Physics 2D Lecture Slides
Nov 5

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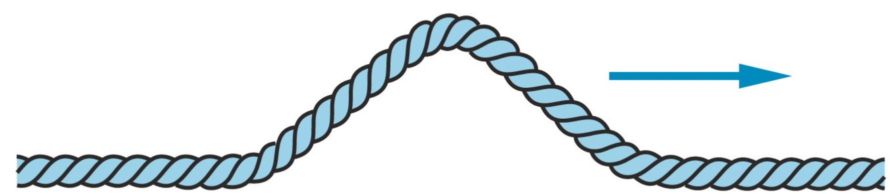
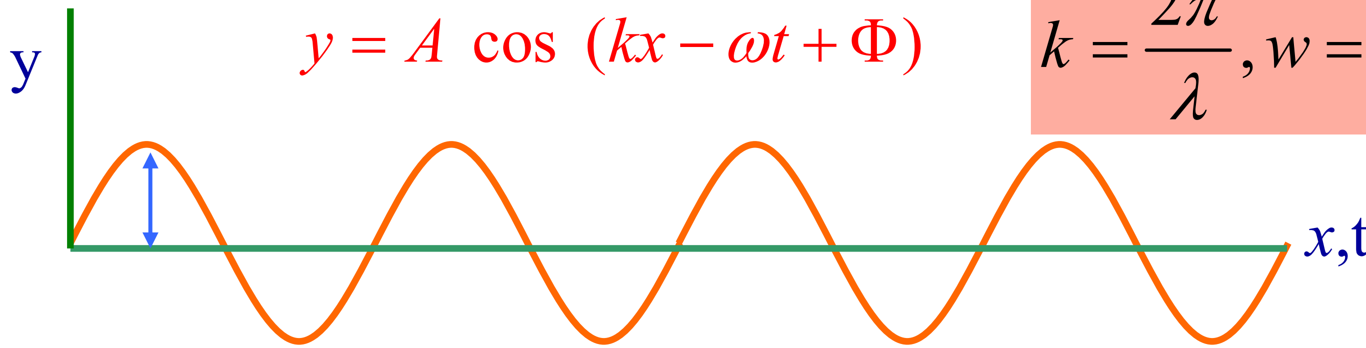
Just What is Waving in Matter Waves ?

- For waves in an ocean, it's the water that "waves"
- For sound waves, it's the molecules in medium
- For light it's the **E** & **B** vectors
- What's waving for matter waves ?
  - **It's the PROBABLILITY OF FINDING THE PARTICLE that waves !**
  - **Particle can be represented by a wave packet in**
    - **Space**
    - **Time**
    - **Made by superposition of many sinusoidal waves of different λ**
    - **It's a "pulse" of probability**

Imagine Wave pulse moving along a string: its localized in time and space (unlike a pure harmonic wave)



What Wave Does Not Describe a Particle



$$y = A\cos(kx - \omega t + \Phi)$$

$$k = \frac{2\pi}{\lambda}, w = 2\pi f$$

- What wave form can be associated with particle's pilot wave?
- A traveling sinusoidal wave? $y = A\cos(kx - \omega t + \Phi)$
- Since de Broglie "pilot wave" represents particle, it must travel with same speed as particle ……(like me and my shadow)

Phase velocity (v_p) of sinusoidal wave: $v_p = \lambda f$

In Matter:

$$(a)\quad \lambda = \frac{h}{p} = \frac{h}{\gamma m v}$$

$$(b)\quad f = \frac{E}{h} = \frac{\gamma m c^2}{h}$$

$$\Rightarrow v_p = \lambda f = \frac{E}{p} = \frac{\gamma m c^2}{\gamma m v} = \frac{c^2}{v} > c!$$

