Physics 2D Lecture Slides
Nov 4

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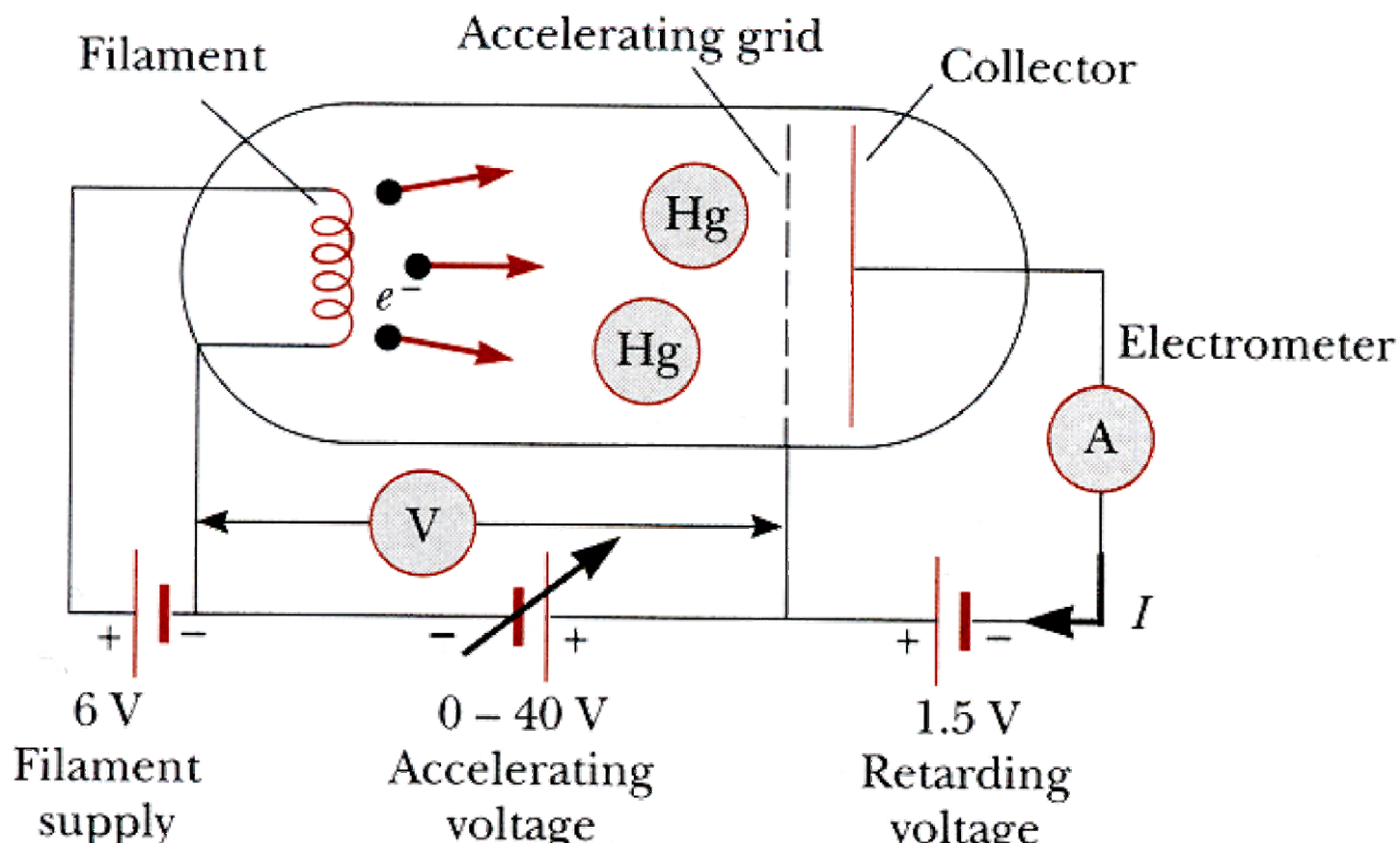
Bohr’s Explanation of Hydrogen like atoms

- Bohr’s Semiclassical theory explained some spectroscopic data → Nobel Prize : 1922
- The “hotch-potch” of clasical & quantum attributes left many (Einstein) unconvinced
  - “appeared to me to be a miracle – and appears to me to be a miracle today ...... One ought to be ashamed of the successes of the theory”
- Problems with Bohr’s theory:
  - Failed to predict INTENSITY of spectral lines
  - Limited success in predicting spectra of Multi-electron atoms (He)
  - Failed to provide “time evolution ” of system from some initial state
  - Overemphasized Particle nature of matter-could not explain the wave-particle duality of light
  - No general scheme applicable to non-periodic motion in subatomic systems
- “Condemned” as a one trick pony ! Without fundamental insight …raised the question : Why was Bohr successful?

