Physics 2D Lecture Slides
Oct 1

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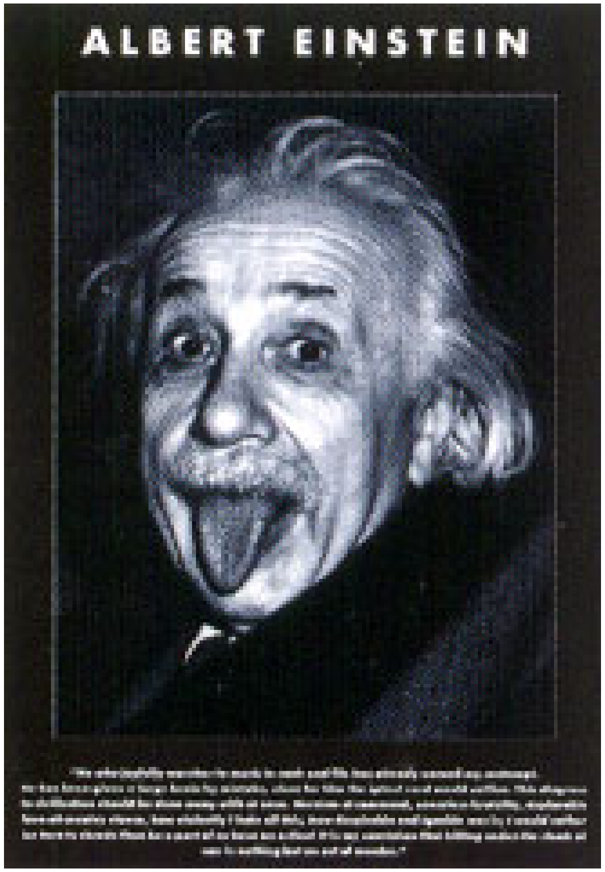
UCSD Physics

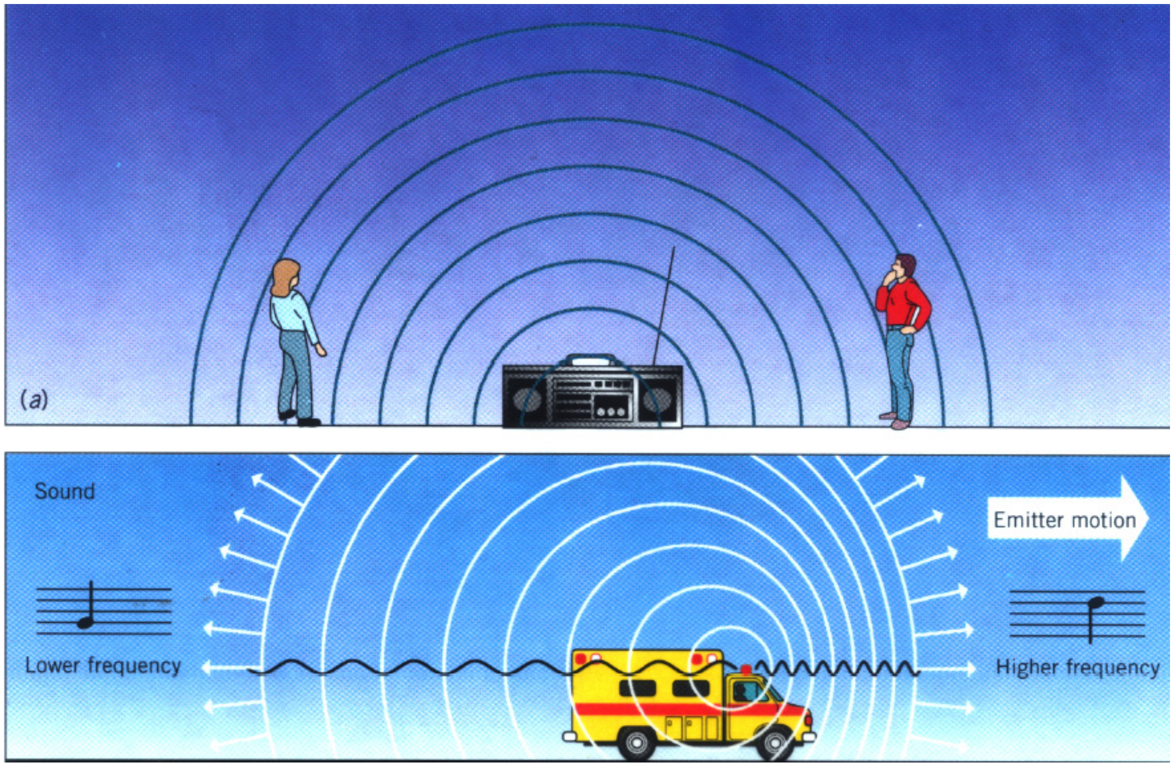
Einstein's Special Theory of Relativity



Einstein's Postulates of SR

- The laws of physics must be the same in all inertial reference frames
- The speed of light in vacuum has the same value ($c = 3.0 \times 10^8$ m/s) , in all inertial frames, regardless of the velocity of the observer or the velocity of the source emitting the light.

Doppler Effect In Sound : Reminder from 2A

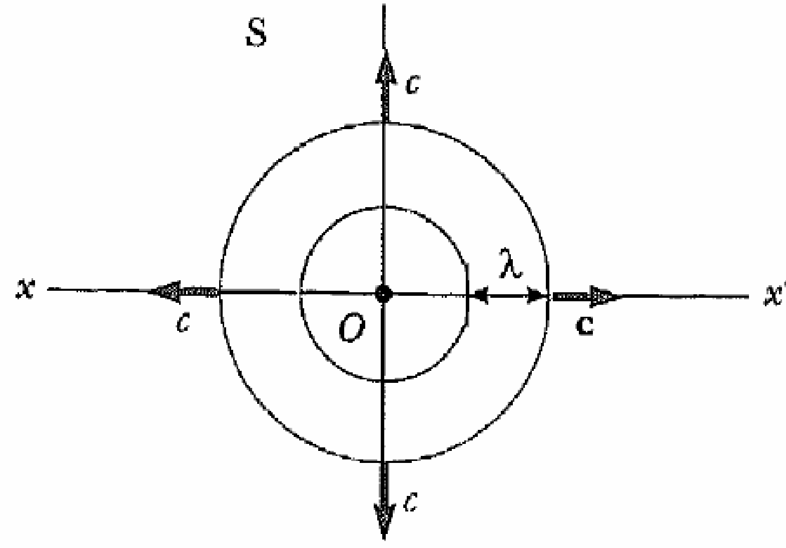


Observed Frequency of sound INCREASES if emitter moves towards the Observer
Observed Wavelength of sound DECREASES if emitter moves towards the Observer

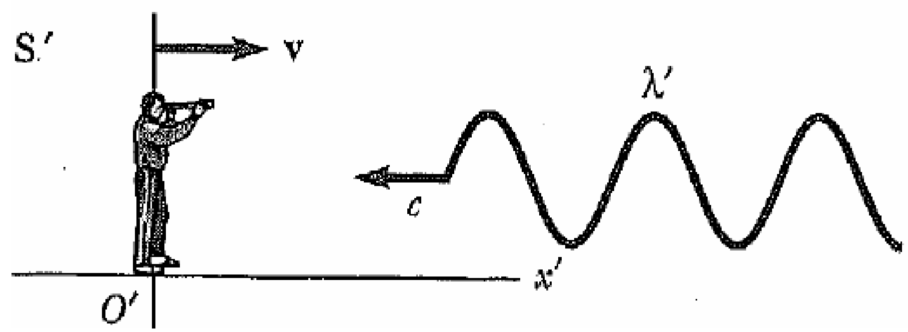
$$v = f\lambda$$

Time Dilation Example: Relativistic Doppler Shift

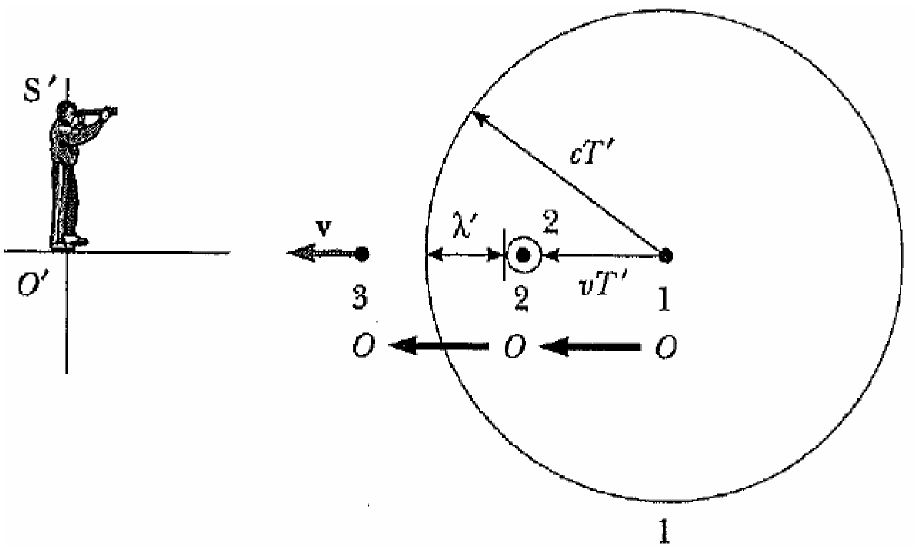
- Light: velocity c = f λ , f =1/T
- A source of light S at rest
- Observer S'approches S with velocity v
- S' measures f' or λ', c = f' λ'
- Expect f' > f since more wave crests are being crossed by Observer S' due to its approach direction than if it were at rest w.r.t source S



