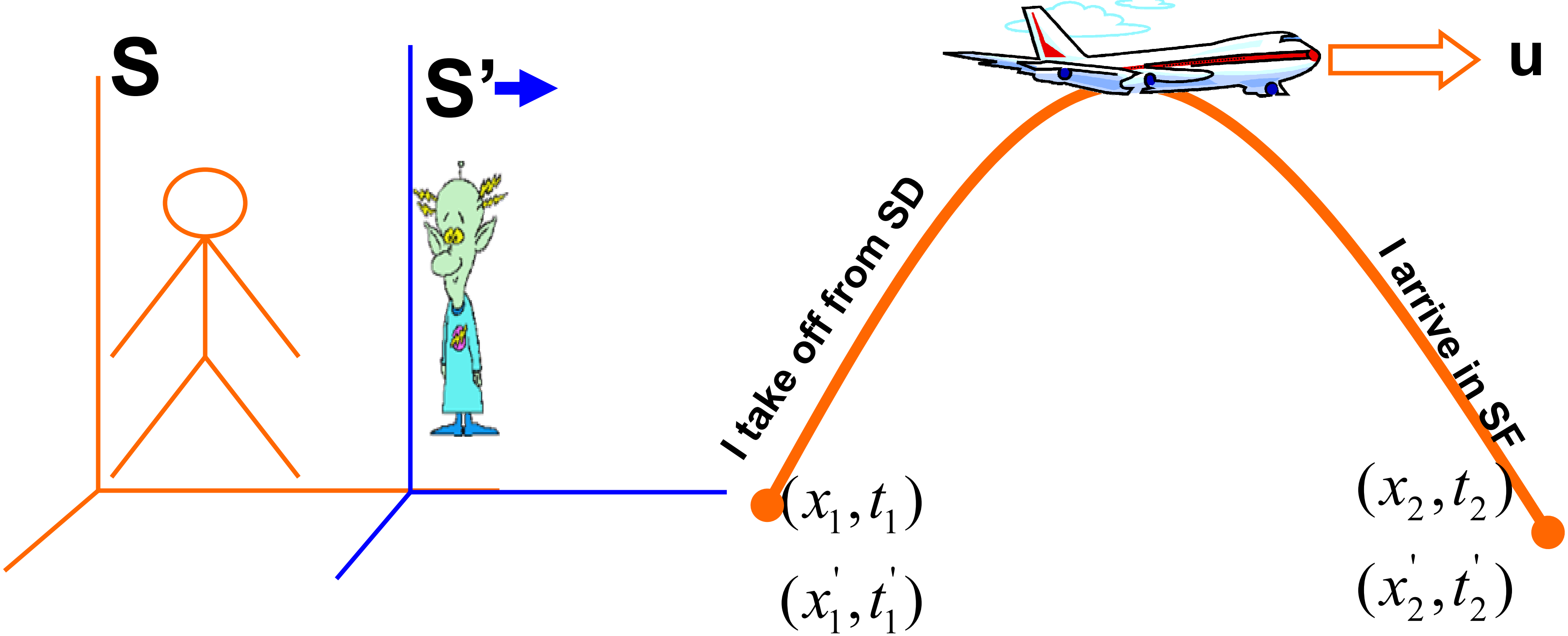


IT'S NOTHING PERSONAL.
TERMINATOR 2
JUDGMENT DAY

BACK TO THE FUTURE

Terminator : Can you be **seen** to be born before your mother?

