Physics 2D Lecture Slides
Oct 15

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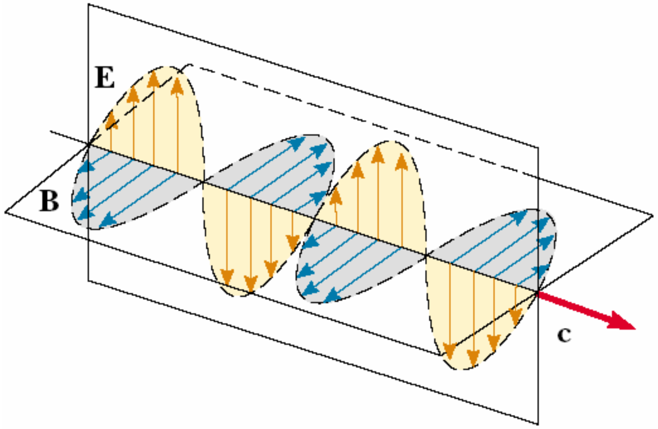
Properties of EM Waves: Maxwell's Equations



Energy Flow in EM Waves :

Poynting Vector $\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$

Power incident on an area A $= \vec{S}.\vec{A} = \frac{1}{\mu_0}\left(AE_0 B_0 Sin^2(kx - \omega t)\right)$

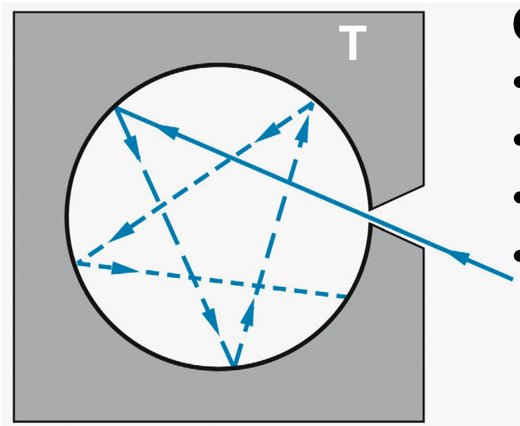
Intensity of Radiation $I = \frac{1}{2\mu_0 c} E_0^2$

Larger the amplitude of Oscillation
More intense is the radiation

Disasters in Classical Physics (1899-1922)

- Disaster → Experimental observation that could not be explained by Classical theory (Phys 2A, 2B, 2C)
  - Disaster # 1 : Nature of Blackbody Radiation from your BBQ grill
  - Disaster # 2: Photo Electric Effect
  - Disaster # 3: Scattering light off electrons (Compton Effect)
- Resolution of Experimental Observation will require radical changes in how we think about nature
  - → QUANTUM MECHANICS
    - The Art of Conversation with Subatomic Particles

Blackbody Radiator: An Idealization



Classical Analysis:

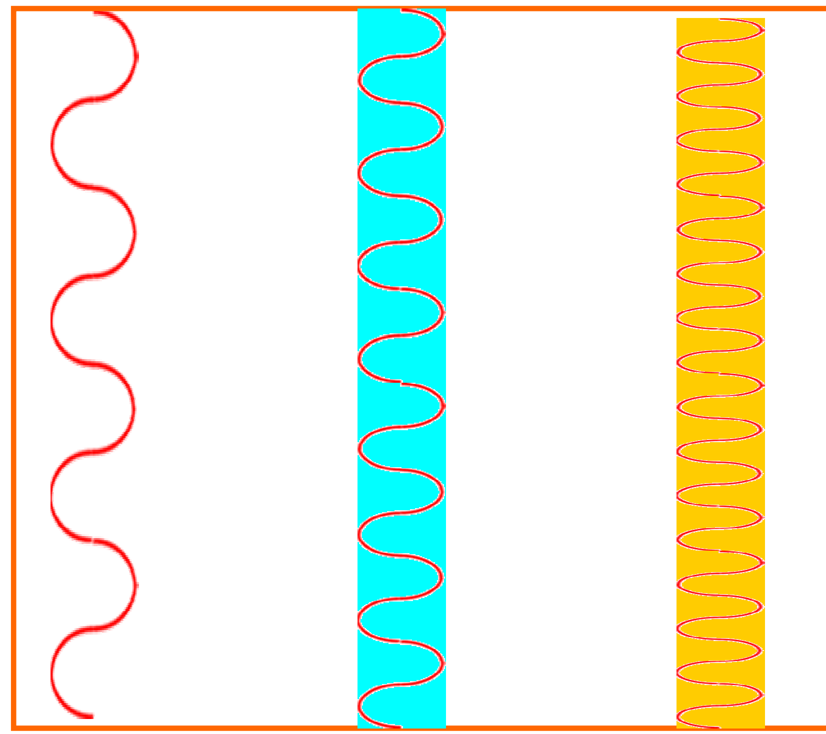
- **Box is filled with EM standing waves**
- **Radiation reflected back-and-forth between walls**
- **Radiation in thermal equilibrium with walls of Box**
- **How may waves of wavelength λ can fit inside the box ?**

Blackbody Absorbs everything
Reflects nothing
All light entering opening gets absorbed (ultimately) by the cavity wall

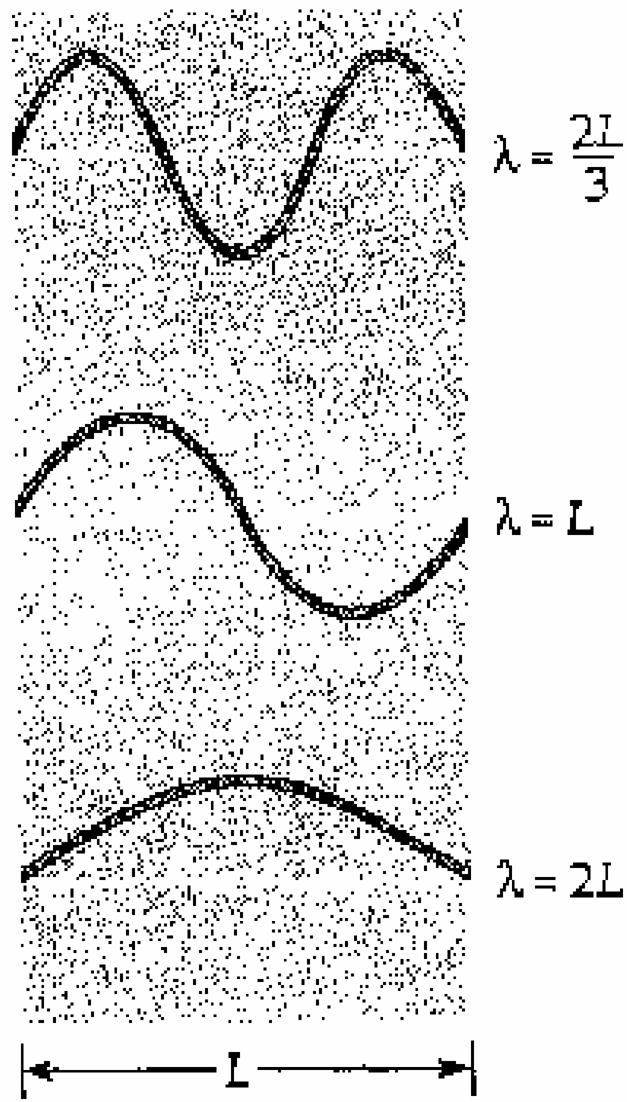
Cavity in equilibrium T w.r.t. surrounding. So it radiates everything It absorbs

Emerging radiation is a sample of radiation inside box at temp T

Predict nature of radiation inside Box ?