(a)



Relativistic Doppler Shift



Examine two successive wavefronts emitted by S at location 1 and 2

In S' frame, T' = time between two wavefronts

In time T', the Source moves by cT' w.r.t 1

Meanwhile Light Source moves a distance vT'

Distance between successive wavefront

$$\lambda' = cT' - vT'$$

$\lambda' = cT' - vT'$, use $f = c/\lambda$

$$f' = \frac{c}{(c-v)T'}, \quad T' = \frac{T}{\sqrt{1-(v/c)^2}}$$

Substituting for T', use f=1/T

$$\Rightarrow f' = \frac{\sqrt{1-(v/c)^2}}{1-(v/c)}$$

$$\Rightarrow f' = \frac{\sqrt{1+(v/c)}}{\sqrt{1-(v/c)}}\, f$$

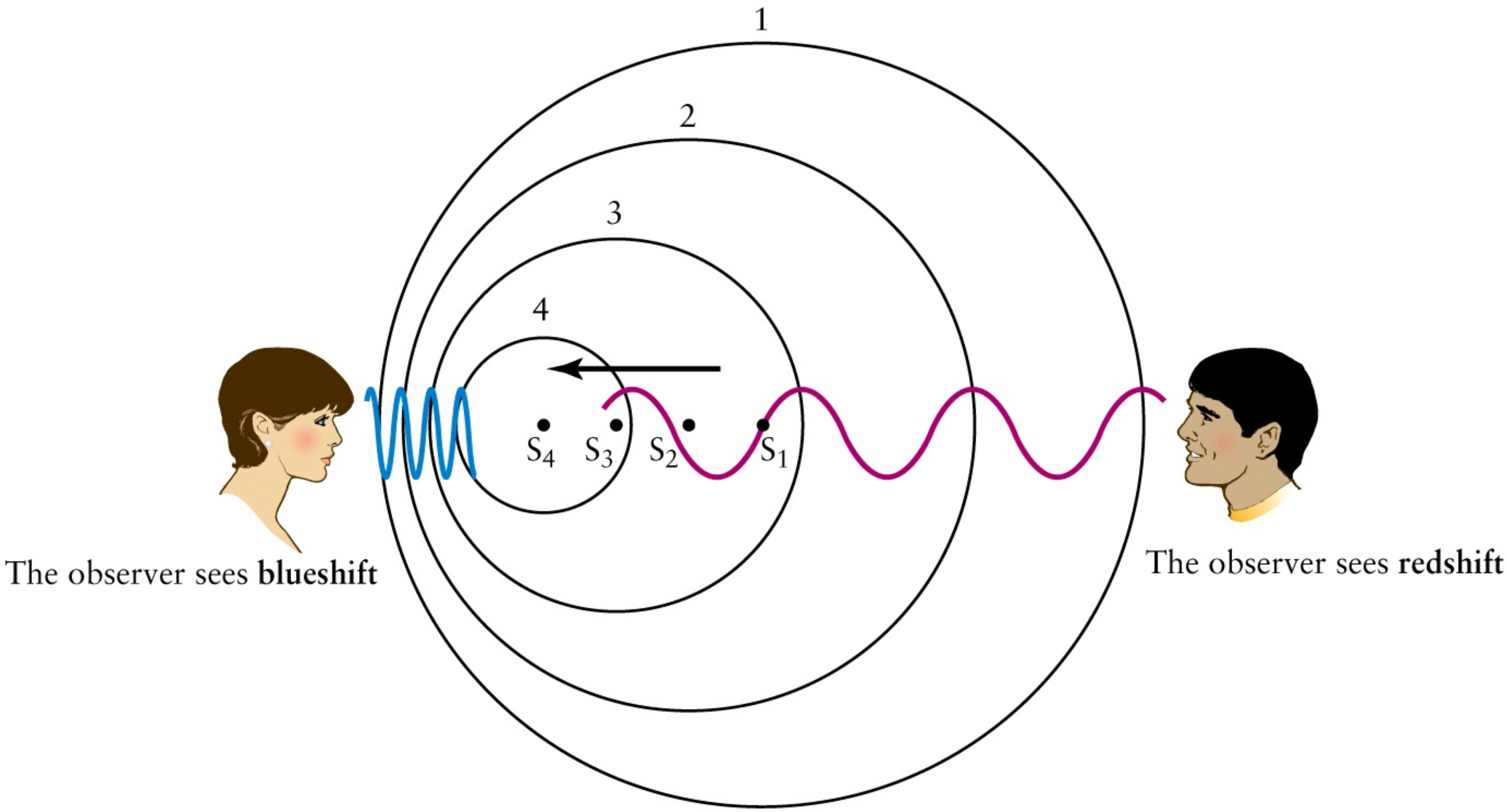
better remembered as:

$$f_{obs} = \frac{\sqrt{1+(v/c)}}{\sqrt{1-(v/c)}}\, f_{source}$$

f_{obs} = Freq measured by observer approching light source

Relativistic Doppler Shift

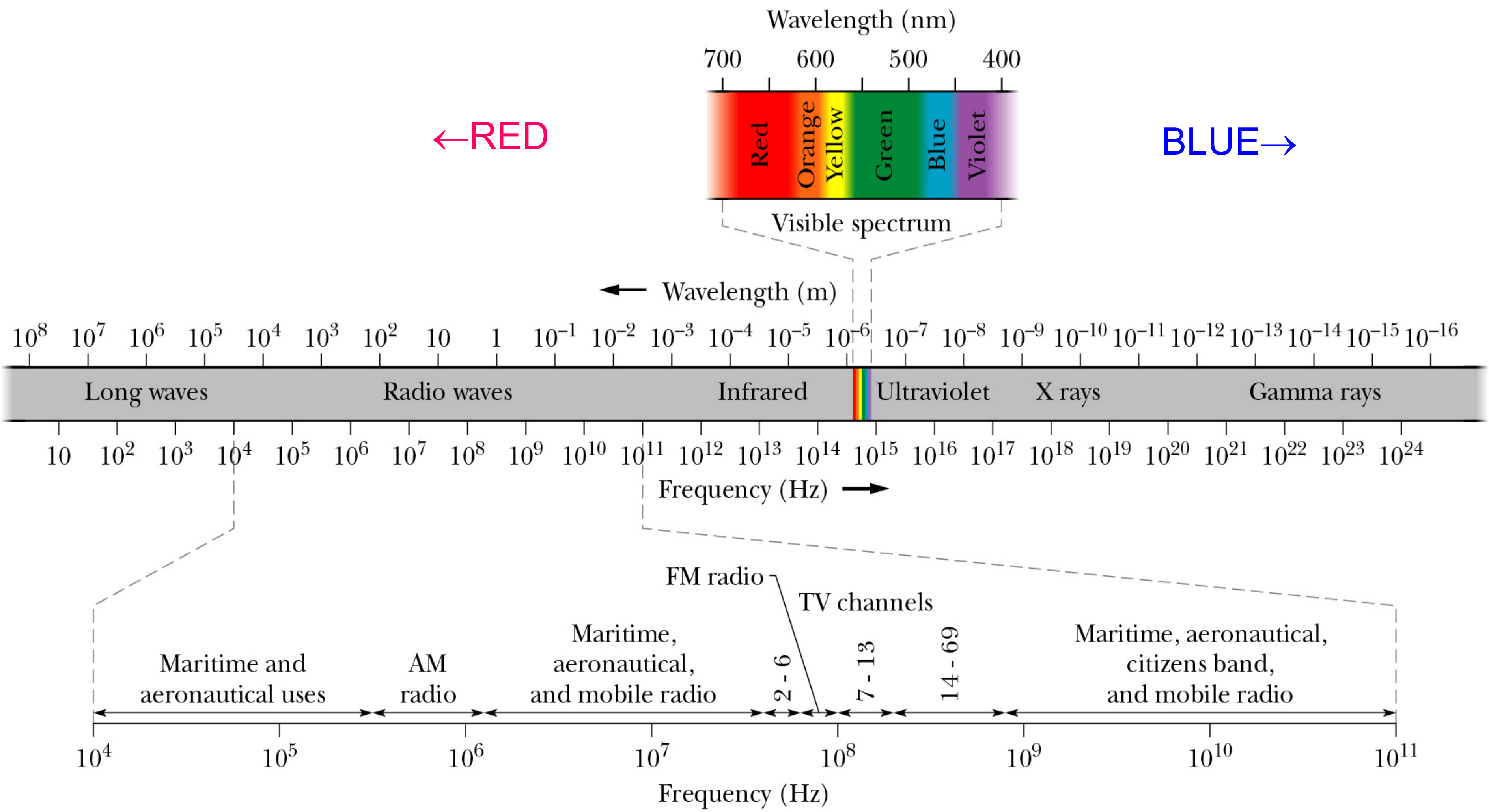
$$f_{obs} = \frac{\sqrt{1+(v/c)}}{\sqrt{1-(v/c)}} f_{source}$$