Conflicts with Relativity → Unphysical

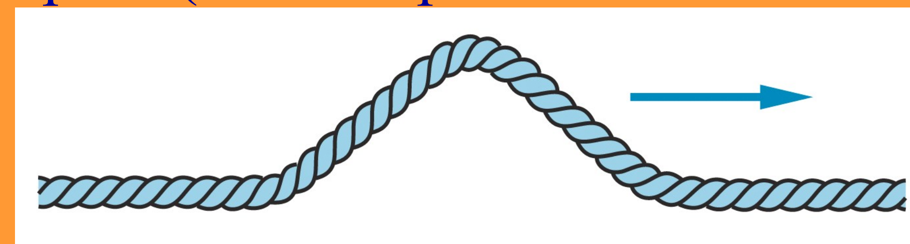
Single sinusoidal wave of infinite extent does not represent particle localized in space

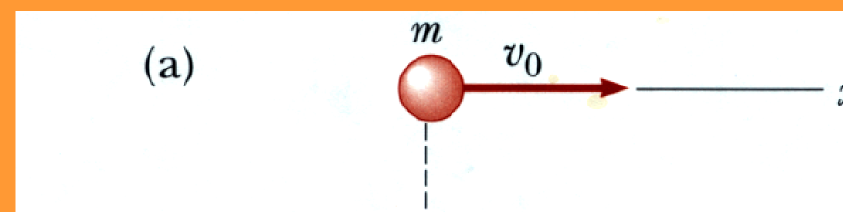
Need "wave packets" localized Spatially (x) and Temporally (t)

Wave Group or Wave Pulse

- Wave Group/packet:
  - Superposition of many sinusoidal waves with different wavelengths and frequencies
  - Localized in space, time
  - Size designated by
    - Δx or Δt
  - Wave groups travel with the speed $v_g = v_0$ of particle
- Constructing Wave Packets
  - Add waves of diff λ,
  - For each wave, pick
    - Amplitude
    - Phase
  - Constructive interference over the space-time of particle
  - Destructive interference elsewhere !

Imagine Wave pulse moving along a string: its localized in time and space (unlike a pure harmonic wave)





Wave packet represents particle prob

localized

Making Wave Packets: Simple Model with 2 waves

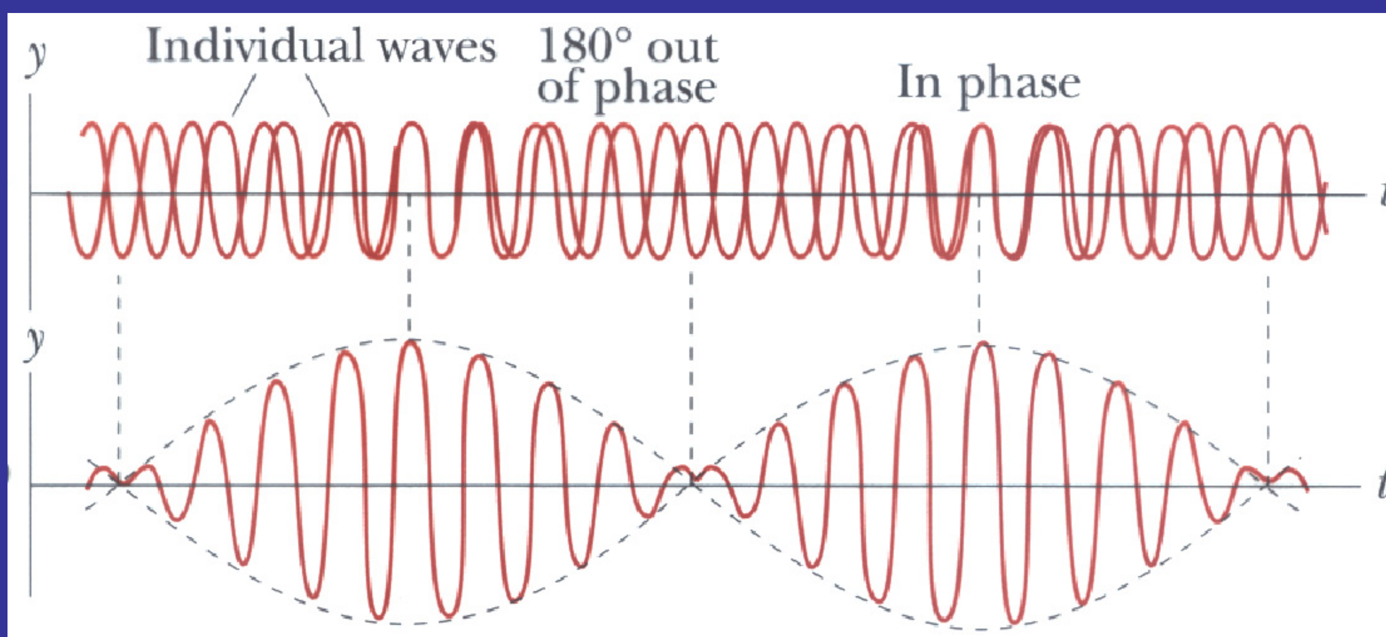
Ex: Phenomenon of "Beating" in Sound:

Add two waves of slightly different λ, f

$$\Rightarrow \text{Wave with : } f_{ave} = \left(\frac{f_1 + f_2}{2}\right), \text{ Amplitude A} \propto \left(\frac{f_1 - f_2}{2}\right)$$

Start with two waves

$$y_1 = ACos(k_1 x - w_1 t), \;\; y_2 = ACos(k_2 x - w_2 t) : k = \frac{2\pi}{\lambda}, w = 2\pi f$$



Resulting wave's "displacement" $y = y_1 + y_2$:

$$y = A\left[\cos(k_1 x - w_1 t) + \cos(k_2 x - w_2 t)\right]$$

Trignometry : $\cos A + \cos B = 2\cos(\frac{A+B}{2})\cos(\frac{A-B}{2})$

$$\therefore y = 2A\left[\left(\cos(\frac{k_2 - k_1}{2}x - \frac{w_2 - w_1}{2}t)\right)\left(\cos(\frac{k_2 + k_1}{2}x - \frac{w_2 + w_1}{2}t)\right)\right]$$

since $k_2 \cong k_1 \cong k_{ave}$, $w_2 \cong w_1 \cong w_{ave}$, $\Delta k \ll k$, $\Delta w \ll w$

$$\therefore \boxed{y = 2A\left[\left(\cos(\frac{\Delta k}{2}x - \frac{\Delta w}{2}t)\right)\cos(kx - wt)\right]} \equiv y = A' \cos(ks - wt),$$