Standing Waves



The Beginning of The End ! How BBQ Broke Physics

Classical Calculation

of standing waves between Wavelengths λ and λ+dλ are

$$N(\lambda)d\lambda = \frac{8\pi V}{\lambda^4} \bullet d\lambda \; ; \; V = \text{Volume of box} = L^3$$

Each standing wave contributes energy E= kT to radiation in Box

Energy density $u(\lambda)$= [# of standing waves/volume] × Energy/Standing Wave

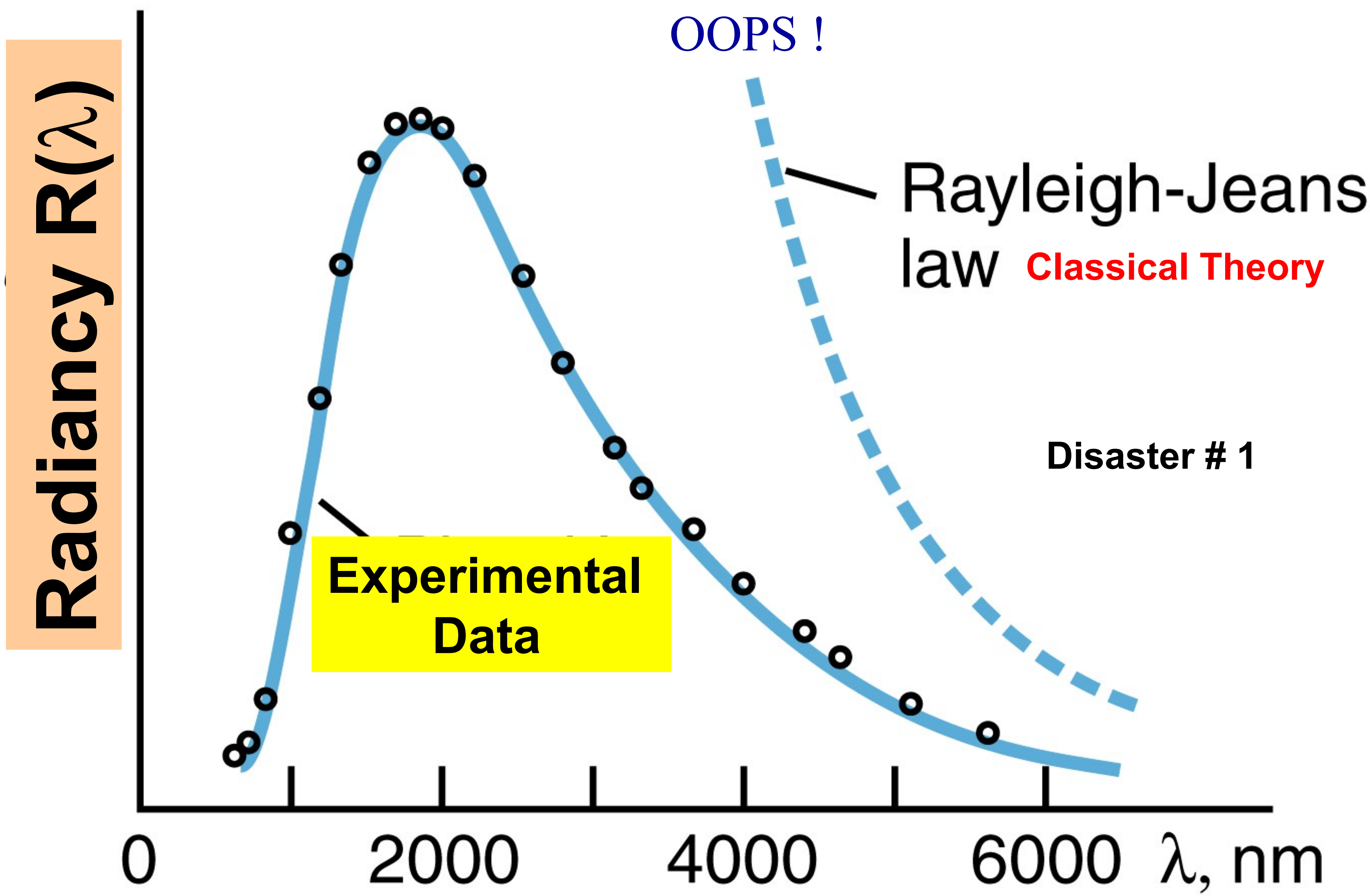
$$= \frac{8\pi V}{\lambda^4} \times \frac{1}{V} \times kT = \frac{8\pi}{\lambda^4} kT$$

$$\boxed{\text{Radiancy } R(\lambda) = \frac{c}{4} u(\lambda) = \frac{c}{4}\frac{8\pi}{\lambda^4} kT = \frac{2\pi c}{\lambda^4} kT}$$

Radiancy is Radiation intensity per unit λ interval: Lets plot it

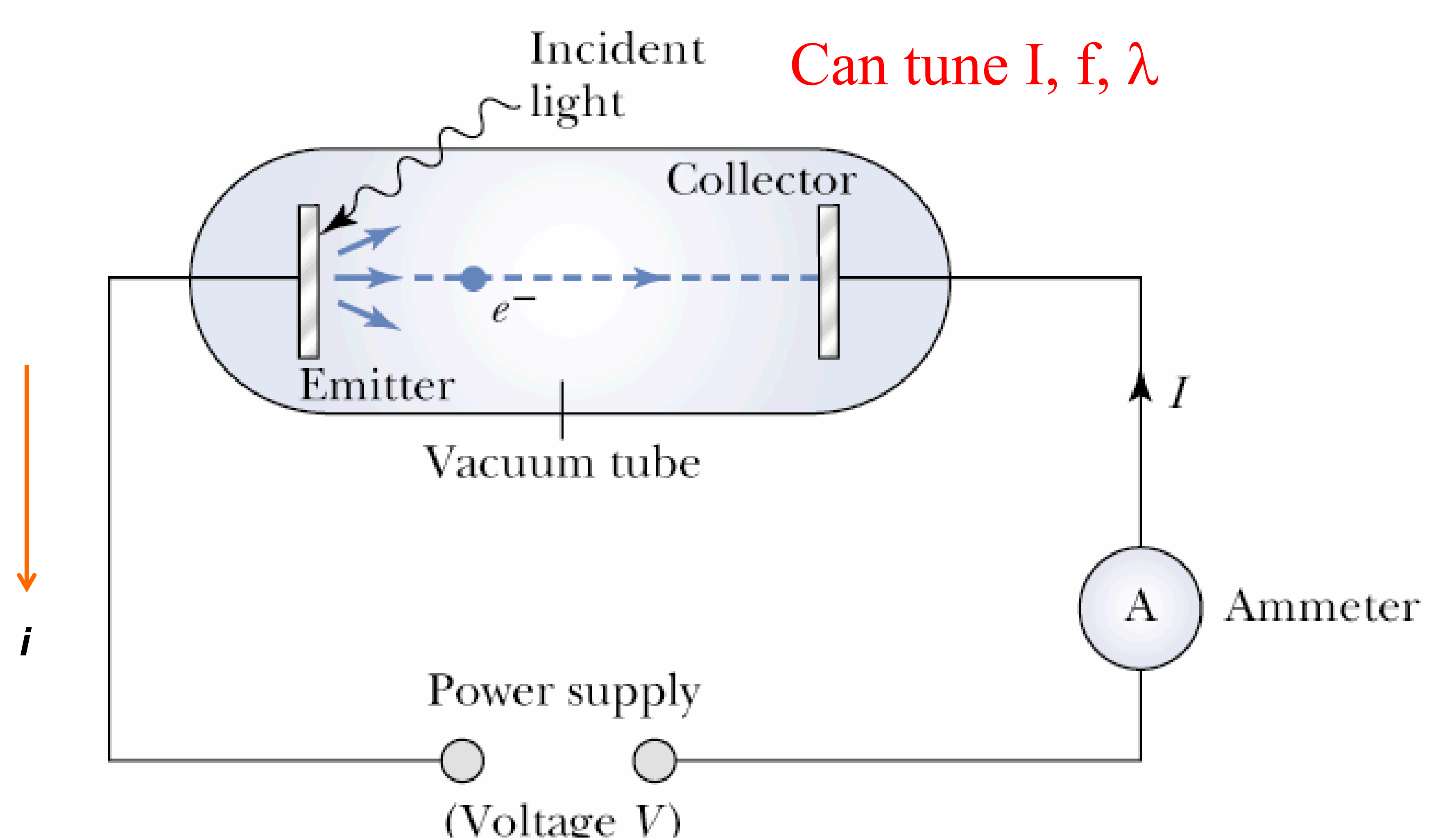
Prediction : as $\lambda \to 0$ (high frequency) $\Rightarrow R(\lambda) \to$ Infinity !
Oops !