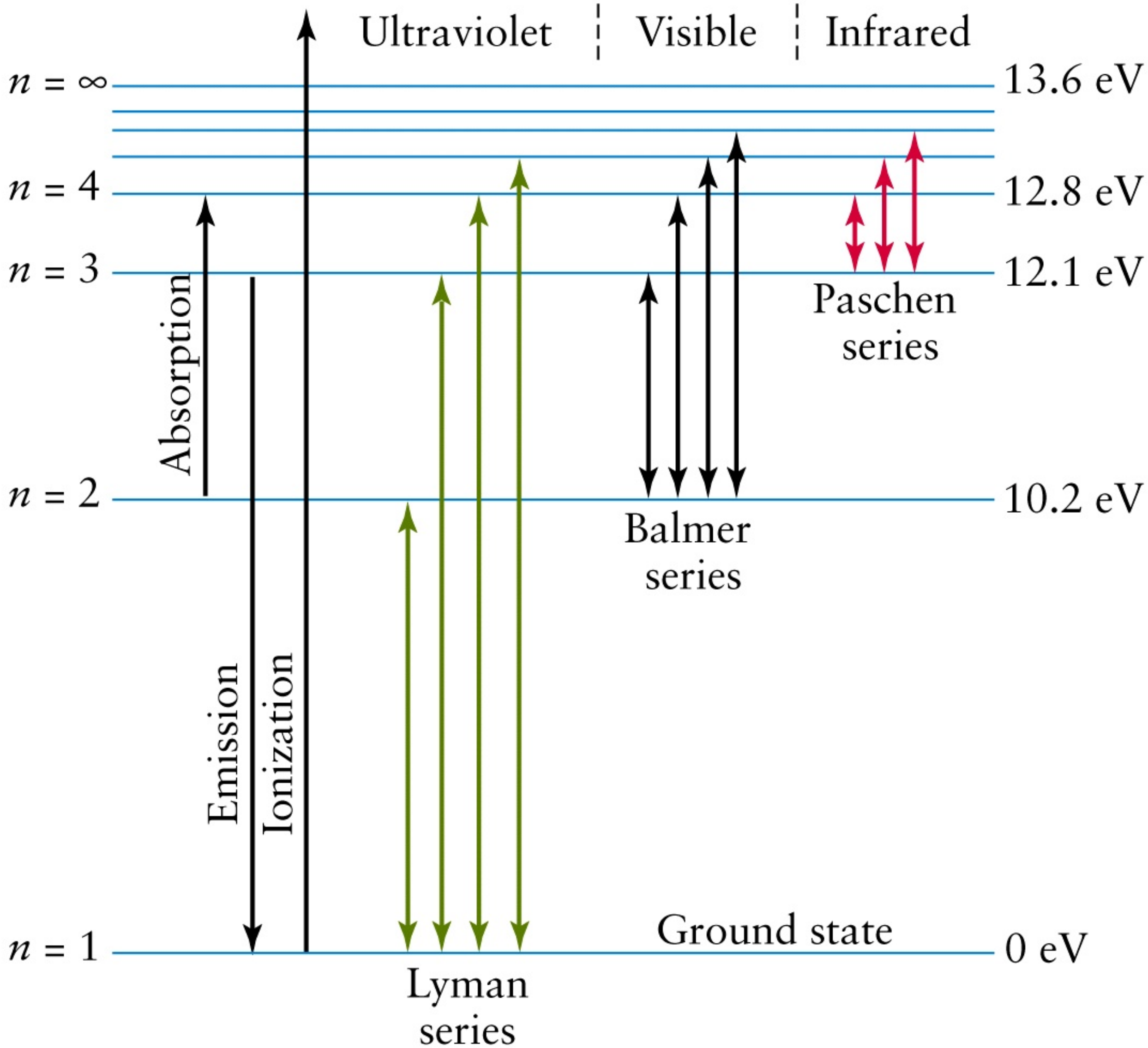
Doppler Shift & Electromagnetic Spectrum