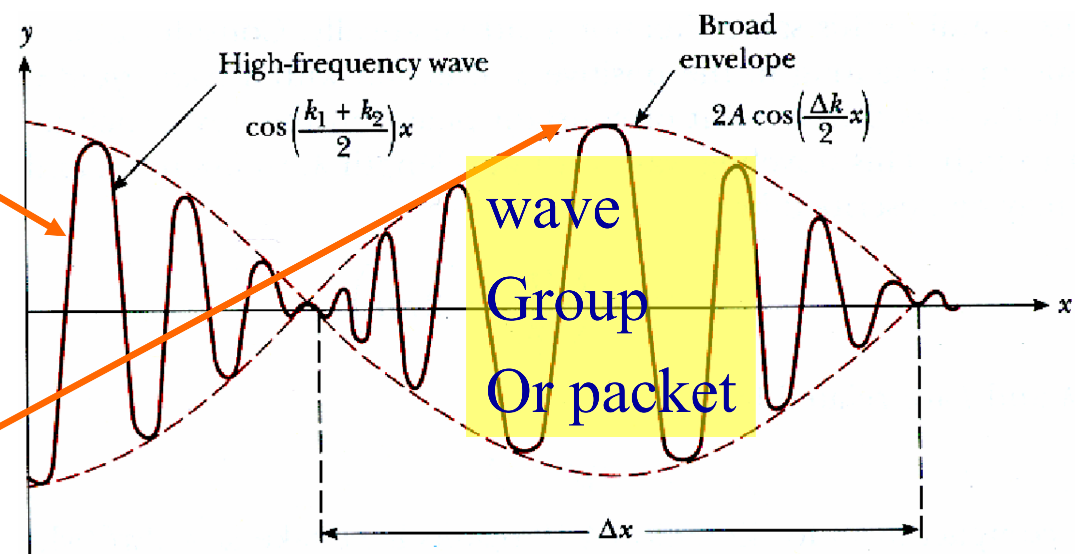
A' oscillates in x,t

$$A' = 2A\left(\cos(\frac{\Delta k}{2}x - \frac{\Delta w}{2}t)\right) = \text{modulated amplitude}$$

Phase Vel $V_p = \frac{w_{ave}}{k_{ave}}$

Group Vel $V_g = \frac{\Delta w}{\Delta k}$

V_g : Vel of envelope $= \frac{dw}{dk}$



Wave Packet : Localization

- Finite # of diff. Monochromatic waves always produce INFINTE sequence of repeating wave groups → can't describe (localized) particle
- To make localized wave packet, add " infinite" # of waves with Well chosen Ampl A, Wave# k, ang.

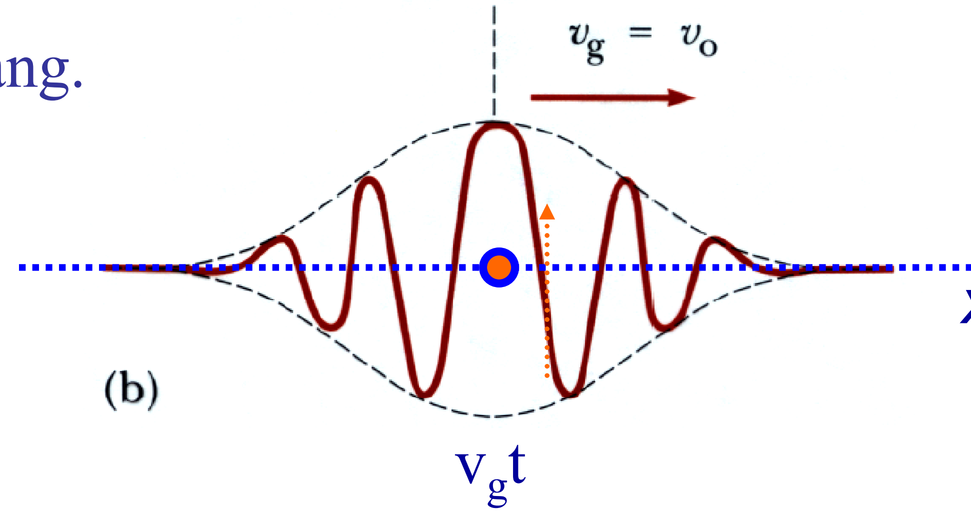
$$\psi(x,t) = \int_{-\infty}^{\infty} A(k)\, e^{i(kx-wt)} dk$$

