Physics 2D Lecture Slides
Oct 6

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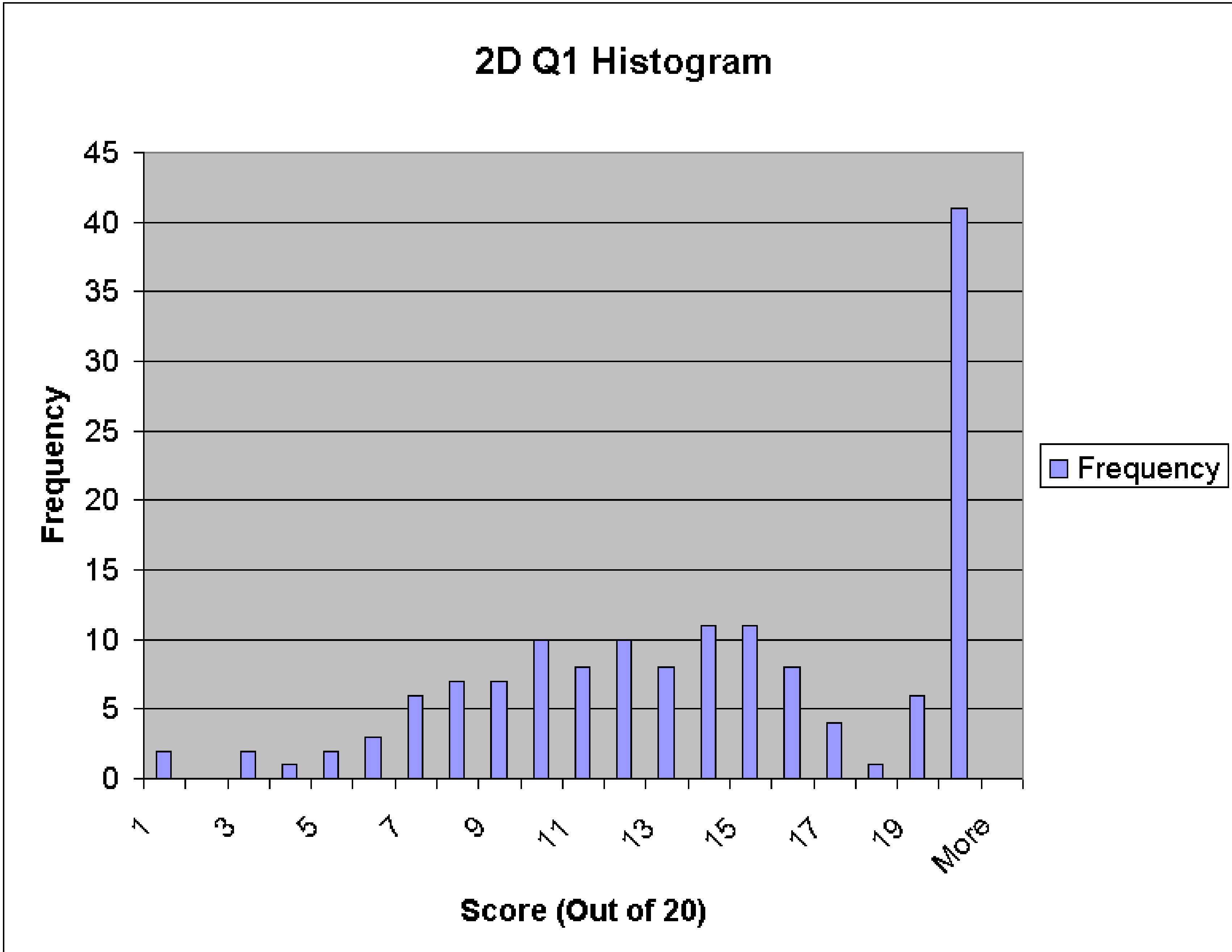
UCSD Physics


2D Q1 Histogram
Frequency
45
40
35
30
25
20
15
10
5
0
1
3
5
7
9
11
13
15
17
19
More
Score (Out of 20)
Frequency

New Rules of Coordinate Transformation Needed

- The Galilean/Newtonian rules of transformation could not handles frames of refs or objects traveling fast
  - $V \approx C$ (like v = 0.1 c or 0.8c or 1.0c)
- Einstein's postulates led to
  - Destruction of concept of simultaneity ($\Delta t \neq \Delta t'$)
  - Moving clocks run slower
  - Moving rods shrink
- Lets formalize this in terms of general rules of coordinate transformation : Lorentz Transformation
  - Recall the Galilean transform
    - $X' = (x-vt)$
    - $T' = T$
      - These rules that work ok for mule carts now must be modified for rocket ships with $V \approx C$

Discovering The Correct Transformation Rule

$x' = x - vt$ guess $\rightarrow$ $x' = G(x - vt)$

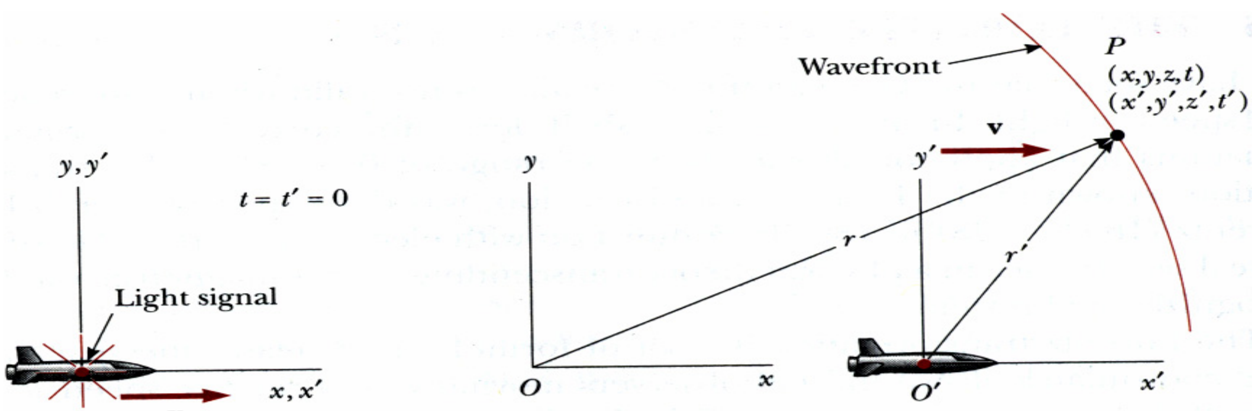
$x = x' + vt'$ guess $\rightarrow$ $x = G(x' + vt')$

Need to figure out functional form of G !