Ultra Violet (Frequency) Catastrophe



Disaster # 2 : Photo-Electric Effect

Light of intensity I, wavelength λ and frequency ν incident on a photo-cathode



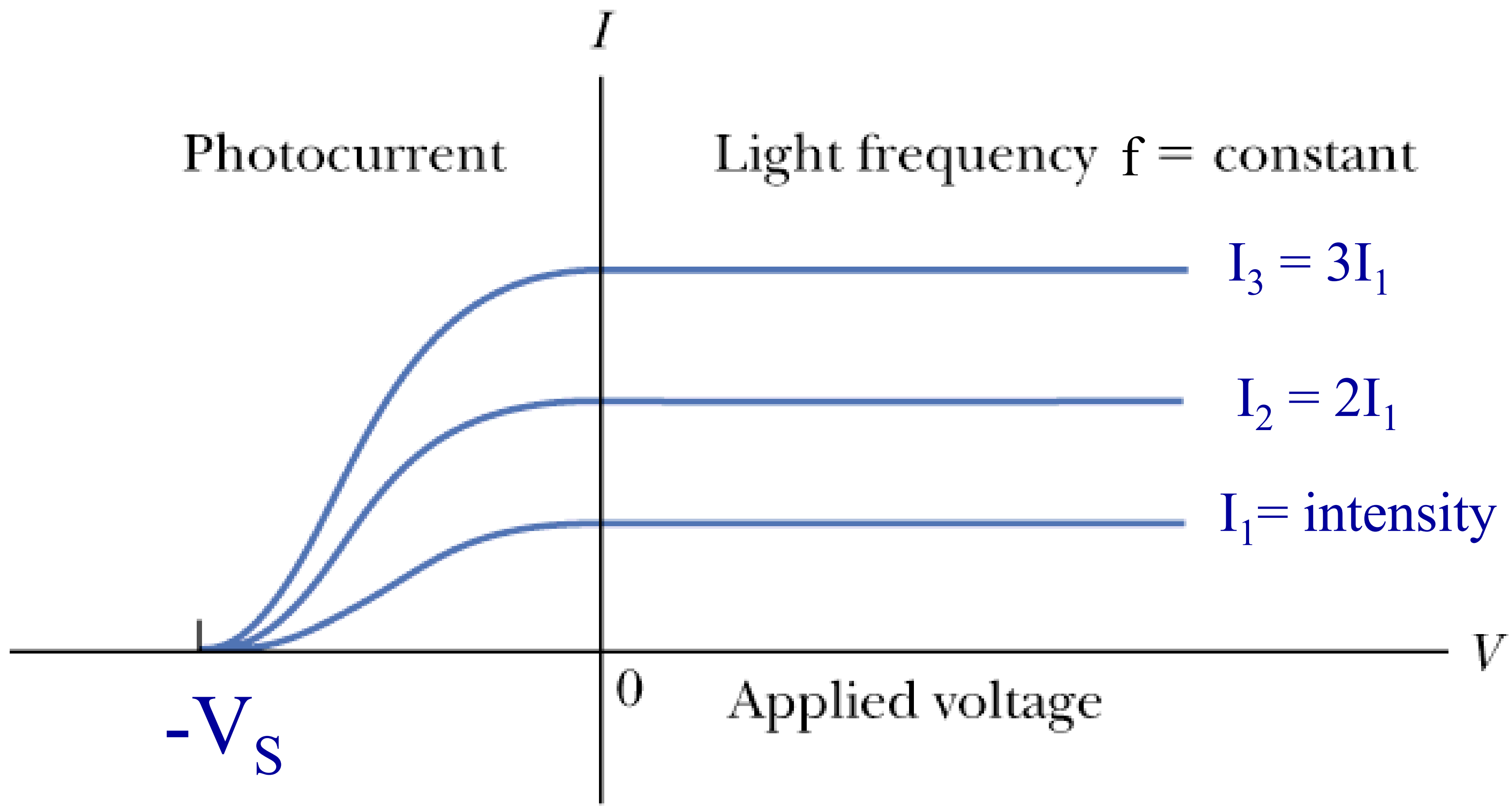
Measure characteristics of current in the circuit as a fn of I, f, λ

Photo Electric Effect: Measurable Properties

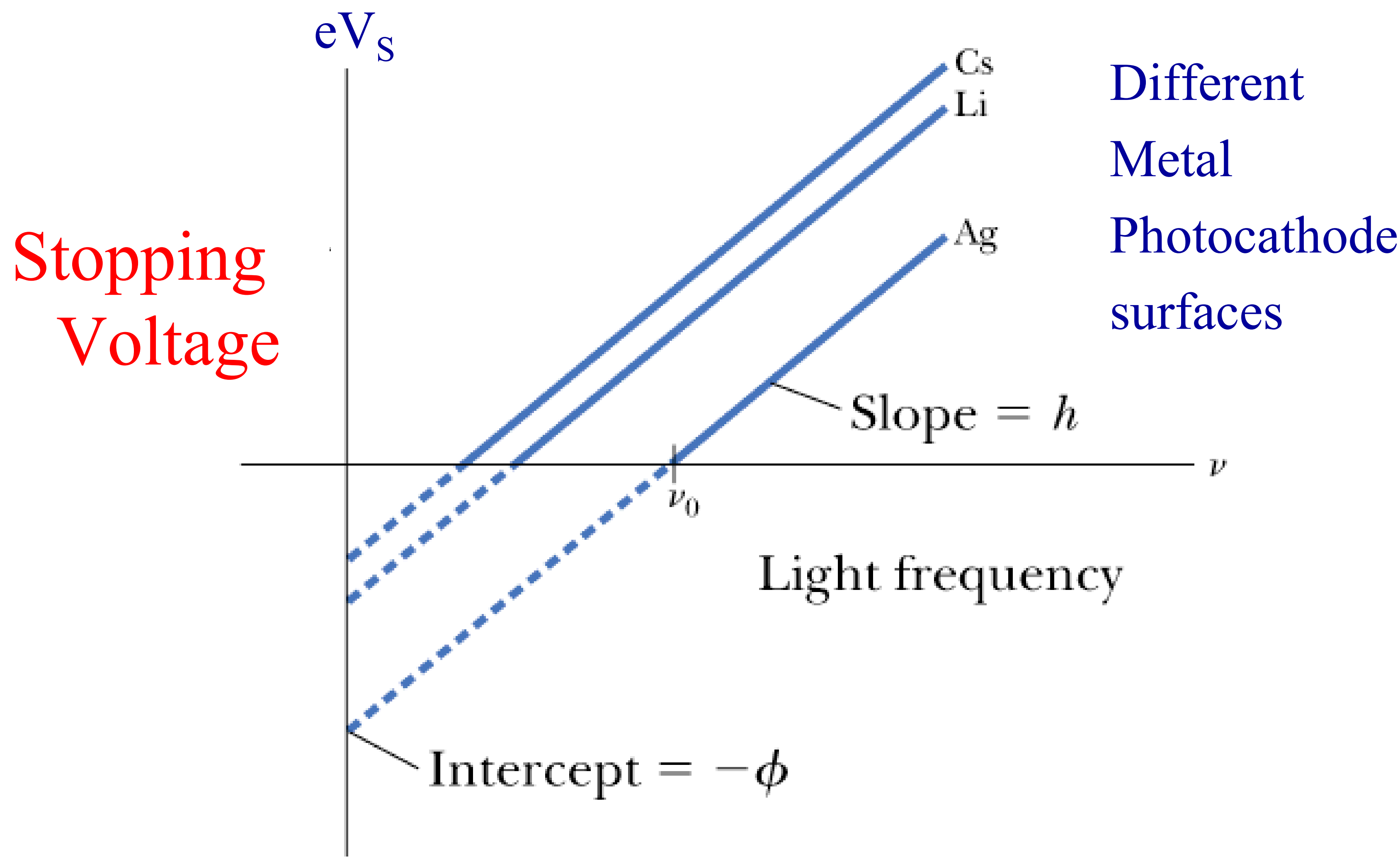
- Rate of electron emission from cathode
  - From current *i* seen in ammeter

- Maximum kinetic energy of emitted electron
  - By applying retarding potential on electron moving towards Collector plate
    - » **$K_{MAX} = eV_S$ (V_S = Stopping voltage)**
    - » **Stopping voltage → no current flows**
- Effect of different types of photo-cathode metal
- Time between shining light and first sign of photo-current in the circuit