Atomic Excitation by Electrons: Franck-Hertz Expt

Other ways of Energy exchange are also quantized ! Example:

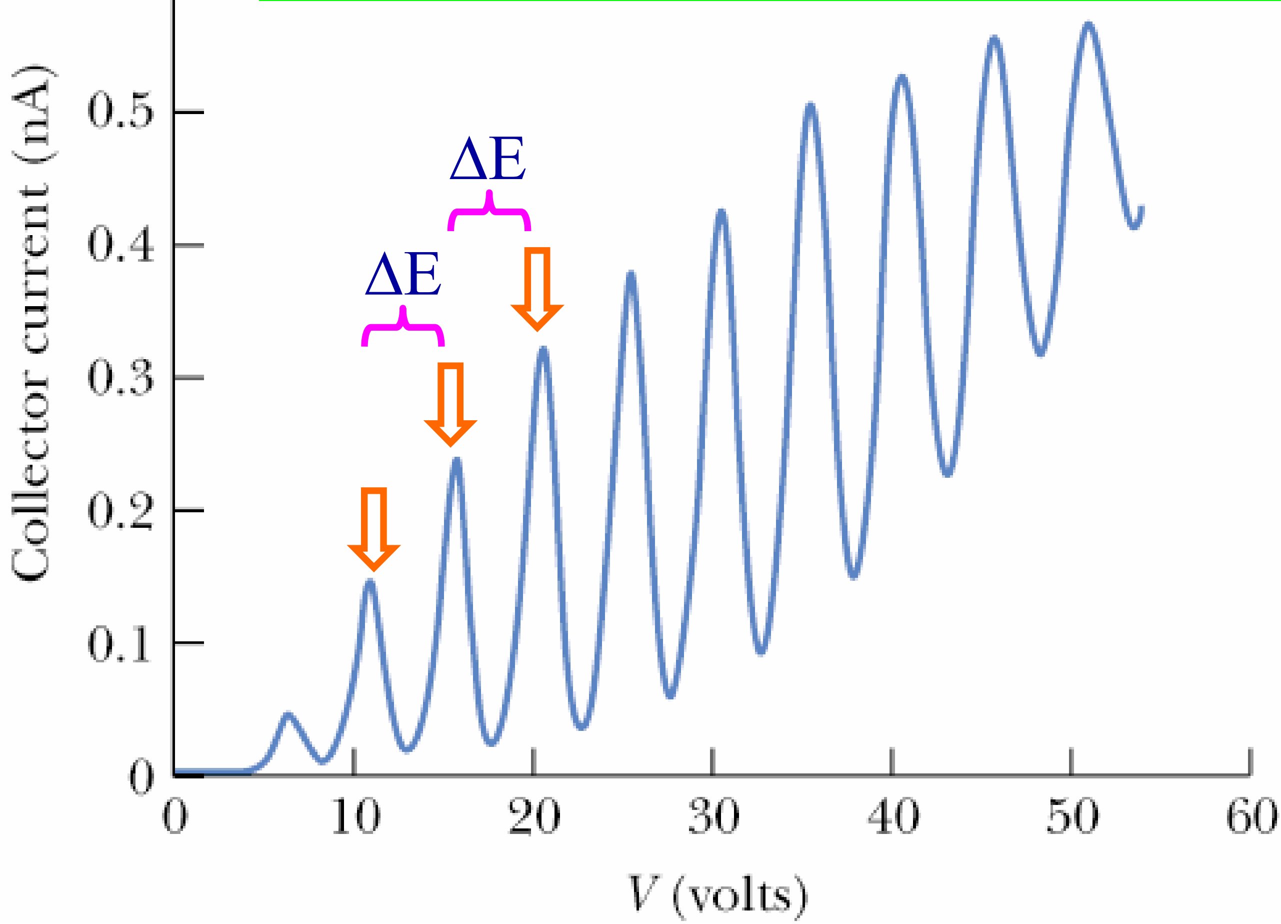
- Transfer energy to atom by colliding electrons on it
- Accelerate electrons, collide with Hg atoms, measure energy transfer in inelastic collision (retarding voltage)



Atomic Excitation by Electrons: Franck-Hertz Expt

Plot # of electrons/time (current) overcoming the retarding potential (V)

Equally spaced Maxima and minima in I-V curve



Atoms accept only discrete amount of Energy,
no matter the fashion in which energy is transffered