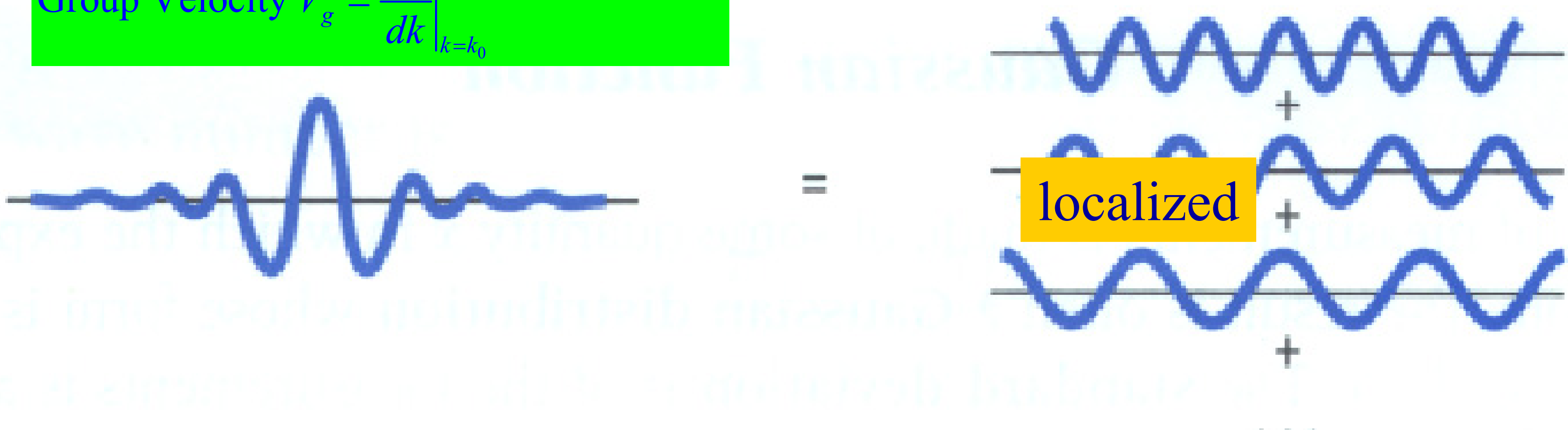
$A(k)$ = Amplitude Fn

⇒ diff waves of diff k have different amplitudes A(k)

w = w(k), depends on type of wave, media

Group Velocity $V_g = \left.\frac{dw}{dk}\right|_{k=k_0}$





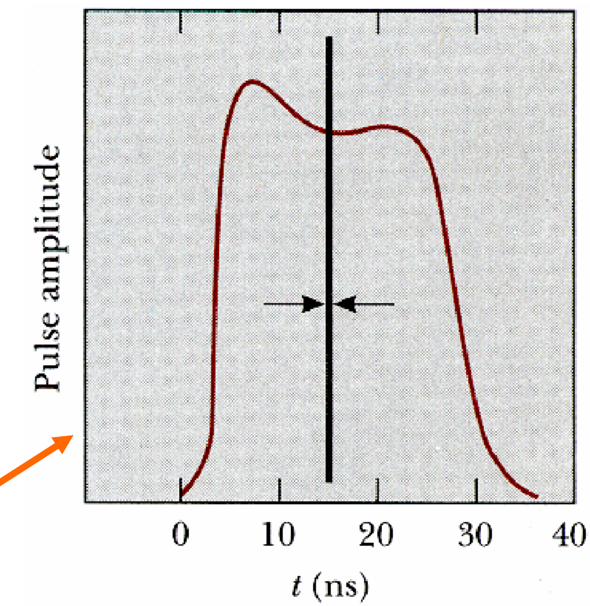
Group, Velocity, Phase Velocity and Dispersion

In a Wave Packet: $w = w(k)$

Group Velocity $V_g = \left.\frac{dw}{dk}\right|_{k=k_0}$

Since $V_p = \frac{w}{k}$ (def) $\Rightarrow w = kV_p$

$$\therefore V_g = \left.\frac{dw}{dk}\right| = V_p\Big|_{k=k_0} + k\left.\frac{dV_p}{dk}\right|_{k=k_0}$$



1ns laser pulse disperse

By x30 after travelling

1km in optical fiber

usually $V_p = V_p(k \text{ or } \lambda)$

Material in which V_p varies with λ are said to be Dispersive

Individual harmonic waves making a wave pulse travel at different V_p thus changing shape of pulse and become spread out

In non-dispersive media, $V_g = V_p$

In dispersive media $V_g \neq V_p$, depends on $\frac{dV_p}{dk}$

Matter Wave Packets

Consider An Electron:

mass = m velocity = v, momentum = p

Energy $E = hf = \gamma mc^2$; $\omega = 2\pi f = \frac{2\pi}{h}\gamma mc^2$

Wavelength $\lambda = \frac{h}{p}$; $k = \frac{2\pi}{\lambda} \Rightarrow k = \frac{2\pi}{h}\gamma mv$