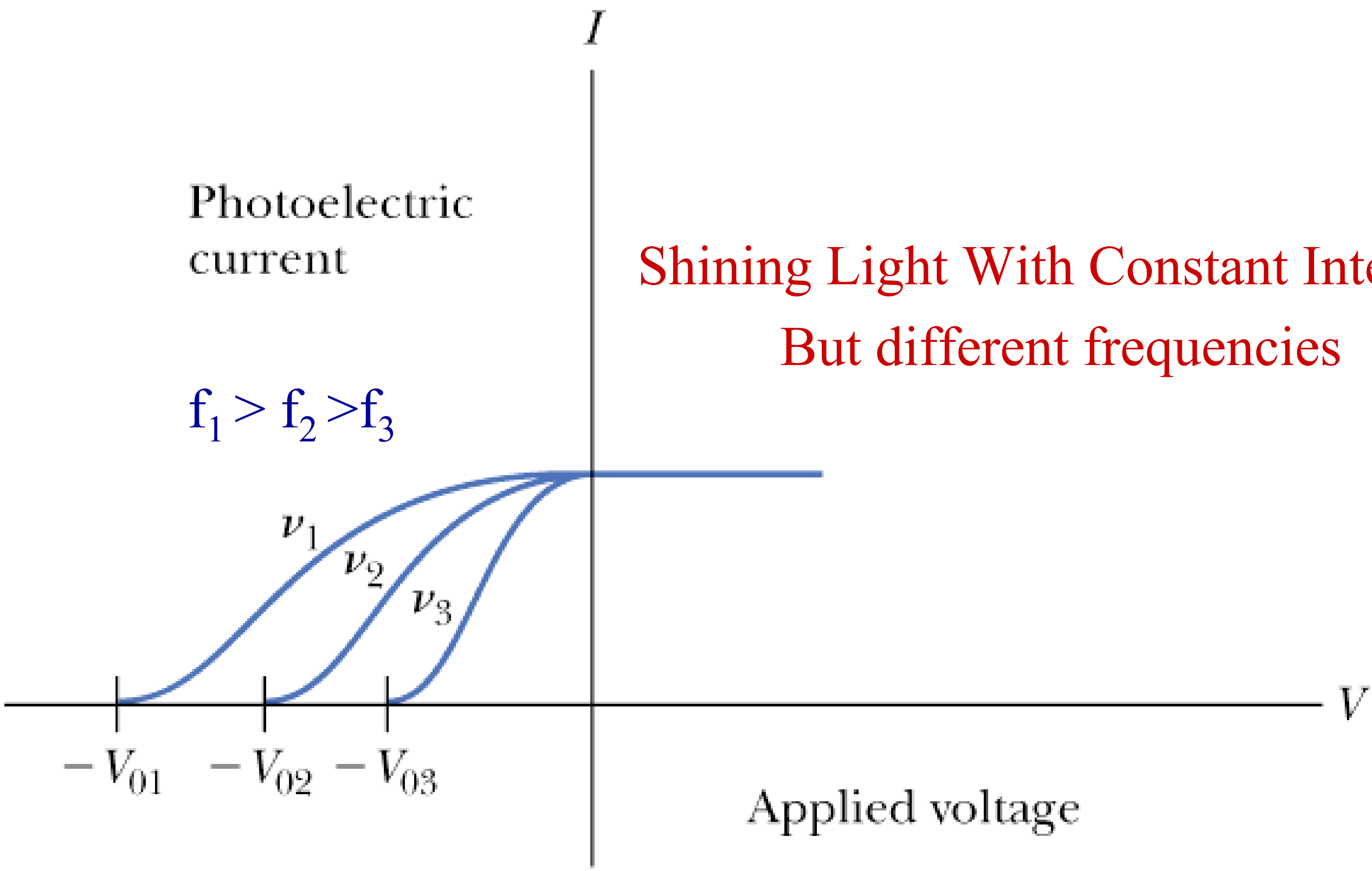
Observations : Current Vs Frequency of Incident Light



Stopping Voltage V_s Vs Incident Light Frequency



Retarding Potential Vs Light Frequency



Shining Light With Constant Intensity
But different frequencies

Time Elapsed between Shining Light & Current

- Time between
  - Light shining on photo-cathode
  - And first photo-electons ejected → current in circuit

  - Depends on distance between light source & cathode surface

  - Seems instantaneous ($< 10^{-9}$ Seconds by the experimenter's watch)

Conclusions from the Experimental Observation

- Max Kinetic energy K_{MAX} independent of Intensity I for light of same frequency
- No photoelectric effect occurs if light frequency f is below a threshold no matter how high the intensity of light
- For a particular metal, light with $f > f_0$ causes photoelectric effect IRRESPECTIVE of light intensity.
  - f_0 is characteristic of that metal
- Photoelectric effect is instantaneous !...not time delay

Can one Explain all this Classically !

Classical Explanation of Photo Electric Effect

- As light Intensity increased $\Rightarrow$ $\vec{E}$ field amplitude larger
  - E field and electrical force seen by the "charged subatomic oscillators" Larger
    - $\vec{F} = e\vec{E}$
    - More force acting on the subatomic charged oscillator
    - $\Rightarrow$ More energy transferred to it
    - $\Rightarrow$ Charged particle "hooked to the atom" should leave the surface with more Kinetic Energy KE !! The intensity of light shining rules !
- As long as light is intense enough , light of ANY frequency f should cause photoelectric effect
- Because the Energy in a Wave is uniformly distributed over the Spherical wavefront incident on cathode, thould be a noticeable time lag ΔT between time is incident & the time a photo-electron is ejected : Energy absorption time
  - How much time ? Lets calculate it classically.

Classical Physics: Time Lag in Photo-Electric Effect

- Electron absorbs energy incident on a surface area where the electron is confined ≅ size of atom in cathode metal
- Electron is "bound" by attractive Coulomb force in the atom, so it must absorb a minimum amount of radiation before its stripped off
- Example : Laser light Intensity $I = 120 W/m^2$ on Na metal
  - Binding energy = 2.3 eV= "Work Function"
  - Electron confined in Na atom, size ≅ 0.1nm ..how long before ejection ?
  - Average Power Delivered $P_{AV} = \mathbf{I} \cdot \mathbf{A}$, $A = \pi r^2 \cong 3.1 \times 10^{-20}\ m^2$
  - If all energy absorbed then $\Delta E = P_{AV} \cdot \Delta T \Rightarrow \Delta T = \Delta E / P_{AV}$

$$\Delta T = \frac{(2.3eV)(1.6\times10^{-19} J/eV)}{(120W/m^2)(3.1\times10^{-20} m^2)} = 0.10\ S$$

  - Classical Physics predicts Measurable delay even by the primitive clocks of 1900
  - But in experiment, the effect was observed to be instantaneous !!
  - Classical Physics fails in explaining all results & goes to DOGHOUSE !

Max Planck & Birth of Quantum Physics

Back to Blackbody Radiation Discrepancy

Planck noted the UltraViolet Catastrophe at high frequency

"Cooked" calculation with new "ideas" so as bring:

$$R(\lambda) \rightarrow 0 \text{ as } \lambda \rightarrow 0$$

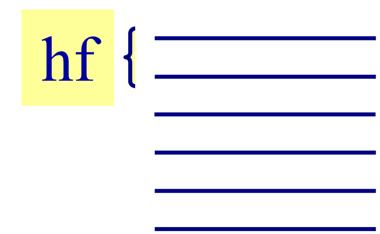
$$f \rightarrow \infty$$

- Cavity radiation as equilibrium exchange of energy between EM radiation & "atomic" oscillators present on walls of cavity
- Oscillators can have any frequency f
- But the Energy exchange between radiation and oscillator NOT continuous and arbitarary…it is discrete …in packets of same amount
- $E = n\,hf$, with $n = 1,2\ 3\ldots\ \infty$

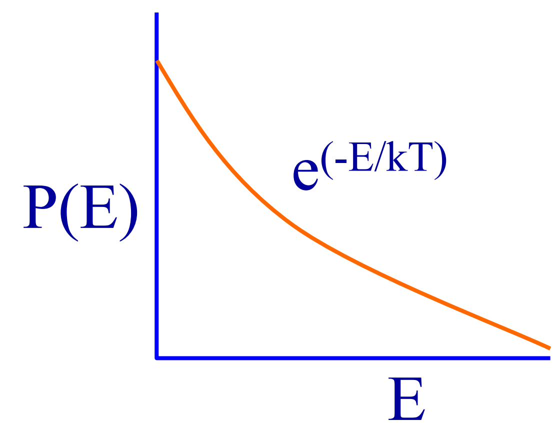
  h = constant he invented, a very small number he made up