←RED

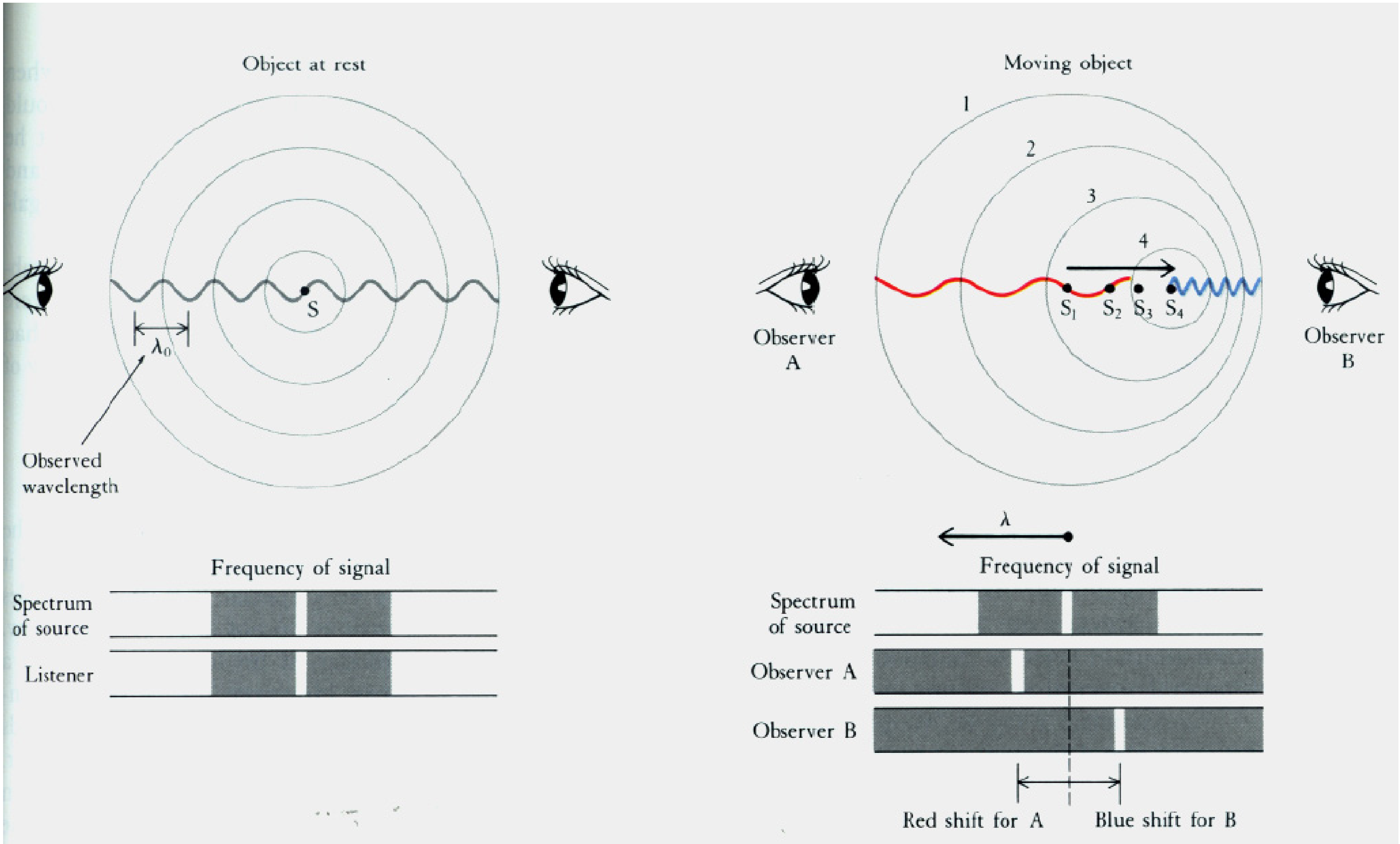
BLUE→



Fingerprint of Elements: Emission & Absorption Spectra



Spectral Lines and Perception of Moving Objects



Doppler Shift in Spectral Lines and Motion of Stellar Objects

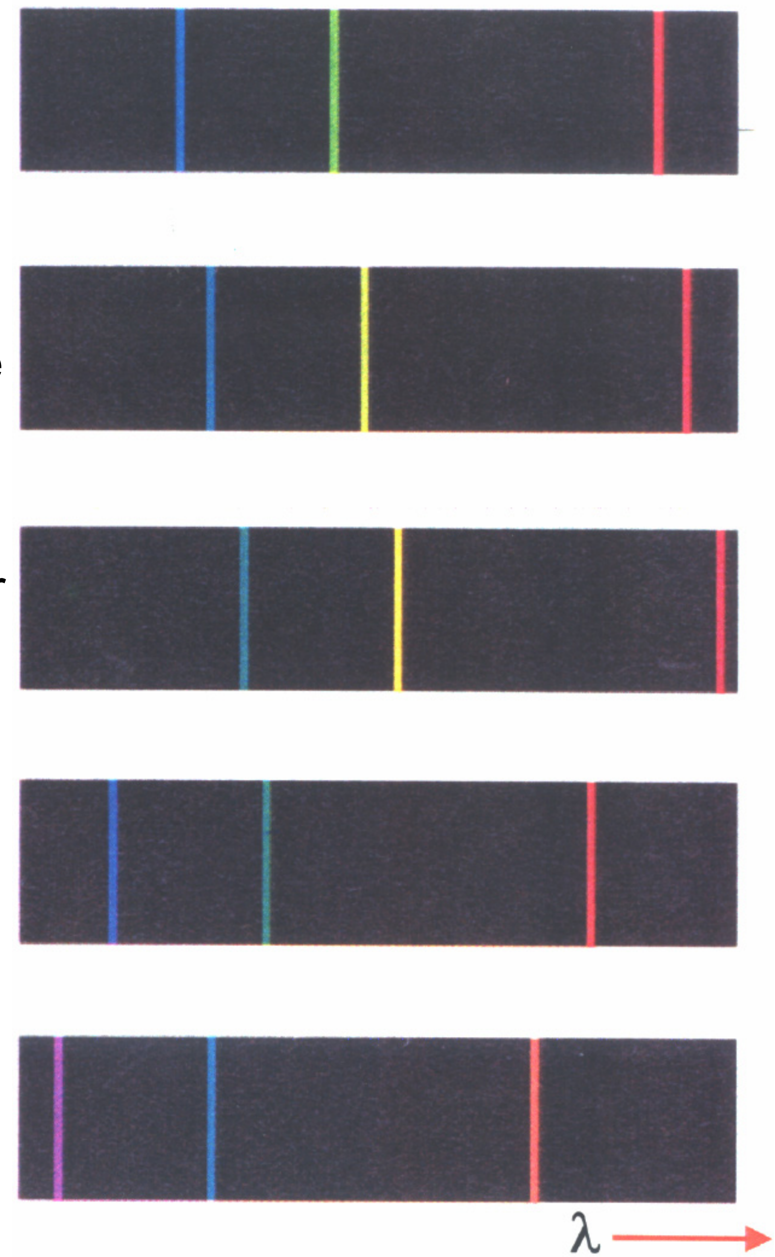
Laboratory Spectrum, lines at rest wavelengths

Lines Redshifted, Object moving away from me

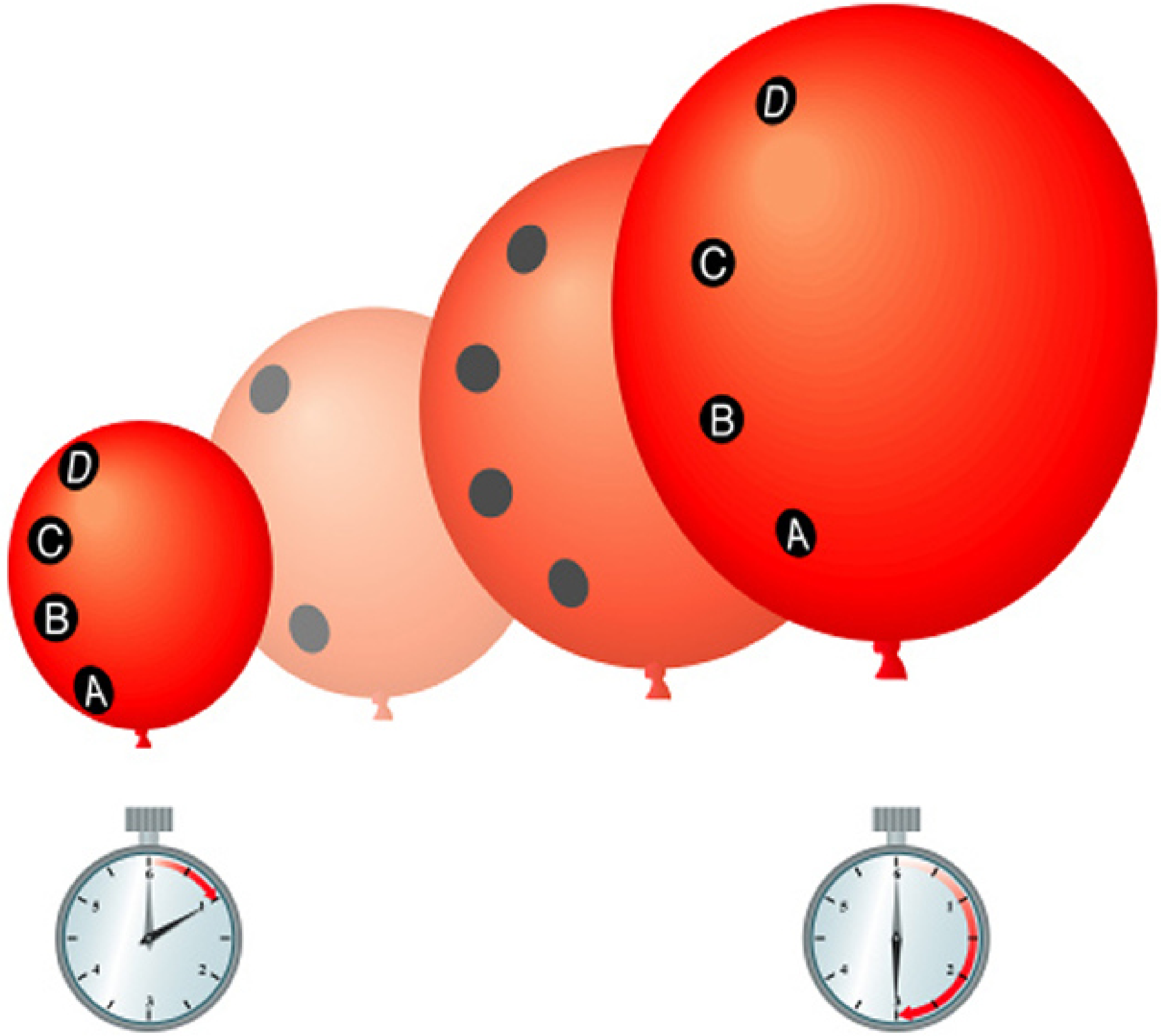
Larger Redshift, object moving away even faster

Lines blueshifted, Object moving towards me

Larger blueshift, object approaching me faster

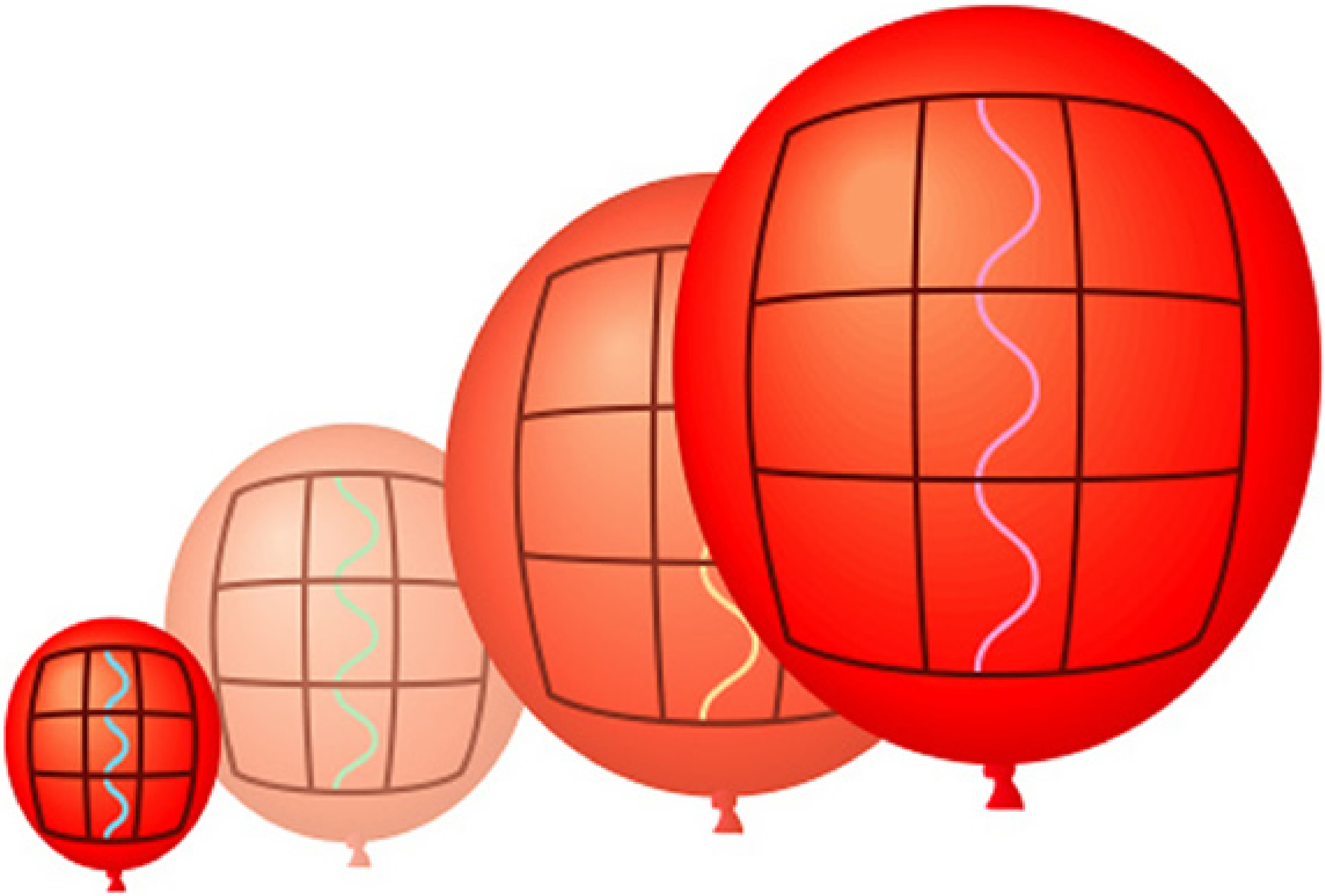


Cosmological Redshift & Discovery of the Expanding Universe:
[Space itself is Expanding]