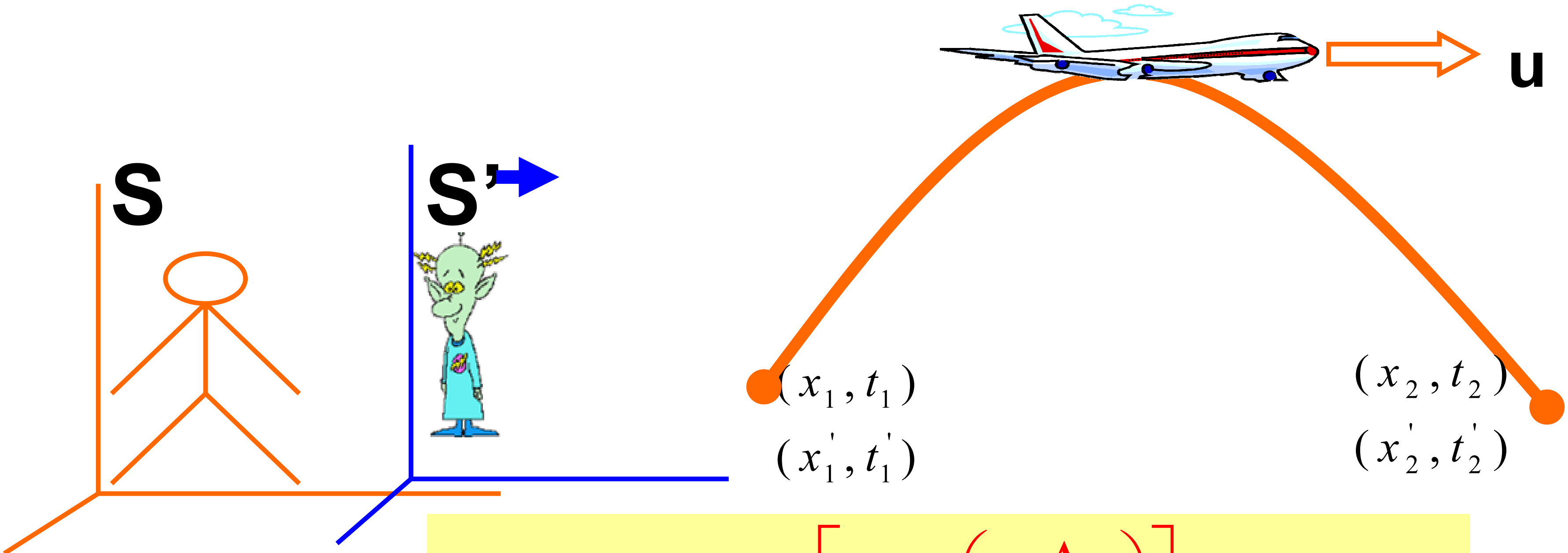
A frame of Ref where sequence of events is REVERSED **?!!**



$$\Delta t' = t_2' - t_1' = \gamma \left[\Delta t - \left(\frac{v\,\Delta x}{c^2} \right) \right]$$

Reversing sequence of events $\Rightarrow \Delta t' < 0$

I Cant 'be seen to arrive in SF before I take off from SD



$$\Delta t' = t_2' - t_1' = \gamma\left[\Delta t - \left(\frac{v\,\Delta x}{c^2}\right)\right]$$

For what value of v can $\Delta t' < 0$

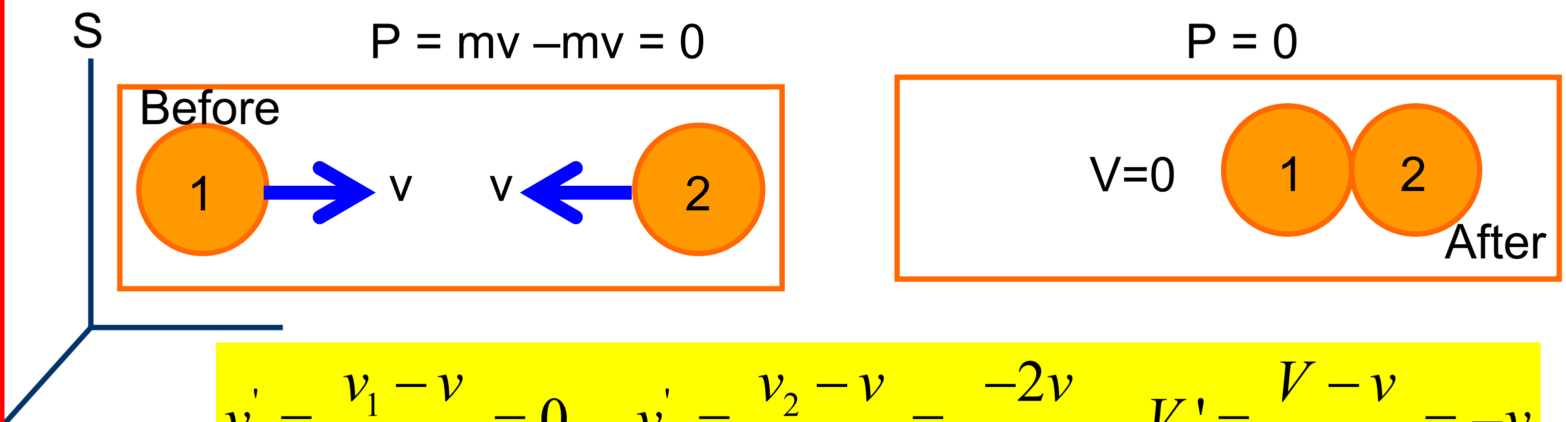
$$\Delta t' < 0 \Rightarrow \Delta t < \frac{v\,\Delta x}{c^2} \quad \Rightarrow 1 < \frac{v\,\Delta x}{c^2 \Delta t} = \frac{v\,u}{c^2}$$