G must be dimensionless
G does not depend on x,y,z,t
But G depends on v/c
G is symmetric
As $v/c \rightarrow 0$, $G \rightarrow 1$

Do a Thought Experiment: Rocket Motion along x axis

Rocket in S' (x',y',z',t') frame moving with velocity v w.r.t observer on frame S (x,y,z,t)
Flashbulb mounted on rocket emits pulse of light at the instant origins of S,S' coincide
That instant corresponds to t = t' = 0 . Light travels as a spherical wave, origin is at O,O'



Speed of light is c for both observers

Examine a point P (at distance r from O and r' from O') on the Spherical Wavefront
The distance to point P from O : r = ct
The distance to point P from O : r' = ct'

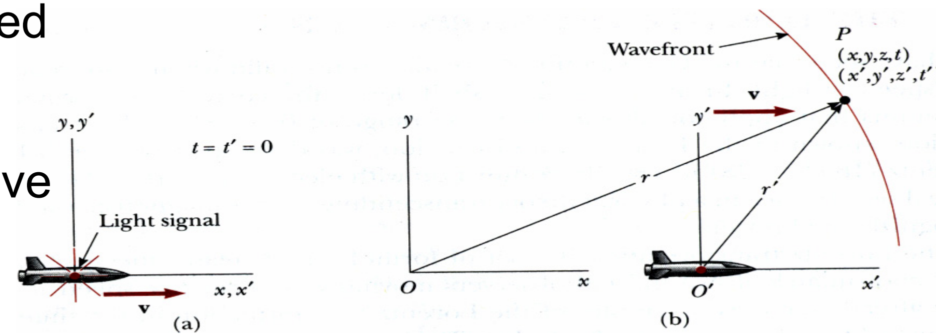
Clearly t and t' must be different
$t \neq t'$

Discovering Lorentz Transfromation for (x,y,z,t)

Motion is along x-x’ axis, so y, z unchanged

y’=y,   z’ = z

Examine points x or x’ where spherical wave crosses the horizontal axes: x = r , x’ =r’



$$x = ct = G(x'+vt')$$

$$x' = ct' = G(x - vt),$$

$$\Rightarrow t' = \frac{G}{c}(x - vt)$$

$$\therefore x = ct = G(ct'+vt')$$

$$\therefore ct = G^2\left[(ct - vt) + vt - \frac{v^2}{c}t\right]$$

$$\Rightarrow c^2 = G^2[c^2 - v^2]$$

$$\text{or } G = \frac{1}{\sqrt{1-(v/c)^2}} = \gamma$$

$$\therefore \quad x' = \gamma(x - vt)$$

$$x' = \gamma(x - vt)\ ,\ x = \gamma(x'+vt')$$

$$\Rightarrow \quad x = \gamma(\gamma(x - vt) + vt')$$

$$\therefore \quad x - \gamma^2 x + \gamma^2 vt = \gamma vt'$$

$$\therefore \quad t' = \left[\frac{x}{\gamma v} - \frac{\gamma^2 x}{\gamma v} + \frac{\gamma^2 vt}{\gamma v}\right] = \gamma\left[\frac{x}{\gamma^2 v} - \frac{x}{v} + t\right]$$

$$\therefore \quad t' = \gamma\left[t + \frac{x}{v}\left(\frac{1}{\gamma^2} - 1\right)\right], \text{ since } \left(\frac{1}{\gamma^2} - 1\right) = -\left(\frac{v}{c}\right)^2$$

$$\Rightarrow t' = \gamma\left[t + \frac{x}{v}[1 - \left(\frac{v}{c}\right)^2 - 1\right] = \gamma\left[t - \left(\frac{vx}{c^2}\right)\right]$$

Lorentz Transformation Between Ref Frames

Lorentz Transformation

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

Inverse Lorentz Transformation

$$x = \gamma(x' + vt')$$

$$y = y'$$

$$z = z'$$

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

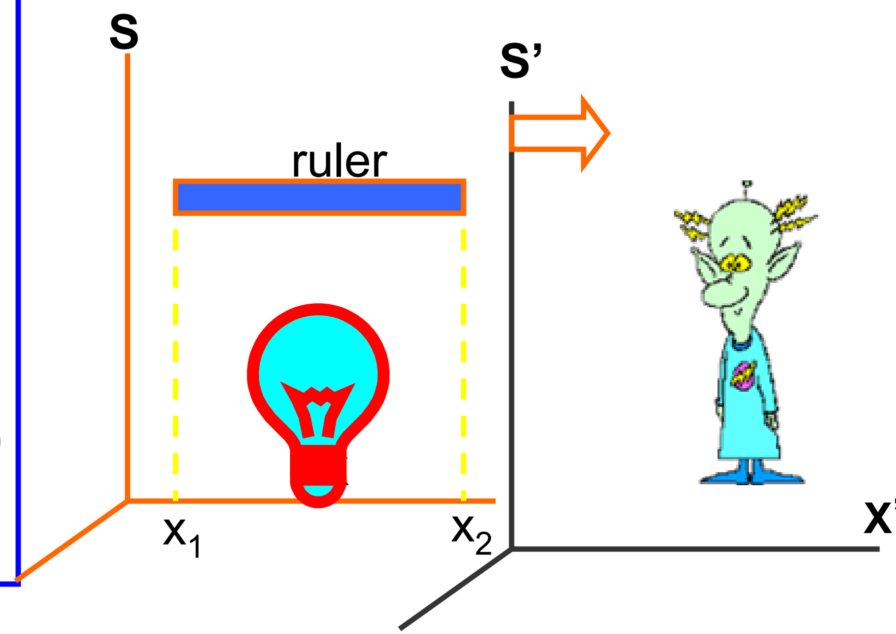
As $v \to 0$, Galilean Transformation is recovered, as per requirement

Notice : SPACE and TIME Coordinates mixed up !!!

Lorentz Transform for Pair of Events

$$\left.\begin{aligned}\Delta x' &= \gamma(\Delta x - v\,\Delta t) \\ \Delta t' &= \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right)\end{aligned}\right\} S \rightarrow S'$$

$$\left.\begin{aligned}\Delta x &= \gamma(\Delta x' + v\,\Delta t') \\ \Delta t &= \gamma\left(\Delta t' + \frac{v}{c^2}\Delta x'\right)\end{aligned}\right\} S' \rightarrow S$$



Can understand Simultaneity, Length contraction & Time dilation formulae from this

Time dilation: Bulb in S frame turned on at t_1 & off at t_2 : What **$\Delta t'$** did S' measure ?