Planck, Quantization of Energy & BB Radiation

- Keep the rule of counting how many waves fit in a BB Volume
- Radiation Energy in cavity is quantized
- EM standing waves of frequency f have energy
  - $E = n\,hf$ ($n = 1, 2, 3 \ldots 10 \ldots. 1000 \ldots$)



- Probability Distribution: At an equilibrium temp T, possible Energy of wave is distributed over a spectrum of states: $P(E) = e^{(-E/kT)}$
- Modes of Oscillation with :
  - Less energy $E=hf$ = favored
  - More energy $E=hf$ = disfavored



By this statistics, large energy, high f modes of EM disfavored

Planck's Calculation

$$R(\lambda) = \left(\frac{c}{4}\right)\left(\frac{8\pi}{\lambda^4}\right)\left[\frac{hc}{\lambda}\left(\frac{1}{e^{\frac{hc}{\lambda kT}} - 1}\right)\right]$$

Odd looking form

When $\lambda \to$ large $\Rightarrow \dfrac{hc}{\lambda kT} \to$ small

Recall $e^{x} = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots.$

$$\Rightarrow e^{\frac{hc}{\lambda kT}} - 1 = (1 + \frac{hc}{\lambda kT} + \frac{1}{2}\left(\frac{hc}{\lambda kT}\right)^2 + \ldots.] - 1$$

$$= \frac{hc}{\lambda kT}$$ plugging this in $R(\lambda)$ eq:

$$R(\lambda) = \left(\frac{c}{4}\right)\left(\frac{8\pi}{\lambda^4}\right)\frac{hc}{\lambda kT}$$

Graph & Compare
With BBQ data

Planck's Formula and Small λ

When λ is small (large f)

$$\frac{1}{e^{\frac{hc}{\lambda kT}}-1} \cong \frac{1}{e^{\frac{hc}{\lambda kT}}} = e^{-\frac{hc}{\lambda kT}}$$

Substituting in R(λ) eqn:

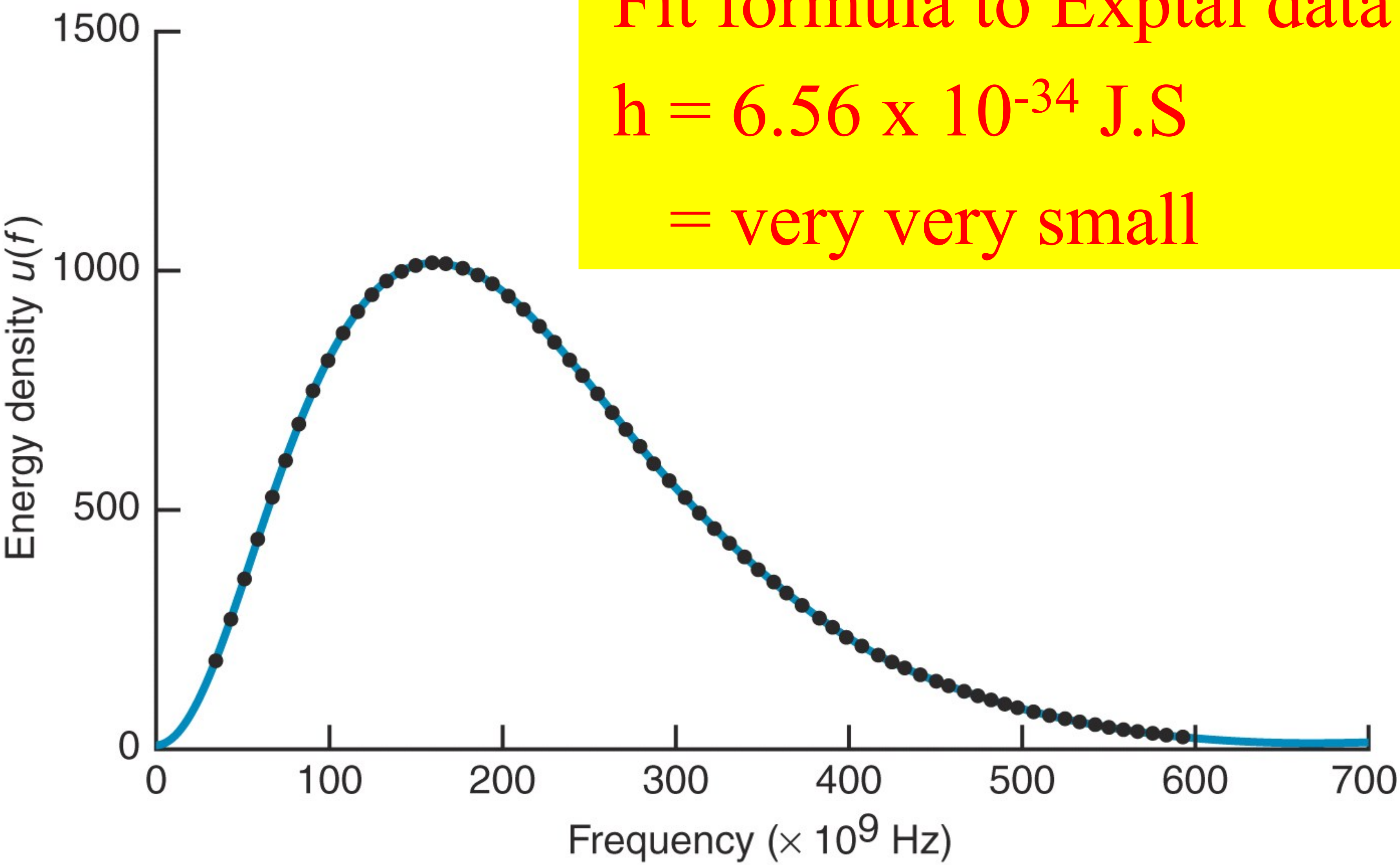
$$R(\lambda) = \left(\frac{c}{4}\right)\left(\frac{8\pi}{\lambda^4}\right)e^{-\frac{hc}{\lambda kT}}$$

As $\lambda \to 0$, $e^{-\frac{hc}{\lambda kT}} \to 0$

$$\Rightarrow R(\lambda) \to 0$$

Just as seen in the experimental data

Planck's Explanation of BB Radiation



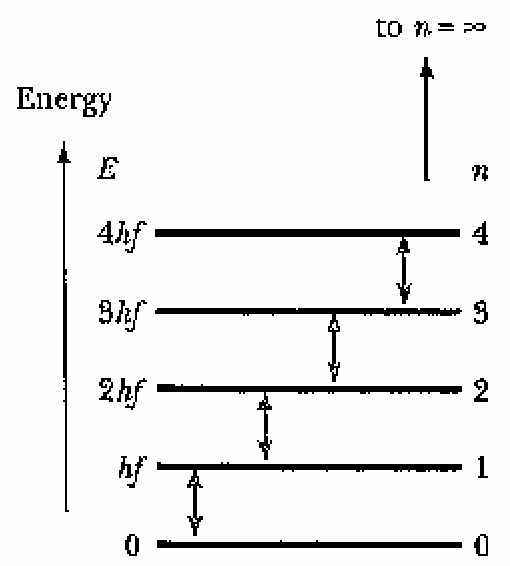
Fit formula to Exptal data

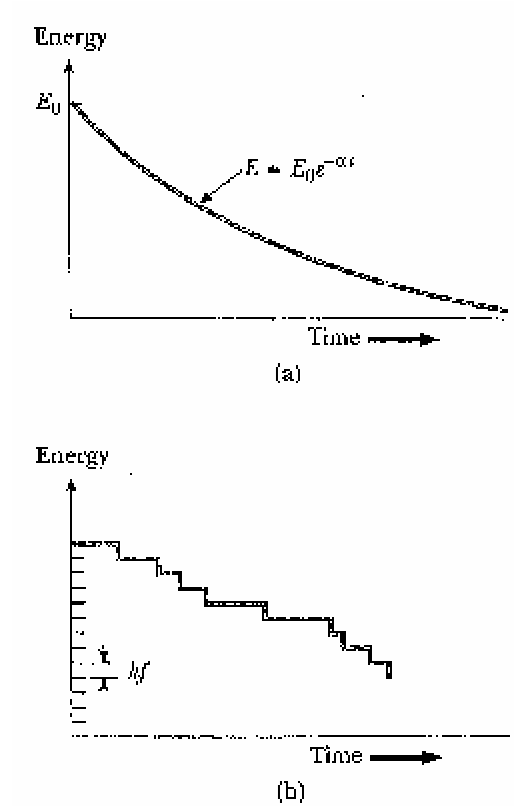
$h = 6.56 \times 10^{-34}$ J.S

= very very small

Consequence of Planck's Formula

Quantization of Energy !