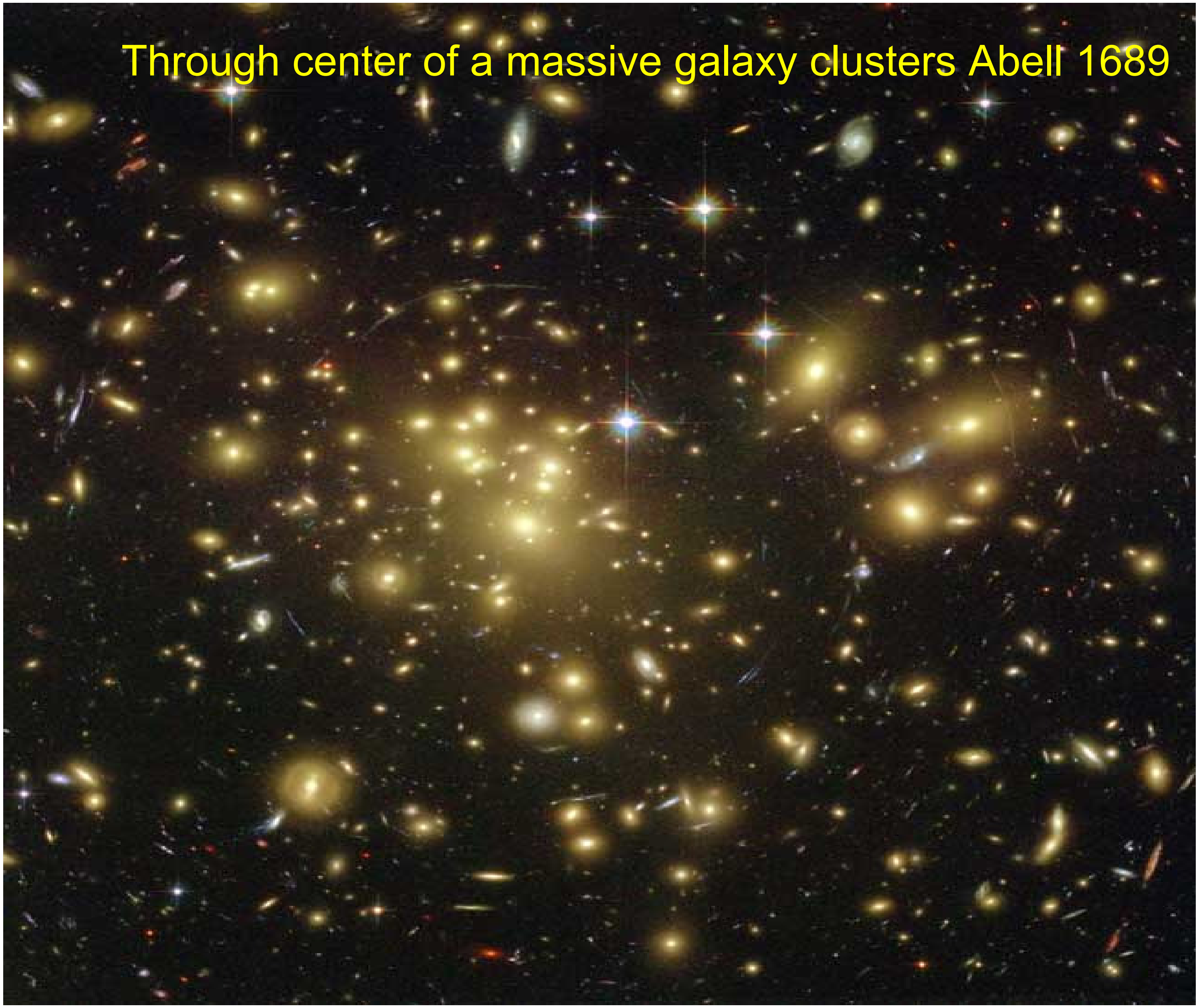


Cosmological Redshift

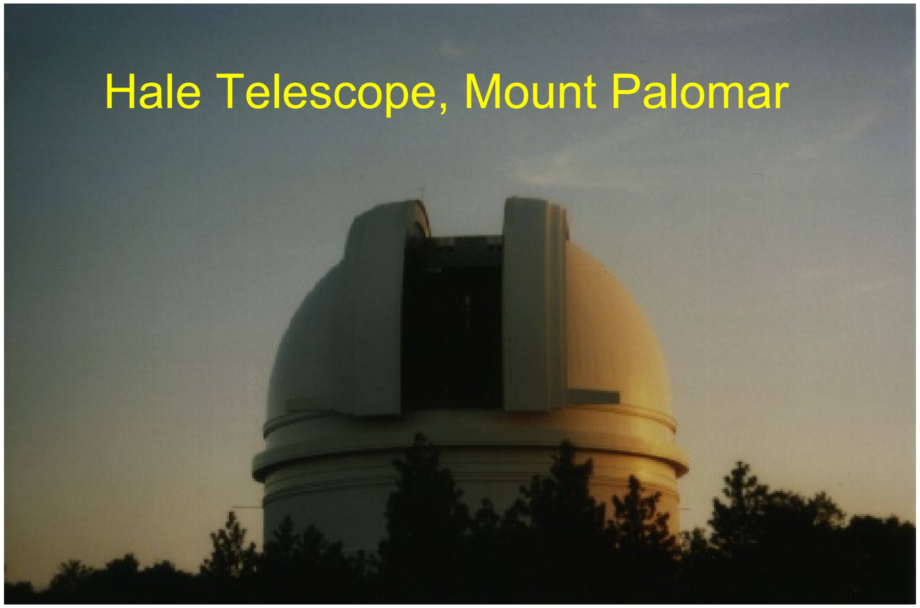
As Universe expands EM waves stretch in Wavelength

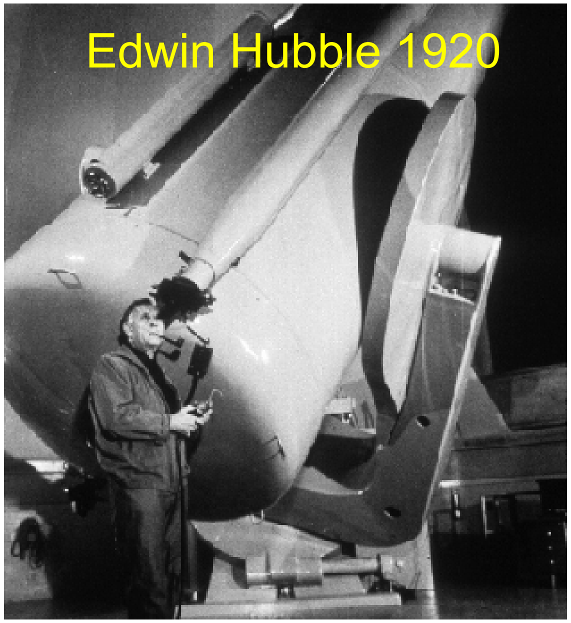


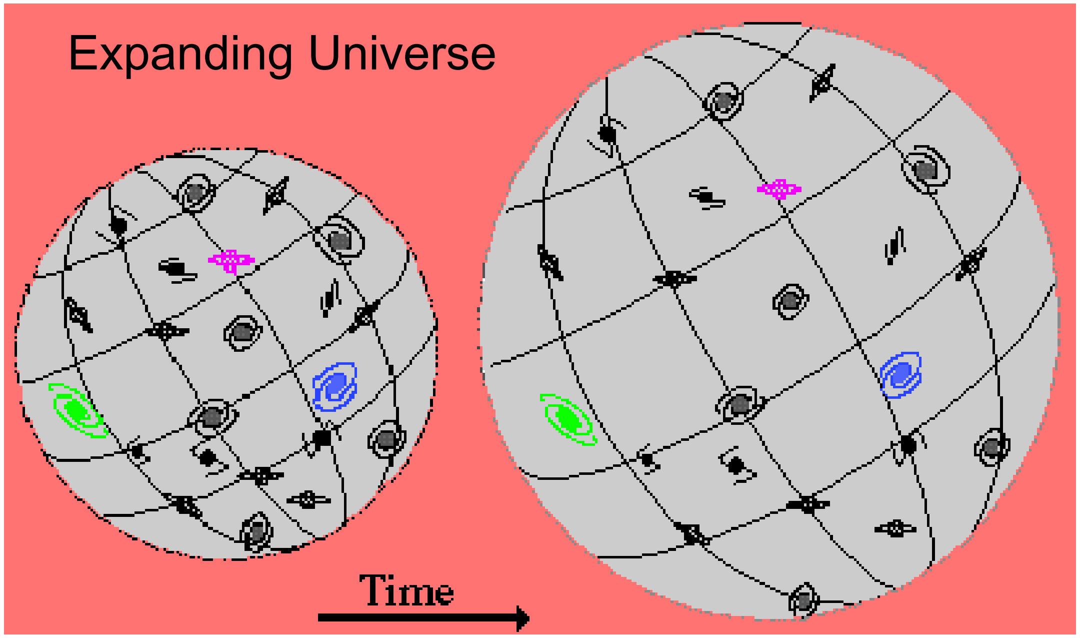
Seeing Distant Galaxies Thru Hubble Telescope