two events occur at same place in S frame => $\Delta x = 0$

$$\Delta t' = \gamma\,\Delta t \quad (\Delta t = \text{proper time})$$

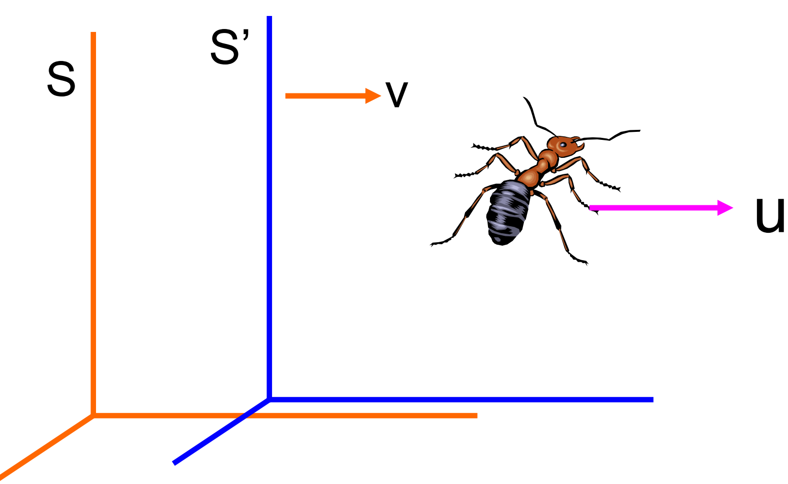
Length Contraction: Ruler measured in S between x_1 & x_2 : What **$\Delta x'$** did S' measure ?

two ends measured at same time in S' frame => $\Delta t' = 0$

$$\Delta x = \gamma\,(\Delta x' + 0) \Rightarrow \Delta x' = \Delta x / \gamma \qquad (\Delta x = \text{proper length})$$

Lorentz Velocity Transformation Rule

S and S' are measuring ant's speed u along x, y, z axes



In S' frame, $u_{x'} = \frac{x_2' - x_1'}{t_2' - t_1'} = \frac{dx'}{dt'}$

$dx' = \gamma(dx - vdt)$, $dt' = \gamma\left(dt - \frac{v}{c^2}dx\right)$

$u_{x'} = \frac{dx - vdt}{dt - \frac{v}{c^2}dx}$, divide by dt'

$u_{x'} = \frac{u_x - v}{1 - \frac{vu_x}{c^2}}$

For $v \ll c$, $u_{x'} = u_x - v$

(Galilean Trans. Restored)

Velocity Transformation Perpendicular to S-S' motion

$$dy' = dy, \quad dt' = \gamma\left(dt - \frac{v}{c^2}dx\right)$$

$$u_y' = \frac{dy'}{dy'} = \frac{dy}{\gamma\left(dt - \frac{v}{c^2}dx\right)}$$

divide by dt on RHS

$$u_y' = \frac{u_y}{\gamma\left(1 - \frac{v}{c^2}u_x\right)}$$

There is a change in velocity in the direction $\perp$ to S-S' motion !

Similarly Z component of Ant's velocity transforms as

$$u_z' = \frac{u_z}{\gamma\left(1 - \frac{v}{c^2}u_x\right)}$$

Inverse Lorentz Velocity Transformation

Inverse Velocity Transform:

$$u_x = \frac{u_{x'} + v}{1 + \frac{v u_{x'}}{c^2}}$$

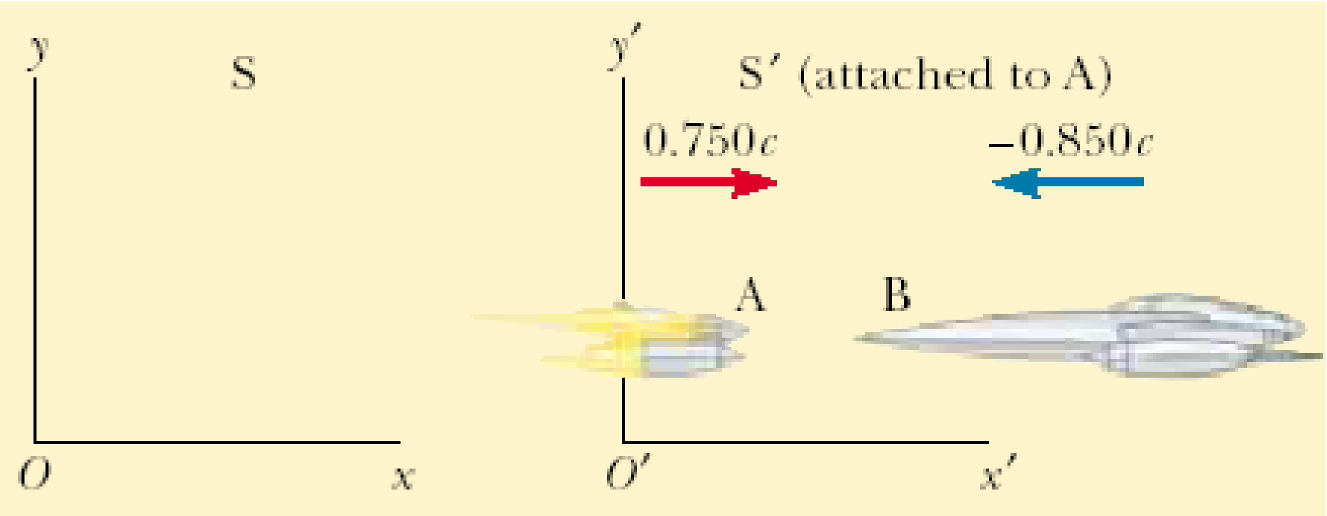
$$u_y = \frac{u_y'}{\gamma\left(1 + \frac{v}{c^2} u_x'\right)}$$

$$u_z = \frac{u_z'}{\gamma\left(1 + \frac{v}{c^2} u_x'\right)}$$

As usual, replace

$v \Rightarrow -v$

Does Lorentz Transform “work” ?



Two rockets travel in opposite directions

An observer on earth (S) measures speeds = 0.75c And 0.85c for A & B respectively