$$\Rightarrow \frac{v}{c} > \frac{c}{u} \qquad \Rightarrow v > c : \text{Not allowed}$$

Relativistic Momentum and Revised Newton's Laws

Need to generalize the laws of Mechanics & Newton to confirm to Lorentz Transform and the Special theory of relativity: Example : $\vec{p} = m\vec{u}$

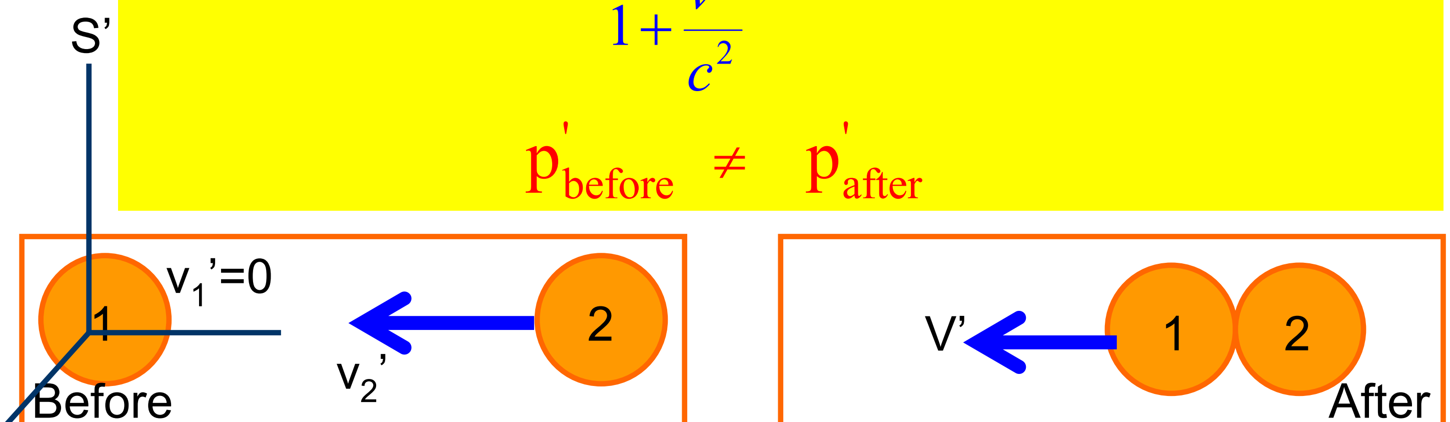
Watching an Inelastic Collision between two putty balls



$$v_1' = \frac{v_1 - v}{1 - \frac{v_1 v}{c^2}} = 0, \quad v_2' = \frac{v_2 - v}{1 - \frac{v_1 v}{c^2}} = \frac{-2v}{1 + \frac{v^2}{c^2}}, \quad V' = \frac{V - v}{1 - \frac{Vv}{c^2}} = -v$$

$$p_{before}' = mv_1' + mv_2' = \frac{-2mv}{1 + \frac{v^2}{c^2}}, \quad p_{after}' = 2mV' = -2mv$$

$$p_{\text{before}}' \neq p_{\text{after}}'$$



Definition (without proof) of Relativistic Momentum

$$\vec{p} = \frac{m\vec{u}}{\sqrt{1-(u/c)^2}} = \gamma m\vec{u}$$

With the new definition relativistic momentum is conserved in all frames of references : Do the exercise