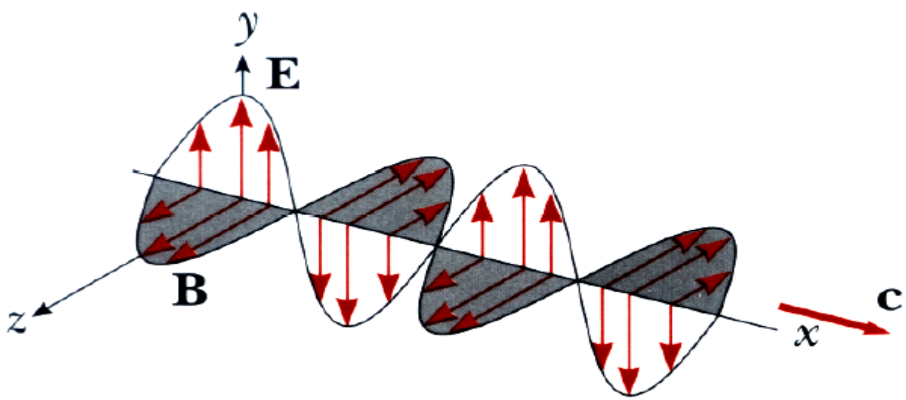


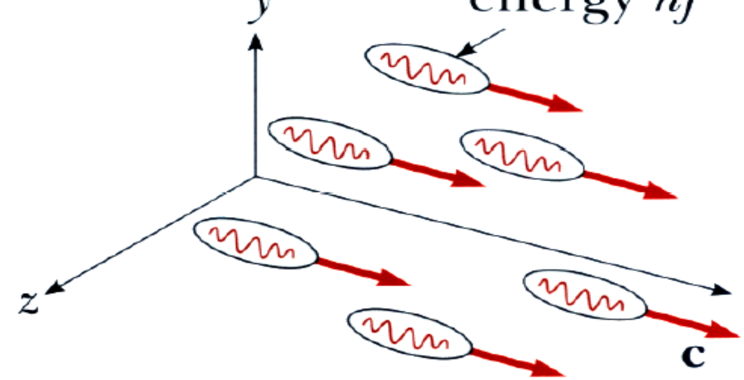


Einstein's Explanation of Photoelectric Effect

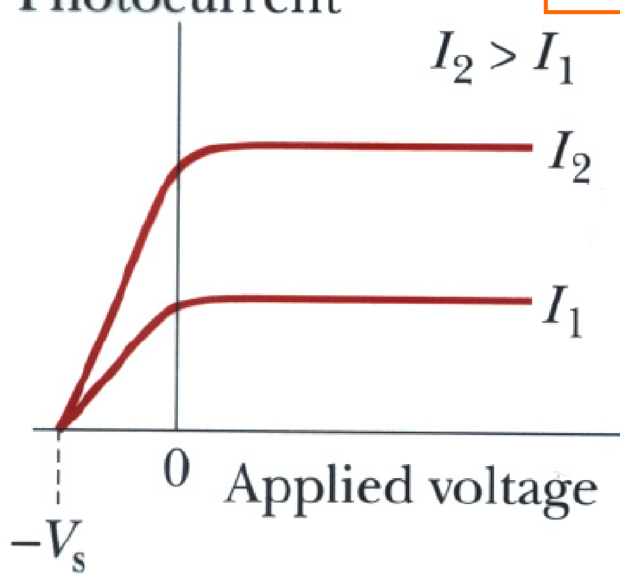
- Energy associated with EM waves in not uniformly distributed over wave-front, rather is contained in packets of "stuff" $\Rightarrow$ PHOTON
- $E = hf = hc/\lambda$ [but is it the same h as in Planck's th.?]
- Light shining on metal emitter/cathode is a stream of photons of energy which depends on frequency f
- Photons knock off electron from metal instantaneously
  - Transfer all energy to electron
  - Energy gets used up to pay for Work Function Φ (Binding Energy)
    - Rest of the energy shows up as KE of electron $KE = hf - \Phi$
- Cutoff Frequency $hf_0 = \Phi$ (pops an electron, $KE = 0$)
- Larger intensity I → more photons incident
- Low frequency light f → not energetic enough to overcome work function of electron in atom

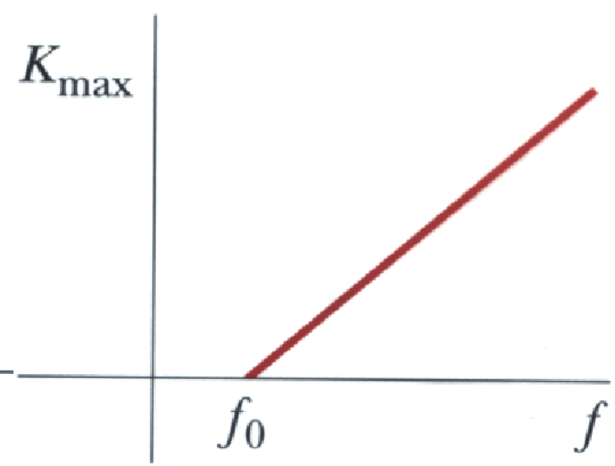
Einstein's Explanation of PhotoElectric Effect





$$V_s e = hf - \phi$$





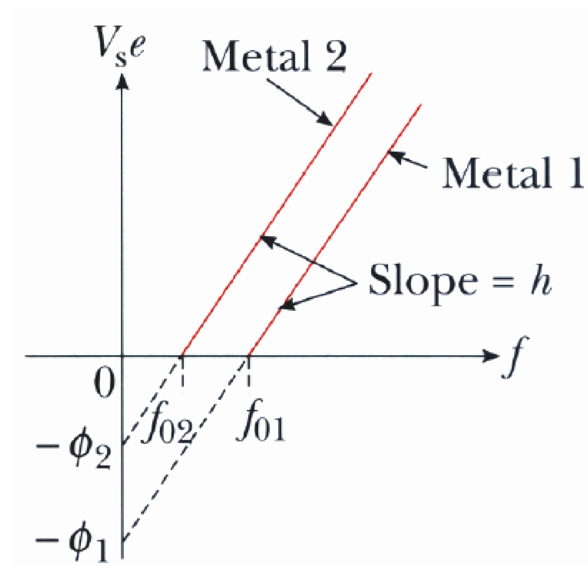


Photo Electric & Einstein (Nobel Prize 1915)