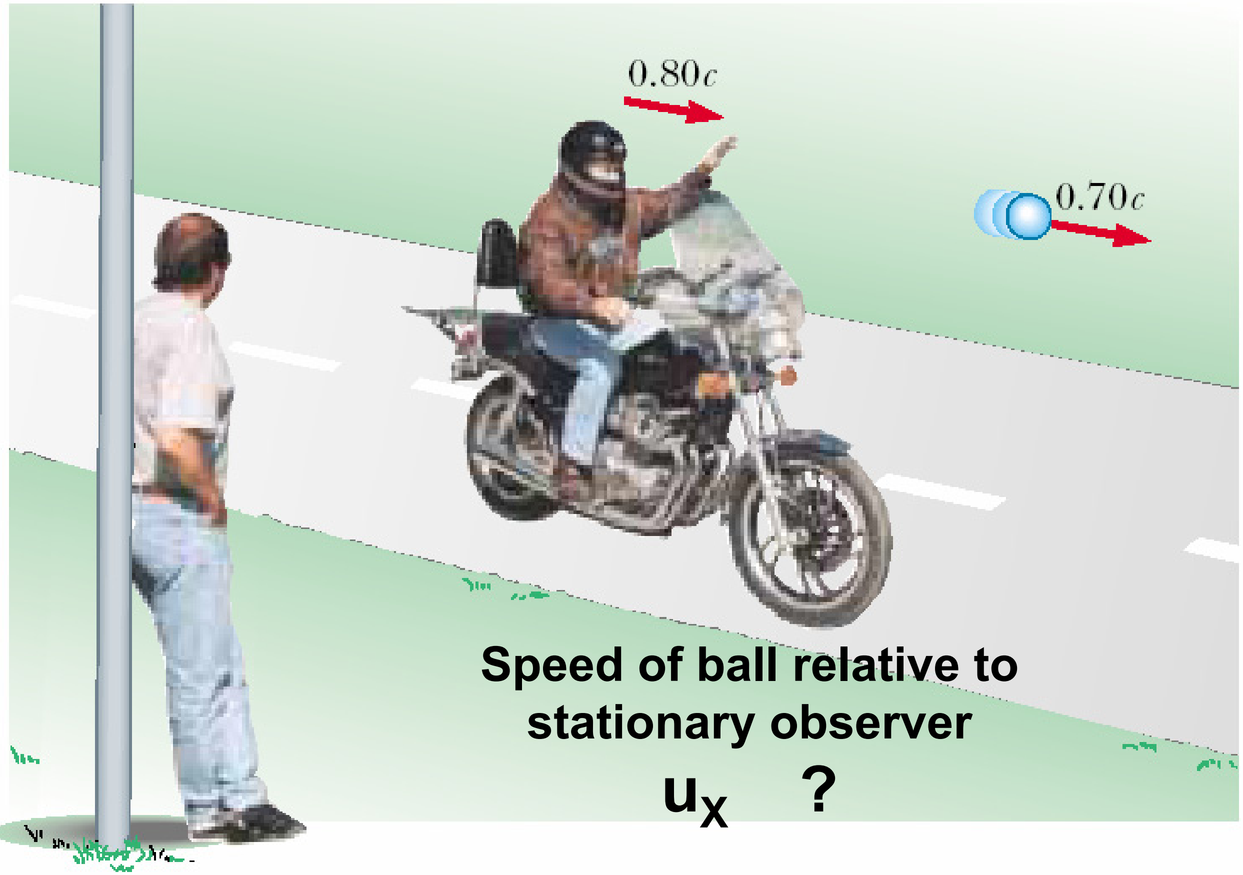
What does A measure as B’s speed?

Place an imaginary S’ frame on Rocket A ⇒ v = 0.75c relative to Earth Observer S

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{-0.850c - 0.750c}{1 - \frac{(-0.850c)(0.750c)}{c^2}} = -0.977c$$

Consistent with Special Theory of Relativity

Example of Inverse velocity Transform



Biker moves with speed = 0.8c past stationary observer

Throws a ball forward with speed = 0.7c

What does stationary observer see as velocity of ball ?

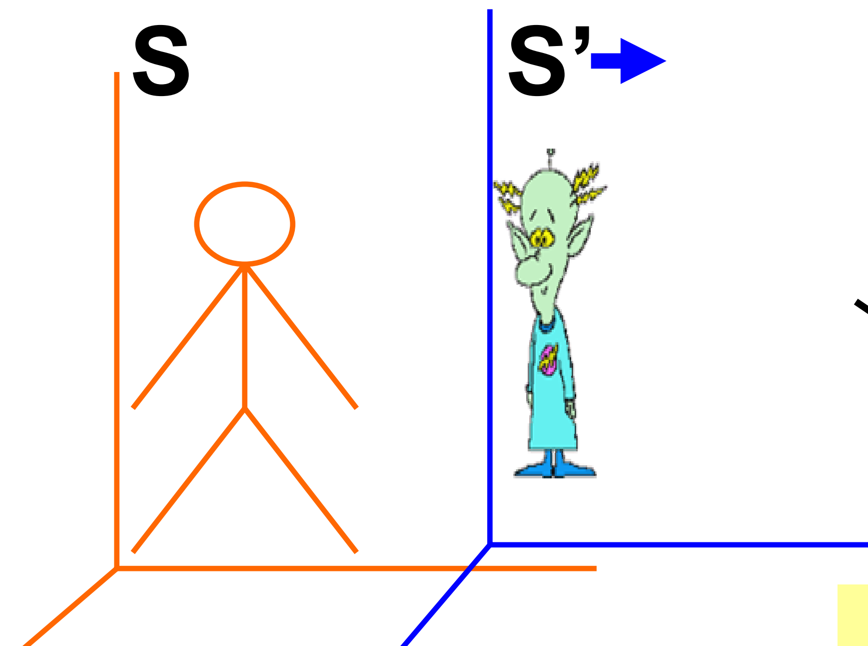
Place S' frame on biker
Biker sees ball speed

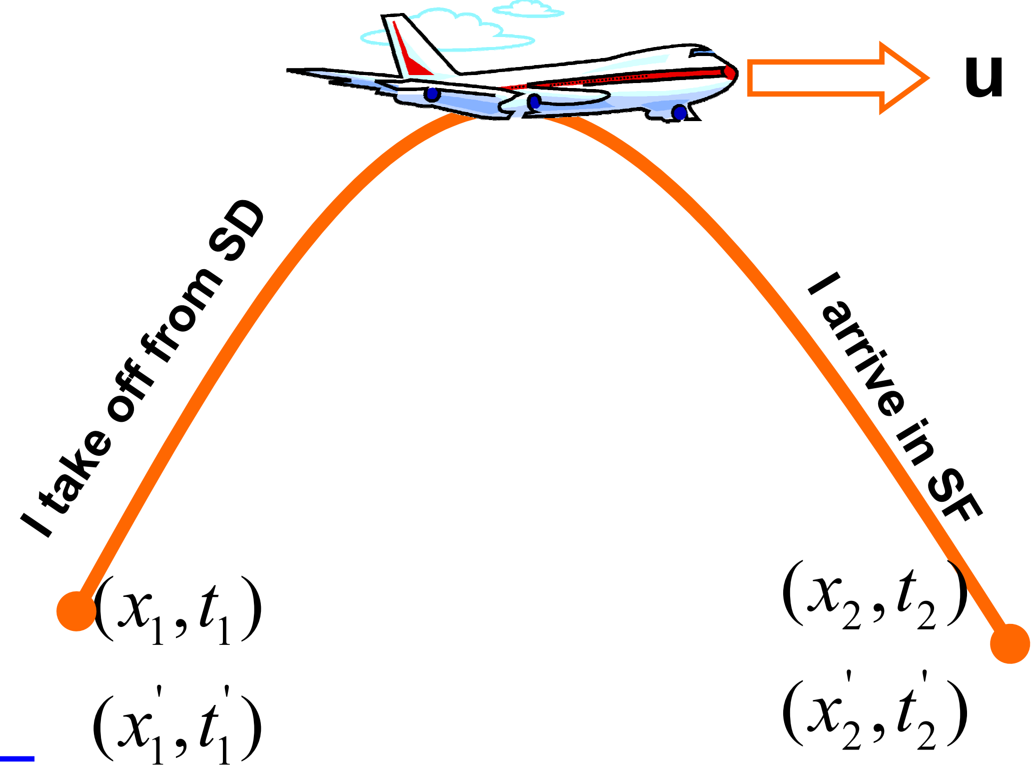
$u_{x'} = 0.7c$

$$u_x = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}} = \frac{0.70c + 0.80c}{1 + \frac{(0.70c)(0.80c)}{c^2}} = 0.96c$$

Can you be **seen** to be born before your mother?