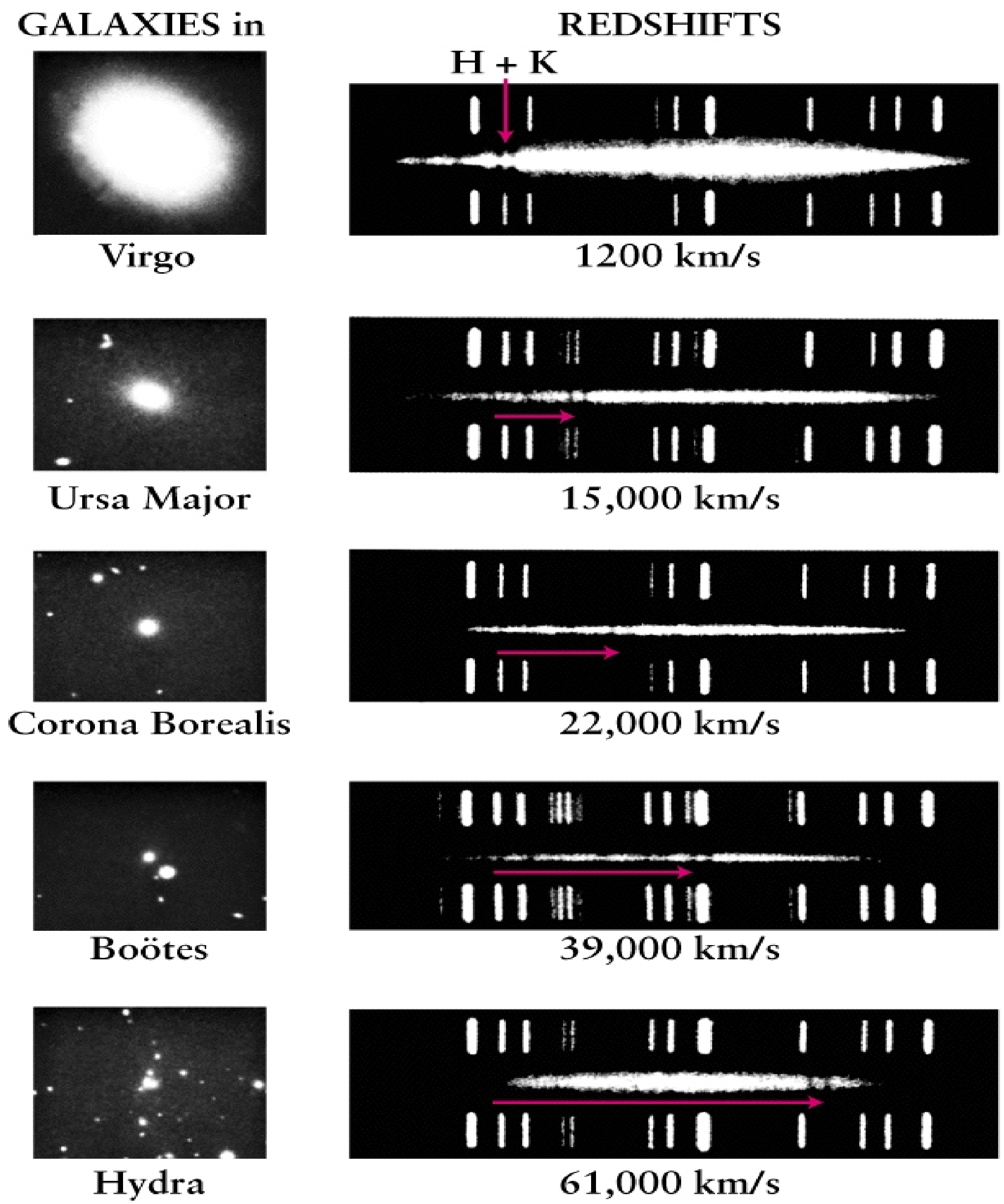
Expanding Universe, Edwin Hubble & Mount Palomar