Prince Louise de Broglie

- Key to Bohr atom was Angular momentum quantization
- Why Quantization $mvr = |L| = nh/2\pi$?
- Invoking symmetry in nature the Prince deBroglie postulated
  - **Because photons have wave and particle like nature → particles must have wave like properties**
  - **Electrons have accompanying "pilot" wave (not EM) which guide particles thru spacetime.**
- Matter Wave :
  - **"Pilot wave" of Wavelength $\lambda = h / p = h / (\gamma m v)$**
  - **frequency $f = E / h$**
- **If matter has wave like properties then there would be interference (destructive & constructive)**
- **Use analogy of standing waves on a plucked string to explain the quantization condition of Bohr orbits**

Matter Waves : How big, how small

1. Wavelength of baseball, m=140g, v=27m/s

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63\times10^{-34}\,J.s}{(.14kg)(27m/s)} = 1.75\times10^{-34}\,m$$

$\Rightarrow$ $\boxed{\lambda_{baseball} <<< \text{ size of nucleus}}$

$\Rightarrow$ Baseball "looks" like a particle

2. Wavelength of electron K=120eV (assume NR)

$$K = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mK}$$

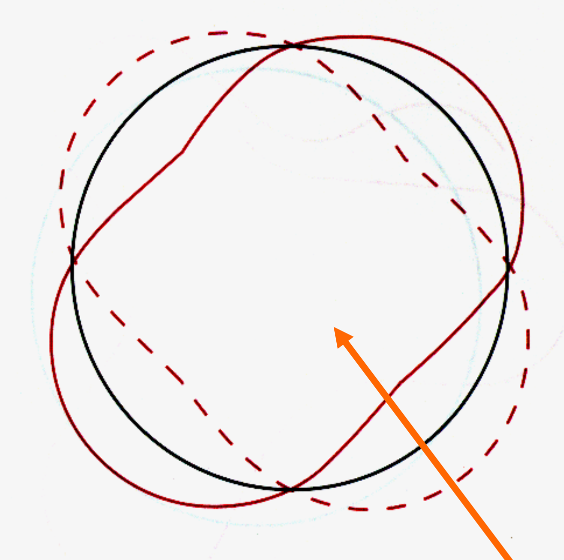
$$= \sqrt{2(9.11\times10^{-31})(120eV)(1.6\times10^{-19})}$$

$$= 5.91\times 10^{-24}\,Kg.m/s$$

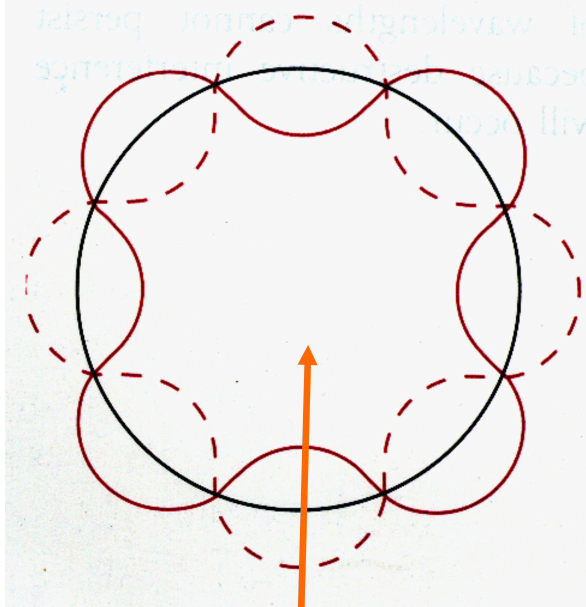
$$\lambda_e = \frac{h}{p} = \frac{6.63\times10^{-34}\,J.s}{5.91\times10^{-24}\,kg.m/s} = 1.12\times10^{-10}\,m$$

$\Rightarrow$ $\boxed{\lambda_e \simeq \text{ Size of atom}}$!!

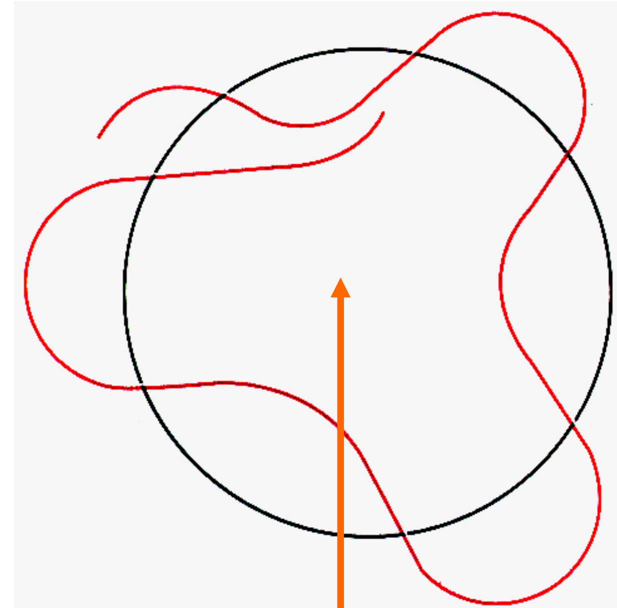
Models of Vibrations on a Loop: Model of e in atom