A frame of Ref where sequence of events is REVERSED **?!!**

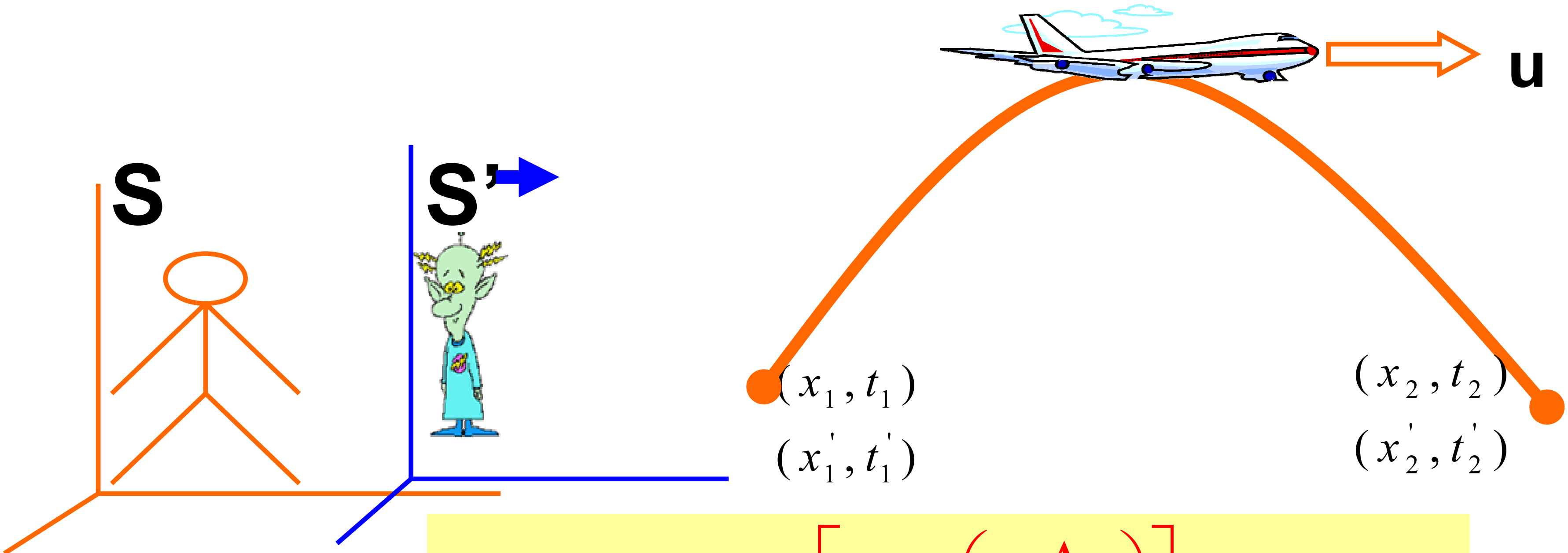




$$\Delta t' = t_2' - t_1' = \gamma\left[\Delta t - \left(\frac{v\,\Delta x}{c^2}\right)\right]$$

For what value of v can $\Delta t' < 0$

I Cant 'be seen to arrive in SF before I take off from SD



$$\Delta t' = t_2' - t_1' = \gamma\left[\Delta t - \left(\frac{v\,\Delta x}{c^2}\right)\right]$$

For what value of v can $\Delta t' < 0$

$$\Delta t' < 0 \Rightarrow \Delta t < \frac{v\,\Delta x}{c^2} \quad \Rightarrow 1 < \frac{v\,\Delta x}{c^2 \Delta t} = \frac{v\,\mathrm{u}}{c^2}$$

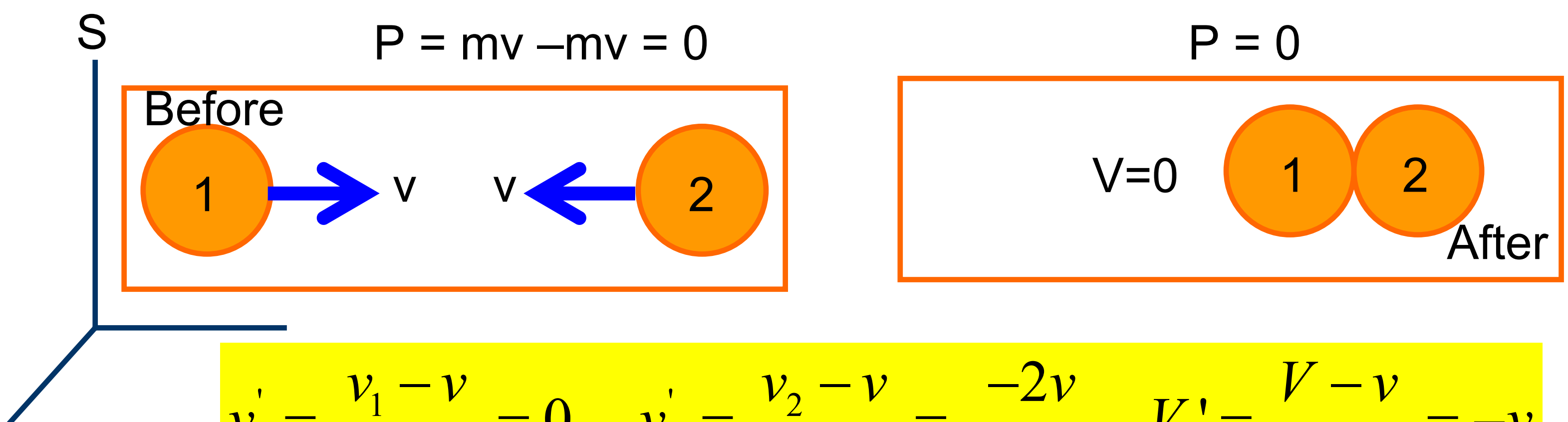
$$\Rightarrow \frac{\mathrm{v}}{\mathrm{c}} > \frac{c}{u} \qquad \Rightarrow v > c : \text{Not allowed}$$

Relativistic Momentum and Revised Newton's Laws

Need to generalize the laws of Mechanics & Newton to confirm to Lorentz Transform and the Special Theory of Relativity: Example : $\vec{p} = m\vec{u}$