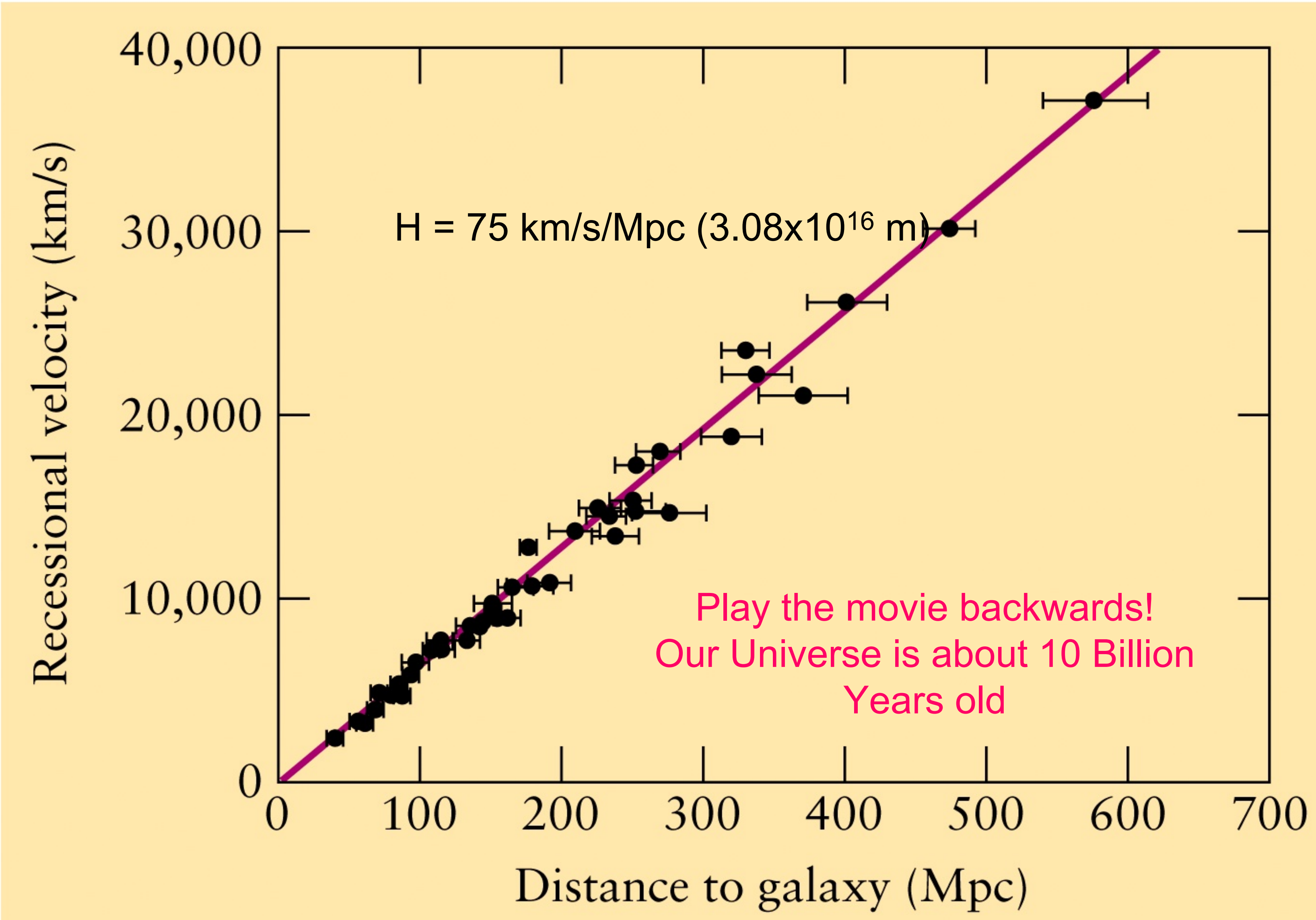


Galaxies at different locations in our Universe travel at different velocities



Hubble's Measurement of Recessional Velocity of Galaxies

V = H d : Farther things are faster they go



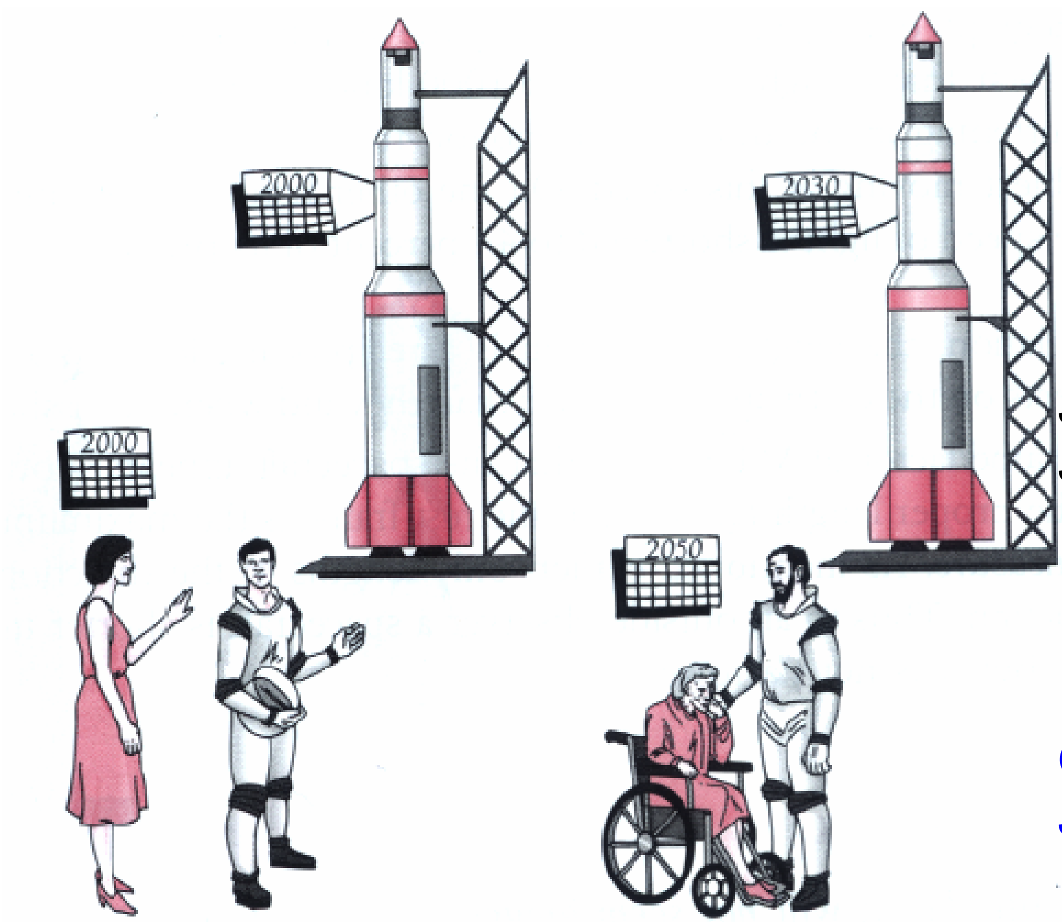
Now for Something Totally Different : Paradox !

A paradox is an apparently self-contradictory statement, the underlying meaning of which is revealed only by careful scrutiny. The purpose of a paradox is to arrest attention and provoke fresh thought

``A paradox is not a conflict within reality. It is a conflict between reality and your feeling of what reality should be like." - Richard Feynman

Construct a few paradoxes in Relativity & analyze them

Jack and Jill's Excellent Adventure: Twin Paradox



Jill sees Jack's heart slow down

Factor : $\sqrt{1-(v/c)^2}$

$$= \sqrt{1-(0.8c/c)^2} = 0.6$$

For every 5 beats of her heart
She sees Jack's beat only 3 !

Jack has only 3 thoughts for 5 that Jill has ! Every things slows!

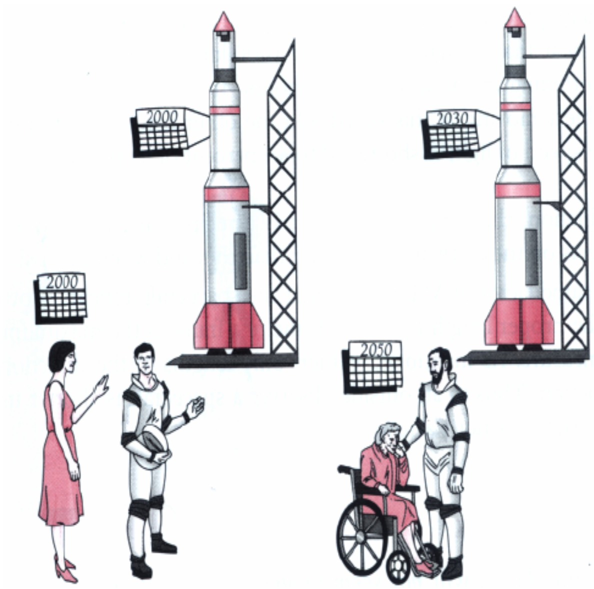
Finally Jack returns after 50 yrs gone by according to Jill's calendar

Only 30 years have gone by Jack
Jack is 50 years old, Jane is 70 !

Jack & Jill are 20 yr old twins, with same heartbeat
Jack takes off with V = 0.8c to a star 20 light years away
Jill stays behind, watches Jack by telescope

Is there a paradox here ??

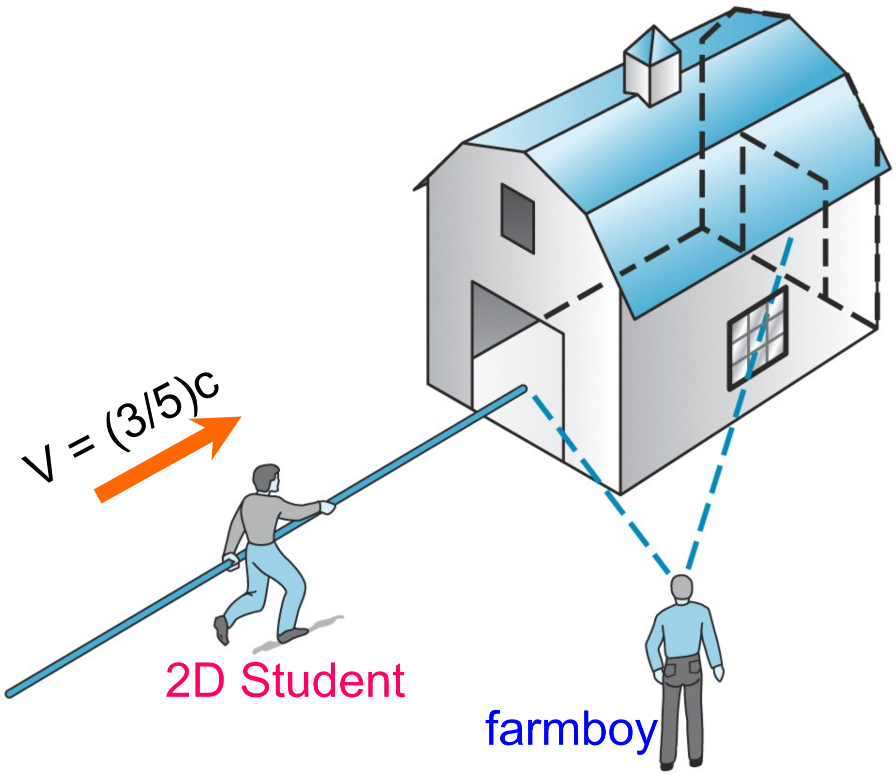
Twin Paradox ?



- Paradox : Turn argument around, motion is relative
- Jack claims he at rest, Jill is moving v=0.8c
- Should not Jill be 50 years old when 70 year old Jack returns from space Odyssey?
- No ! …because Jack is not traveling in a inertial frame of reference
  - TO GET BACK TO EARTH HE HAS TO TURN AROUND => decelerate/accelerate
- But Jill always remained in Inertial frame
- Time dilation formula applies to Jill's observation of Jack but not to Jack's observation of Jill

Non-symmetric aging verified with atomic clocks taken on airplane trip around world and compared with identical clock left behind. Observer who departs from an inertial system will always find its clock slow compared with clocks that stayed in the system

Fitting a 5m pole in a 4m barnhouse



Student with pole runs with v=(3/5)c

farmboy sees pole contraction factor

$$\sqrt{1-(3c/5c)^2} = 4/5$$

says pole just fits in the barn fully!

Student with pole runs with v=(3/5)c

Student sees barn contraction factor

$$\sqrt{1-(3c/5c)^2} = 4/5$$

says barn is only 3.2m long, too short to contain entire 5m pole !

Farmboy says "You can do it"

Student says "Dude, you are nuts"

Is there a contradiction ? Is Relativity wrong?

d
e
c
i

Homework: You figure out who is right, if any and why.