New Concepts

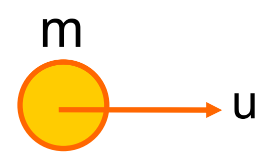
Rest mass = mass of object measured
In a frame of ref. where object is at rest

$$\gamma = \frac{1}{\sqrt{1-(u/c)^2}}$$

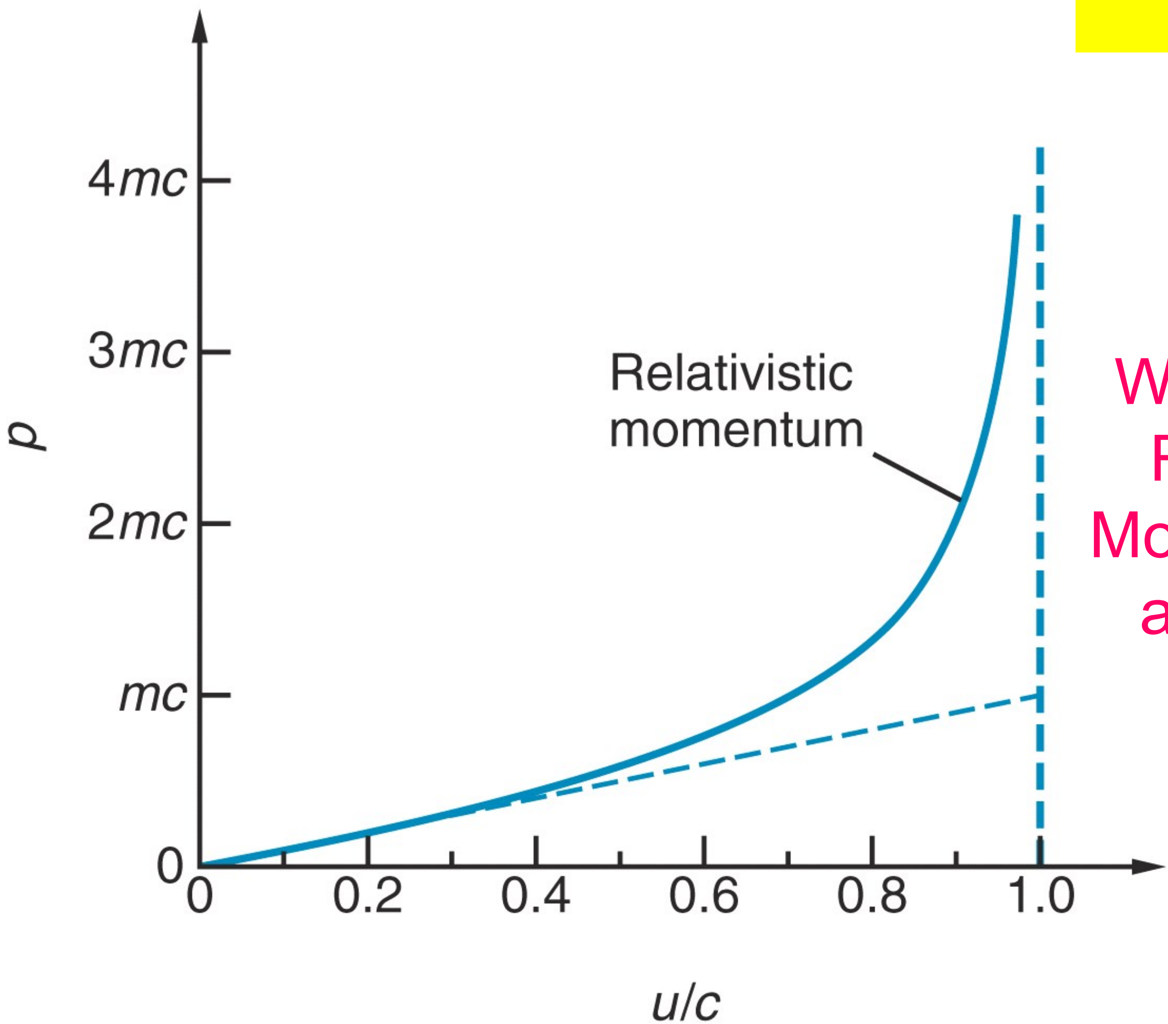
u is velocity of the object

NOT of a reference frame !

Nature of Relativistic Momentum



$$\vec{p} = \frac{m\vec{u}}{\sqrt{1-(u/c)^2}} = \gamma m\vec{u}$$



With the new definition of Relativistic momentum Momentum is conserved in all frames of references