P = mv –mv = 0

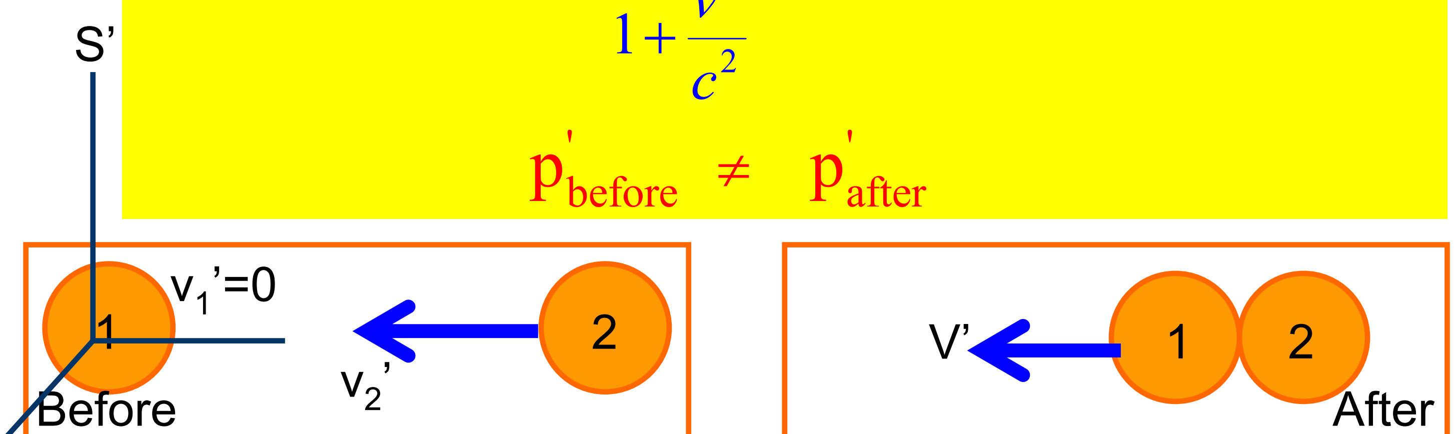
P = 0



$$v_1' = \frac{v_1 - v}{1 - \frac{v_1 v}{c^2}} = 0, \quad v_2' = \frac{v_2 - v}{1 - \frac{v_1 v}{c^2}} = \frac{-2v}{1 + \frac{v^2}{c^2}}, \quad V' = \frac{V - v}{1 - \frac{Vv}{c^2}} = -v$$

$$p_{before}' = mv_1' + mv_2' = \frac{-2mv}{1 + \frac{v^2}{c^2}}, \quad p_{after}' = 2mV' = -2mv$$

$$\mathrm{p}_{\mathrm{before}}' \neq \mathrm{p}_{\mathrm{after}}'$$



Relativistic Momentum Force And Acceleration

$$\vec{p} = \frac{m\vec{u}}{\sqrt{1-(u/c)^2}} = \gamma m\vec{u}$$

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}\left(\frac{m\vec{u}}{\sqrt{1-(u/c)^2}}\right) \quad use \quad \frac{d}{dt} = \frac{du}{dt}\frac{d}{du}$$

$$F = \left[\frac{m}{\sqrt{1-(u/c)^2}} + \frac{mu}{\left(1-(u/c)^2\right)^{3/2}} \times \left(\frac{-1}{2}\right)\left(\frac{-2u}{c^2}\right)\right]\frac{du}{dt}$$

$$F = \left[\frac{mc^2 - mu^2 + mu^2}{c^2\left(1-(u/c)^2\right)^{3/2}}\right]\frac{du}{dt}$$

$$F = \left[\frac{m}{\left(1-(u/c)^2\right)^{3/2}}\right]\frac{du}{dt} \quad \text{: Relativistic Force}$$

Since Acceleration $\vec{a} = \frac{d\vec{u}}{dt}$,

$$\Rightarrow \quad \vec{a} = \frac{\vec{F}}{m}\left[1-(u/c)^2\right]^{3/2}$$

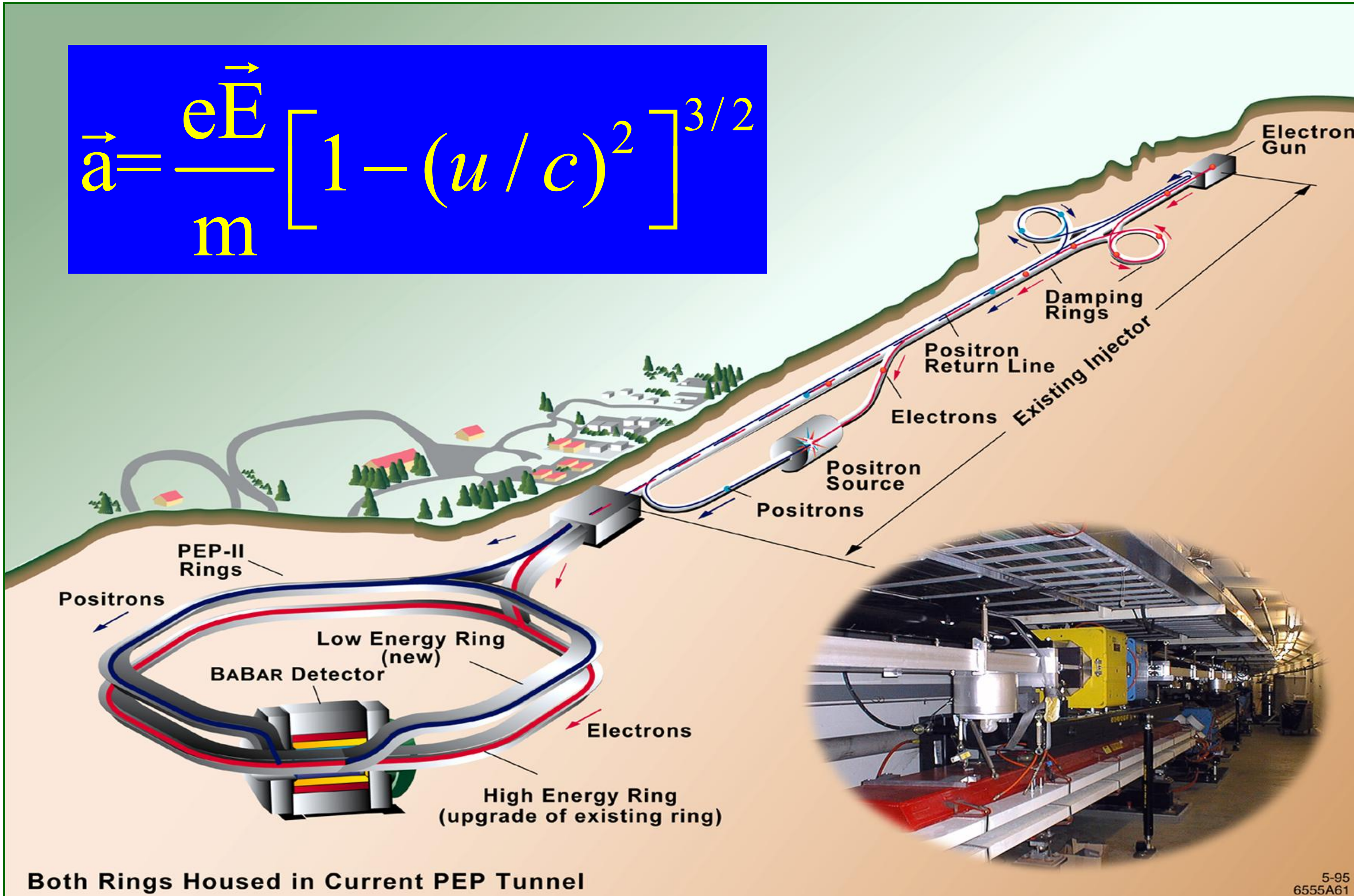
Reason why you cant quite get up to the speed of light !