Light shining on metal cathode is made of photons

Each of the same energy E, depends on frequency f

$$E = hf = h\,(c/\lambda)$$

This QUANTA used to knock off electron & give KE

$$E = hf = KE + \varphi \Rightarrow KE = hf - \varphi$$

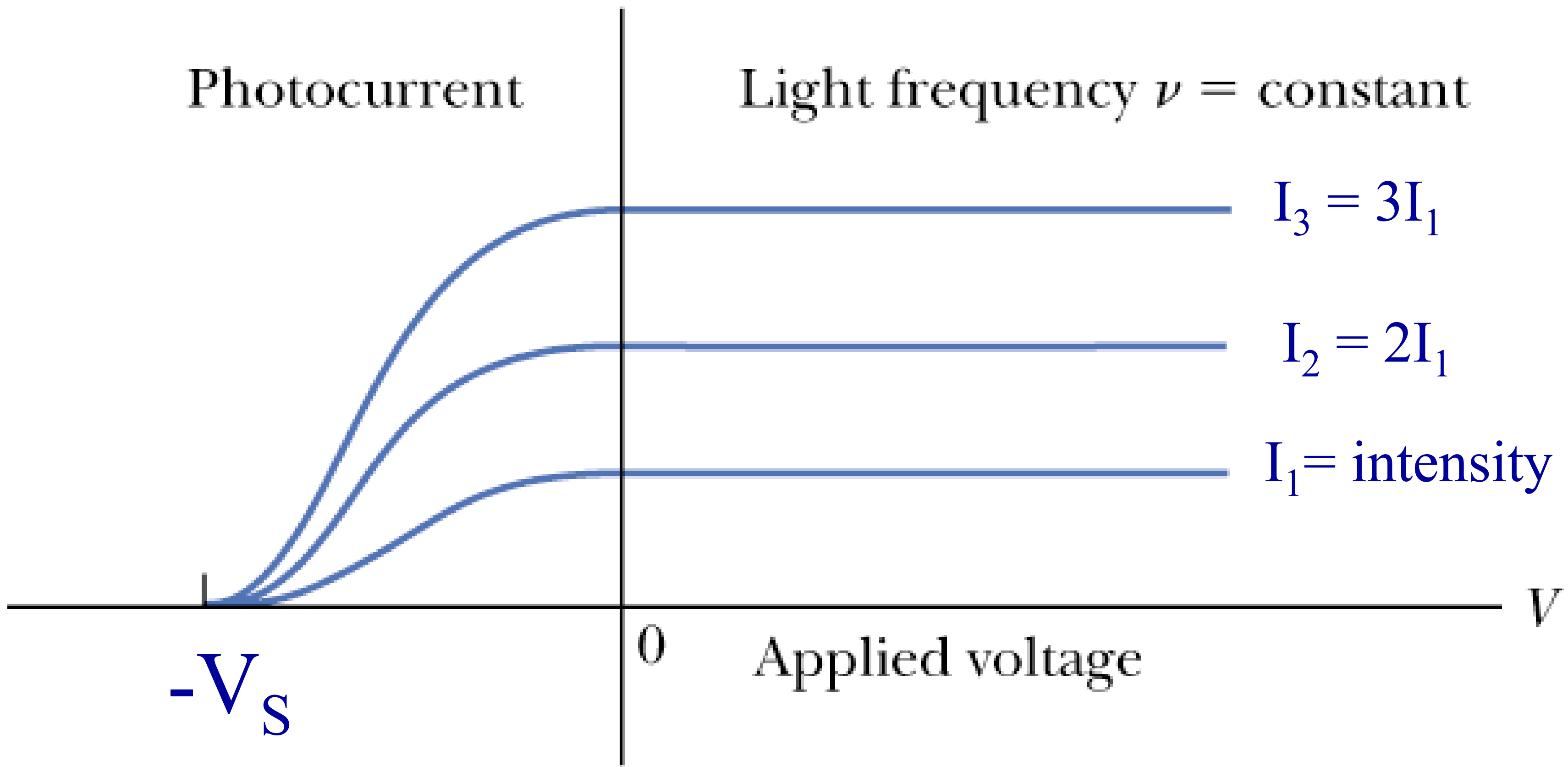


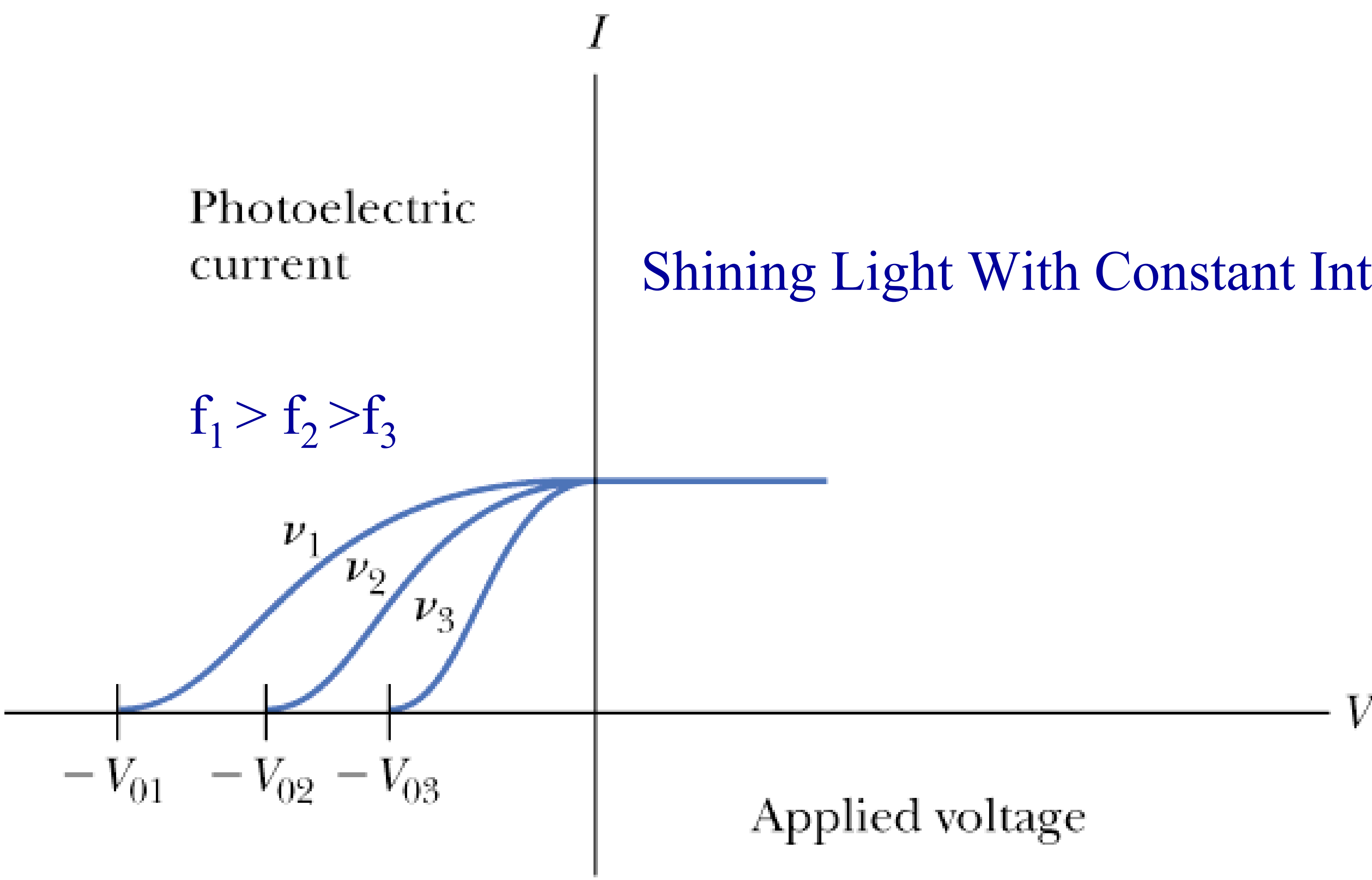
Photo Electric & Einstein (Nobel Prize 1915)