Group Velocity : $V_g = \frac{dw}{dk} = \frac{dw/dv}{dk/dv}$

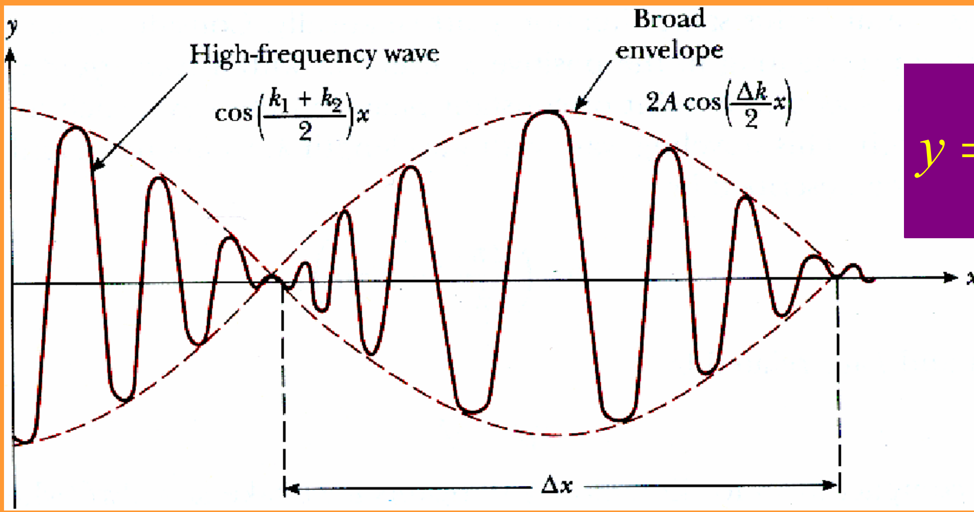
$$\frac{dw}{dv} = \frac{d}{dv}\left[\frac{\frac{2\pi}{h}mc^2}{[1-(\frac{v}{c})^2]^{1/2}}\right] = \frac{2\pi mv}{h[1-(\frac{v}{c})^2]^{3/2}} \quad \& \quad \frac{dk}{dv} = \frac{d}{dv}\left[\frac{2\pi}{h[1-(\frac{v}{c})^2]^{1/2}}mv\right] = \frac{2\pi m}{h[1-(\frac{v}{c})^2]^{3/2}}$$

$V_g = \frac{dw}{dk} = \frac{dw/dv}{dk/dv} = v \Rightarrow$ Group velocity of electron Wave packet "pilot wave"

is same as electron's physical velocity

But velocity of individual waves making up the wave packet $V_p = \frac{w}{k} = \frac{c^2}{v} > c!$ (not physical)

Wave Packets & Uncertainty Principle



$$y = 2A\left[\left(\cos\left(\frac{\Delta k}{2}x - \frac{\Delta w}{2}t\right)\right)\cos(kx - wt)\right]$$

Amplitude Modulation

- Distance ΔX between adjacent minima = $(X_2)_{node}$ - $(X_1)_{node}$
- Define X_1=0 then phase diff from $X_1 \rightarrow X_2 = \pi$

Node at y = 0 = 2A cos $\left(\frac{\Delta w}{2}t - \frac{\Delta k}{2}x\right)$

$\Rightarrow$ $\boxed{\Delta k.\Delta x = 2\pi}$ $\Rightarrow$ Need to combine more k to make small Δx packet

also implies $\Rightarrow$ $\boxed{\Delta p.\Delta x = h}$

and

$\boxed{\Delta w.\Delta t = 2\pi}$ $\Rightarrow$ Need to combine more ω to make small Δt packet

also $\Rightarrow$ $\boxed{\Delta E.\Delta t = h}$

What does This mean?