Note: As $u/c \rightarrow 1$, $\vec{a} \rightarrow 0$!!!!

Its harder to accelerate when you get close to speed of light

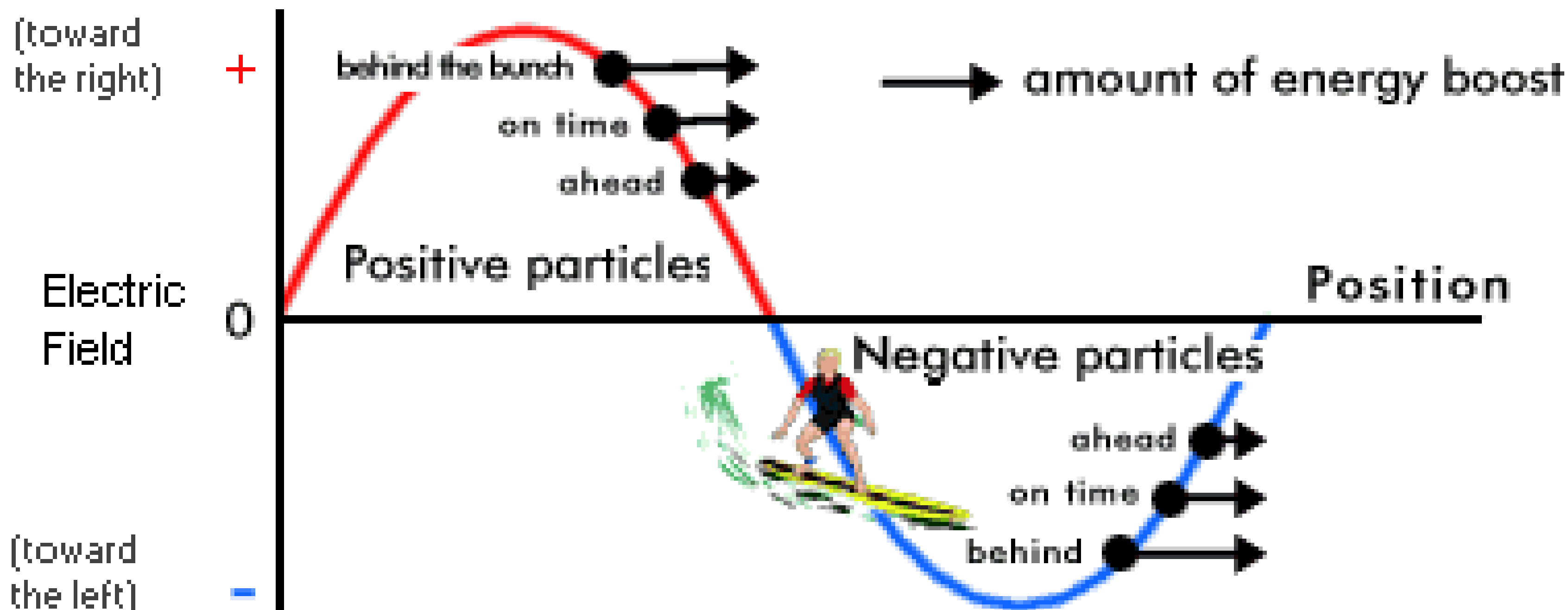
A Linear Particle Accelerator

$$\vec{a} = \frac{e\vec{E}}{m}\left[1-(u/c)^2\right]^{3/2}$$

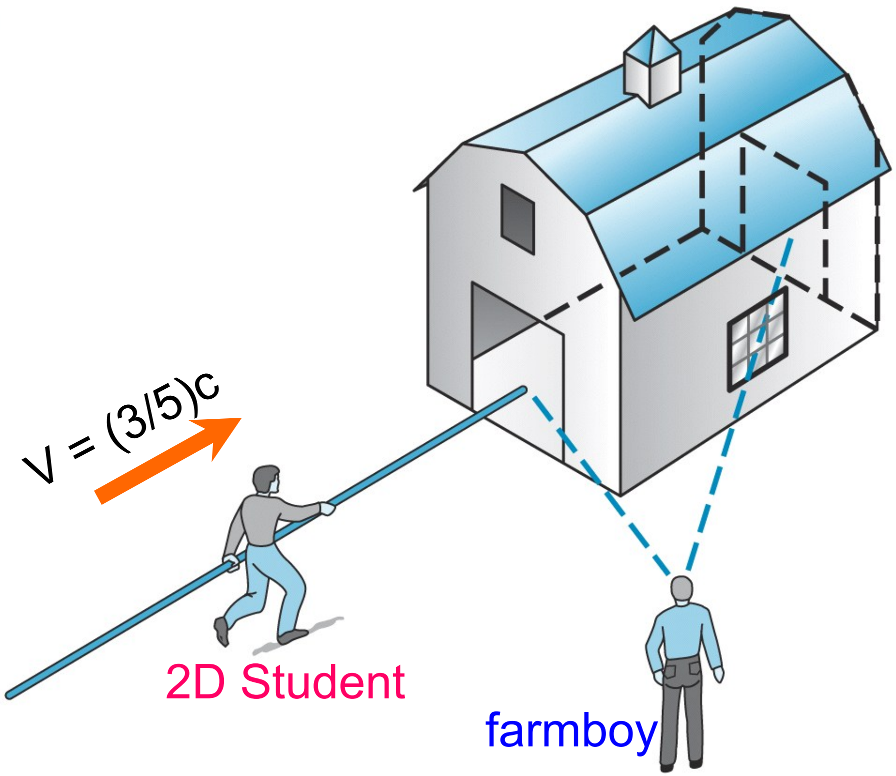


PEP-II accelerator schematic and tunnel view

Accelerating Electrons Thru RF Cavities



Fitting a 5m pole in a 4m barnhouse



Farmboy says “You can do it”

Student says “Dude, you are nuts”

Student with pole runs with v=(3/5)c

farmboy sees pole contraction factor

$$\sqrt{1-(3c/5c)^2} = 4/5$$

says pole just fits in the barn fully!