Light shining on metal cathode is made of photons

Quantum of Energy $E = hf = KE + \varphi \Rightarrow KE = hf - \varphi$

Shining Light With Constant Intensity

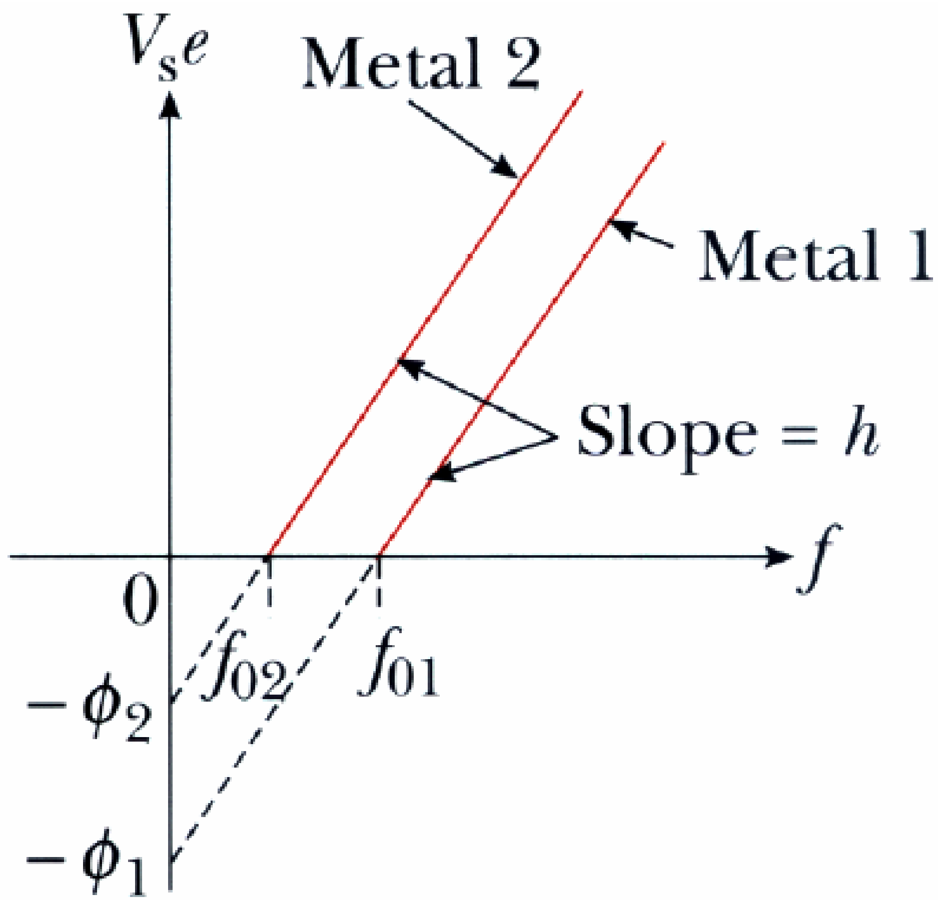
$f_1 > f_2 > f_3$



Is “h” same in Photoelectric Effect as BB Radiation?

Slope h $= 6.626 \times 10^{-34}$ JS

Einstein → Nobel Prize!



No matter where you travel
in the galaxy and beyond…
..no matter what experiment
You do

h : Planck’s constant is same

NOBEL PRIZE FOR PLANCK

Work Function (Binding Energy) In Metals

TABLE 3-1 Photoelectric work functions

Element	ϕ (eV)
Na	2.28
C	4.81
Cd	4.07
Al	4.08
Ag	4.73
Pt	6.35
Mg	3.68
Ni	5.01
Se	5.11
Pb	4.14

Photoelectric Effect on An Iron Surface:

Light of Intensity $I = 1.0\ \mu W/cm^2$ incident on $1.0 cm^2$ surface of Fe

Assume Fe reflects 96% of light

further only 3% of incident light is Violet region ($\lambda = 250nm$)

barely above threshold frequency for Ph. El effect

(a) Intensity available for Ph. El effect $I = 3\% \times 4\% \times (1.0\ \mu W/cm^2)$

(b) how many photo-electrons emitted per second ?

$$\#\text{ of photoelectrons} = \frac{\text{Power}}{h f} = \frac{3\% \times 4\% \times (1.0\ \mu W/cm^2)\ \lambda}{hc}$$

$$= \frac{(250\times10^{-9}\, m)(1.2\times10^{-9}\, J/s)}{(6.6\times10^{-34}\, J\bullet s)(3.0\times10^{8}\, m/s)}$$

$$= 1.5\times10^{9}$$

(c) Current in Ammeter : $i = (1.6\times10^{-19} C)(1.5\times10^{9}) = 2.4\times10^{-10} A$

(d) Work Function $\Phi = hf_0 = (4.14\times10^{-15} eV\bullet s)(1.1\times10^{15} s^{-1})$

$$= 4.5\ eV$$

Photon & Relativity: Wave or a Particle ?

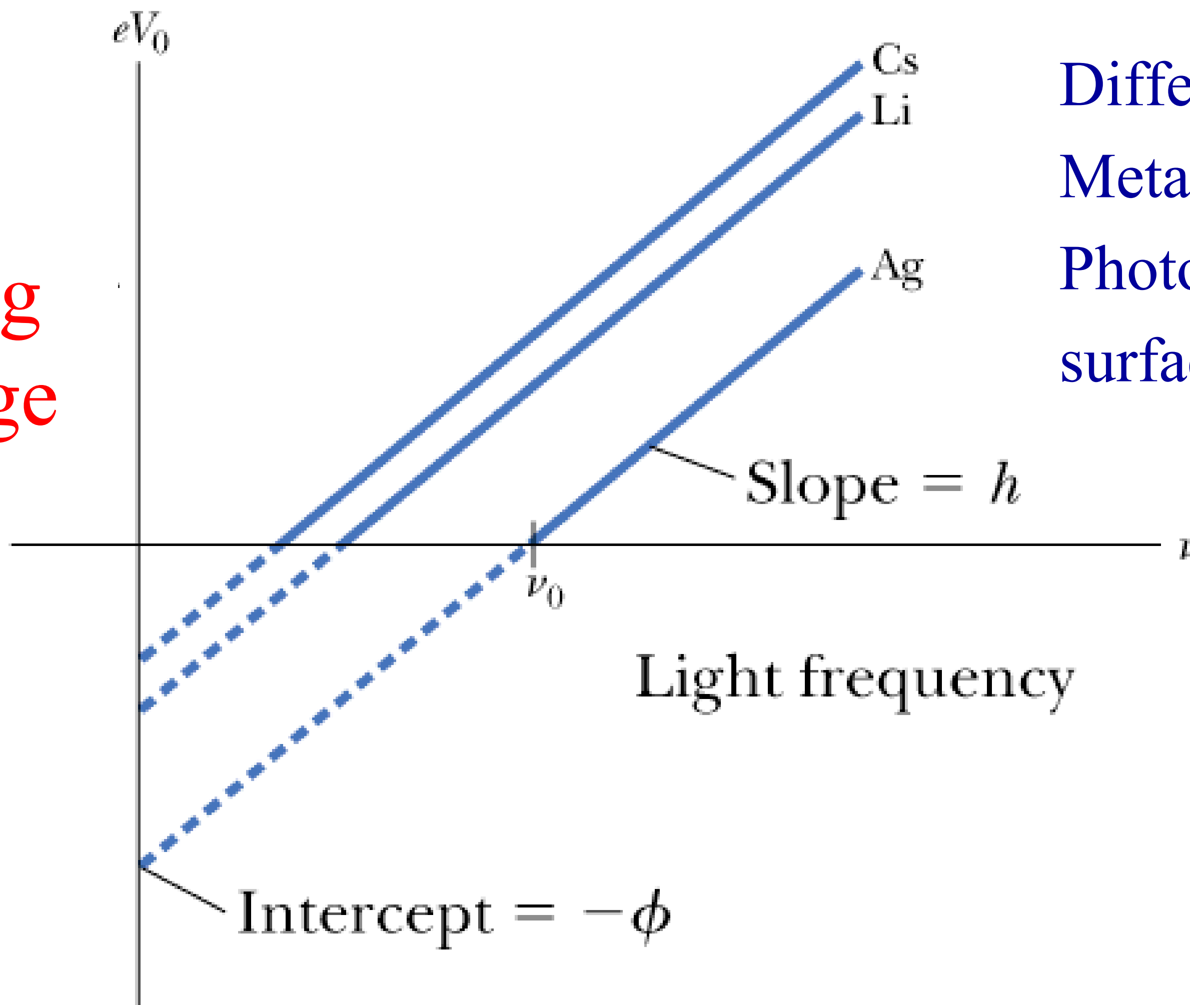
- Photon associated with EM waves, travel with speed =c
- For light (m =0) : Relativity says $E^2 = (pc)^2 + (mc^2)^2$
- $\Rightarrow E = pc$
- But Planck tells us : $E = hf = h\,(c/\lambda)$
- Put them together : $hc/\lambda = pc$
  - $\Rightarrow$ $p = h/\lambda$
  - Momentum of the photon (light) is inversely proportional to λ
- But we associate λ with waves & p with particles ….what is going on??
  - A new paradigm of conversation with the subatomic particles : Quantum Physics

Photo Electric & Einstein (Nobel Prize 1915)

Light shining on metal cathode is made of photons

Quantum of Energy $E = hf = KE + \varphi \Rightarrow KE = hf - \varphi$

Stopping Voltage



Different Metal Photocathode surfaces