Hint: Think in terms of observing two events

Arrival of left end of pole at left end of barn

Arrival of right end of pole at right end of barn

Discovering The Correct Transformation Rule

$x' = x - vt$ guess $\rightarrow$ $x' = G(x - vt)$

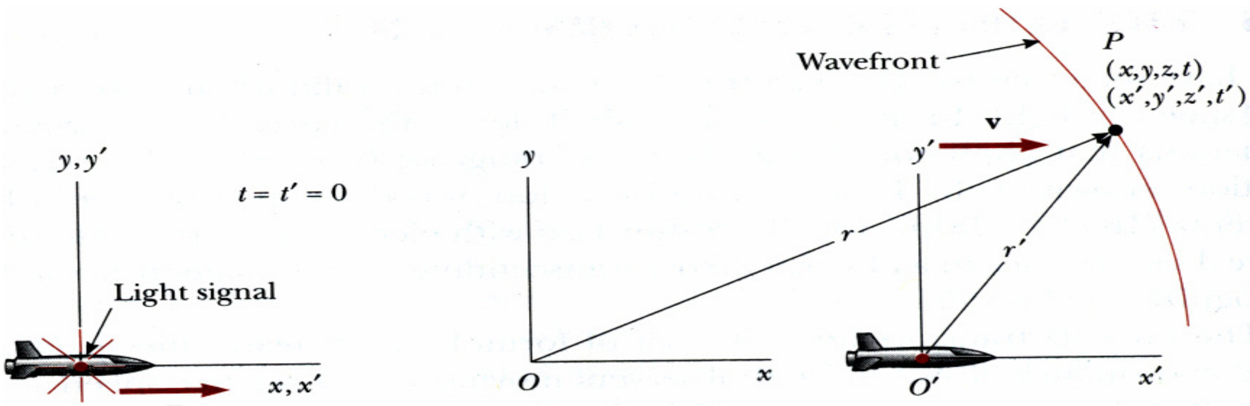
$x = x' + vt'$ guess $\rightarrow$ $x = G(x' + vt')$

Need to figure out functional form of G !

G must be dimensionless
G does not depend on x,y,z,t
But G depends on v/c
G is symmetric
As $v/c \rightarrow 0$, $G \rightarrow 1$

Do a Thought Experiment: Rocket Motion along x axis

Rocket in S' (x',y',z',t') frame moving with velocity v w.r.t observer on frame S (x,y,z,t)
Flashbulb mounted on rocket emits pulse of light at the instant origins of S,S' coincide
That instant corresponds to t = t' = 0 . Light travels as a spherical wave, origin is at O,O'



Speed of light is c for both observers

Examine a point P (at distance r from O and r' from O') on the Spherical Wavefront
The distance to point P from O : r = ct
The distance to point P from O : r' = ct'

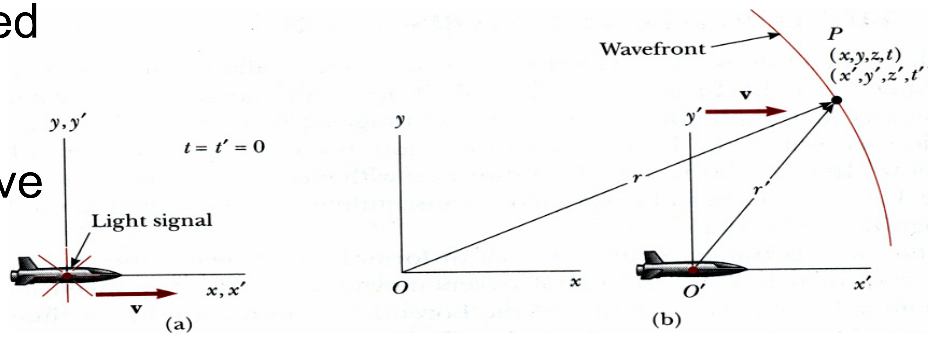
Clearly t and t' must be different
$t \neq t'$

Discovering Lorentz Transfromation for (x,y,z,t)

Motion is along x-x' axis, so y, z unchanged

$$y' = y, \qquad z' = z$$

Examine points x or x' where spherical wave crosses the horizontal axes: x = r , x' =r'



$$x = ct = G(x' + vt')$$

$$x' = ct' = G(x - vt),$$

$$\Rightarrow t' = \frac{G}{c}(x - vt)$$

$$\therefore x = ct = G(ct' + vt')$$

$$\therefore ct = G^2\left[(ct - vt) + vt - \frac{v^2}{c}t\right]$$

$$\Rightarrow c^2 = G^2[c^2 - v^2]$$

$$\text{or } G = \frac{1}{\sqrt{1-(v/c)^2}} = \gamma$$

$$\therefore \quad x' = \gamma(x - vt)$$

$$x' = \gamma(x - vt), \quad x = \gamma(x' + vt')$$

$$\Rightarrow \qquad x = \gamma(\gamma(x - vt) + vt')$$

$$\therefore \quad x - \gamma^2 x + \gamma^2 vt = \gamma vt'$$

$$\therefore \quad t' = \left[\frac{x}{\gamma v} - \frac{\gamma^2 x}{\gamma v} + \frac{\gamma^2 vt}{\gamma v}\right] = \gamma\left[\frac{x}{\gamma^2 v} - \frac{x}{v} + t\right]$$

$$\therefore \quad t' = \gamma\left[t + \frac{x}{v}\left(\frac{1}{\gamma^2} - 1\right)\right], \text{ since } \left(\frac{1}{\gamma^2} - 1\right) = -\left(\frac{v}{c}\right)^2$$

$$\Rightarrow t' = \gamma\left[t + \frac{x}{v}[1 - \left(\frac{v}{c}\right)^2 - 1\right] = \gamma\left[t - \left(\frac{vx}{c^2}\right)\right]$$

Lorentz Transformation Between Ref Frames

Lorentz Transformation

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

Inverse Lorentz Transformation

$$x = \gamma(x' + vt)$$

$$y = y'$$

$$z = z'$$

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

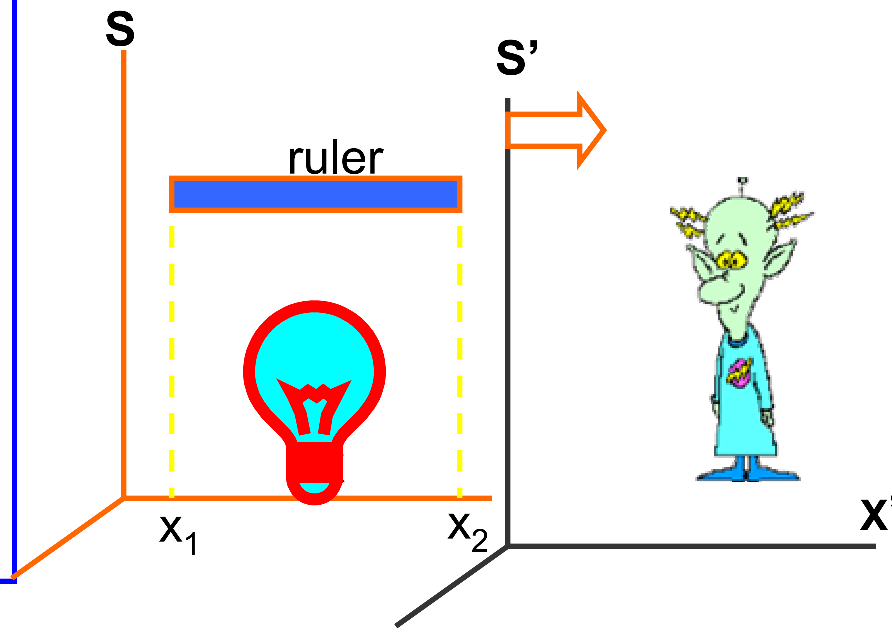
As v→0 , Galilean Transformation is recovered, as per requirement

Notice : SPACE and TIME Coordinates mixed up !!!

Lorentz Transform for Pair of Events

$$\left.\begin{aligned}\Delta x' &= \gamma(\Delta x - v\,\Delta t)\\ \Delta t' &= \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right)\end{aligned}\right\} S \rightarrow S'$$

$$\left.\begin{aligned}\Delta x &= \gamma(\Delta x' + v\,\Delta t')\\ \Delta t &= \gamma\left(\Delta t' + \frac{v}{c^2}\Delta x'\right)\end{aligned}\right\} S' \rightarrow S$$



One Can derive Length Contraction and Time Dilation formulae from this

Time dilation: Bulb in S frame turned on at t_1 & off at t_2 : What $\Delta t'$ did S' measure ?

two events occur at same place in S frame => $\Delta x = 0$

$$\Delta t' = \gamma\,\Delta t \quad (\Delta t = \text{proper time})$$

Length Contraction: Ruler measured in S between x_1 & x_2 : What $\Delta x'$ did S' measure ?

two ends measured at same time in S' frame => $\Delta t' = 0$

$$\Delta x = \gamma\,(\Delta x' + 0\,) \Rightarrow \Delta x' = \Delta x / \gamma \qquad (\Delta x = \text{proper length})$$

Velocity Transformation Rule : Just differentiate

$$u'_x = \frac{dx'}{dt'} \quad \textbf{(39.15)}$$

Using Equation 39.11, we have

$$dx' = \gamma(dx - v\,dt)$$

$$dt' = \gamma\left(dt - \frac{v}{c^2}\,dx\right)$$

Substituting these values into Equation 39.15 gives

$$u'_x = \frac{dx'}{dt'} = \frac{dx - v\,dt}{dt - \dfrac{v}{c^2}\,dx} = \frac{\dfrac{dx}{dt} - v}{1 - \dfrac{v}{c^2}\dfrac{dx}{dt}}$$

But dx/dt is just the velocity component u_x of the object measured by an observer in S, and so this expression becomes

$$\boxed{u'_x = \frac{u_x - v}{1 - \dfrac{u_x v}{c^2}}} \quad \textbf{(39.16)}$$