Student with pole runs with v=(3/5)c

Student sees barn contraction factor

$$\sqrt{1-(3c/5c)^2} = 4/5$$

says barn is only 3.2m long, too short

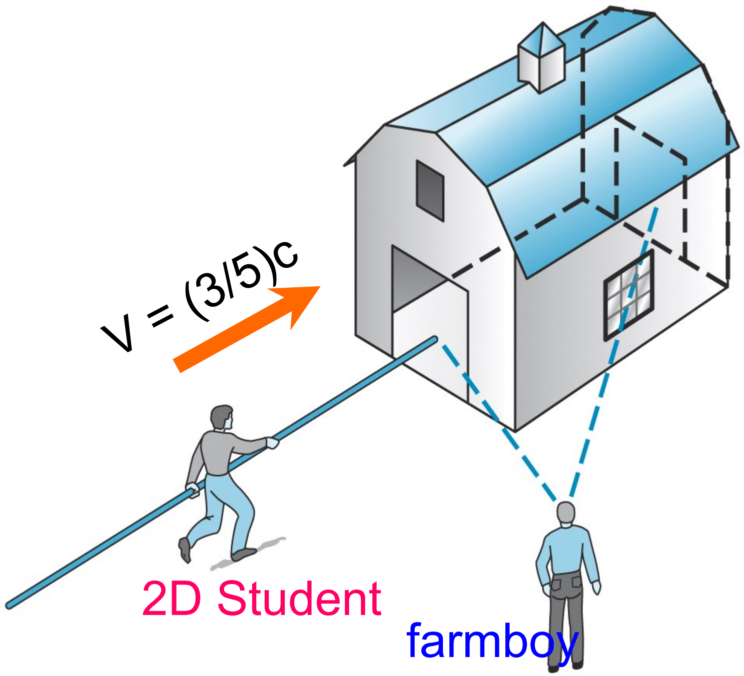
to contain entire 5m pole !

Is there a contradiction ? Is Relativity wrong?

Homework: You figure out who is right, if any and why.

Hint: Think in terms of observing three events

Fitting a 5m pole in a 4m barnhouse?



A: Arrival of right end of pole at left end of barn
B: Arrival of left end of pole at left end of barn
C: Arrival of right end of pole at right end of barn

Let S = Barn frame, S' = student frame

Answer : Simultaneity !

Event A : arrival of right end of pole at left end of barn: (t =0, t'=0) is reference

L_0' = proper length of pole in S'

l_0 = length of barn in S frame $< L_0'$

In S: length of pole $L = L_0'\sqrt{1-(v/c)^2}$

The times in two frames are related:

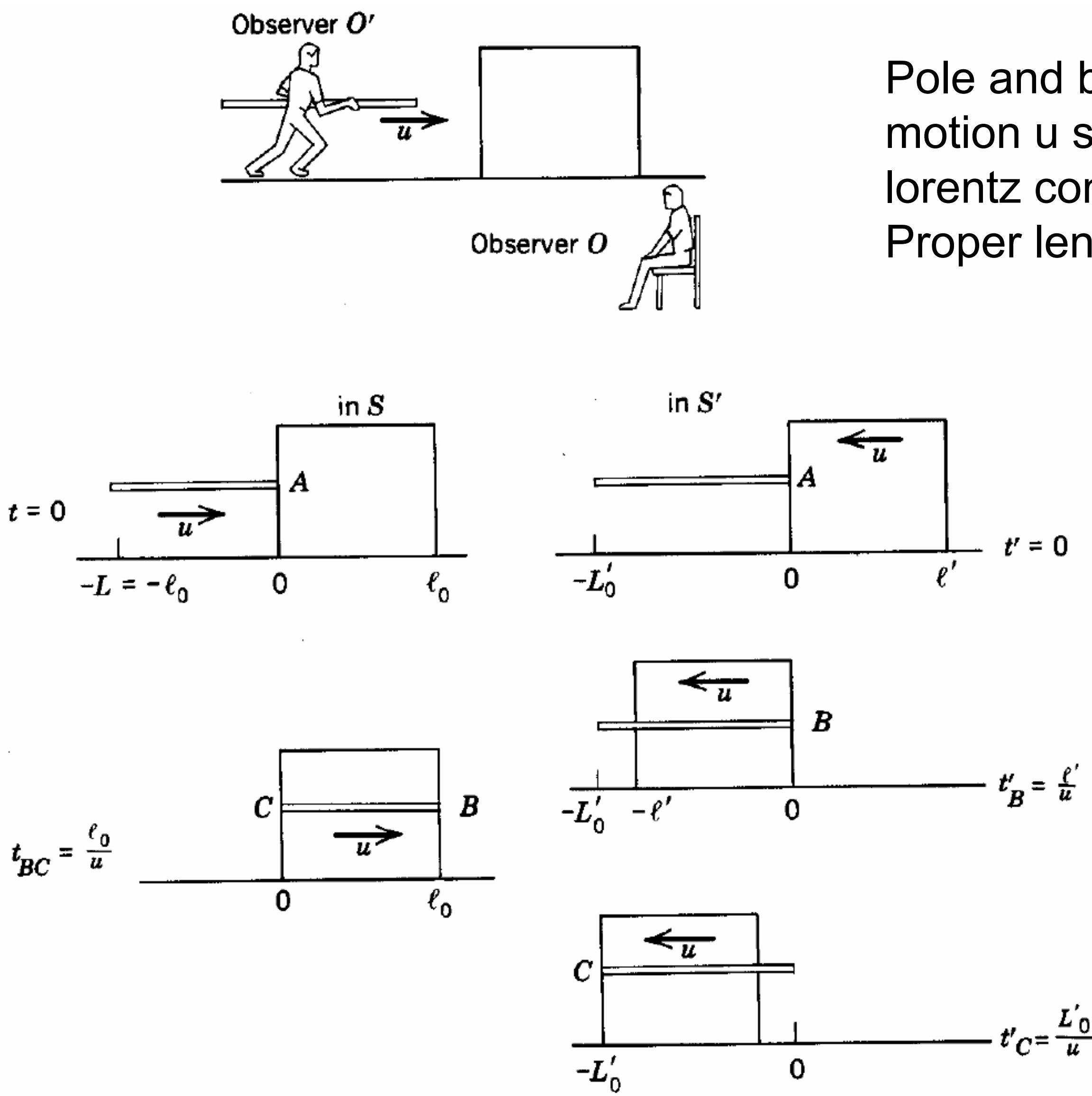
$$t_B' = \frac{l'}{v} = \frac{l_0}{v}\sqrt{1-(v/c)^2} = t_{BC}\sqrt{1-(v/c)^2}$$

$$t_C' = \frac{L_0'}{v} = \frac{l'}{v}\frac{1}{1-(v/c)^2} = \frac{t_{BC}}{\sqrt{1-(v/c)^2}}$$

⇒ Time gap in S' by which events B and C fail to be simultaneous

Farmboy sees two events as simultaneous
2D student can not agree
Fitting of the pole in barn is relative !

Farmboy Vs 2D Student



Pole and barn are in relative motion u such that lorentz contracted length of pole = Proper length of barn

In rest frame of pole, Event B precedes C