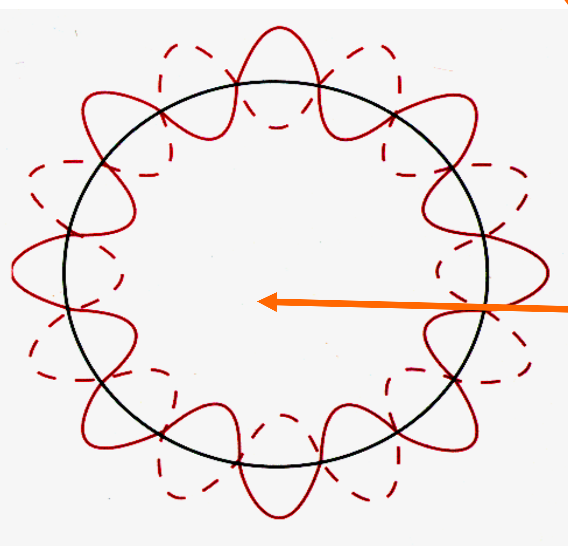


Circumference = 2 wavelengths



Circumference = 4 wavelengths





Circumference = 8 wavelengths

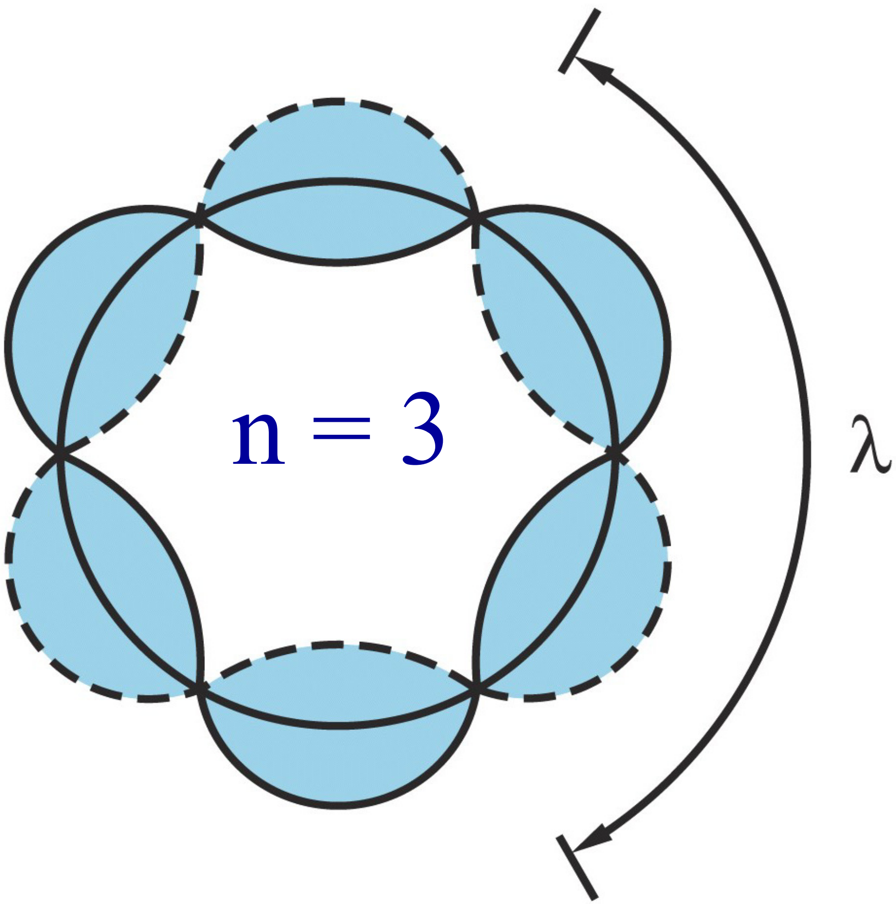
Modes of vibration when a integral # of λ fit into loop

(Standing waves)

vibrations continue Indefinitely

Fractional # of waves in a loop can not persist due to destructive interference

De Broglie's Explanation of Bohr's Quantization



Standing waves in H atom:

Constructive interference when

$$n\lambda = 2\pi r$$

since $\lambda = \dfrac{h}{p} = \dfrac{h}{mv}$ ......(NR)

$$\Rightarrow \frac{nh}{mv} = 2\pi r$$

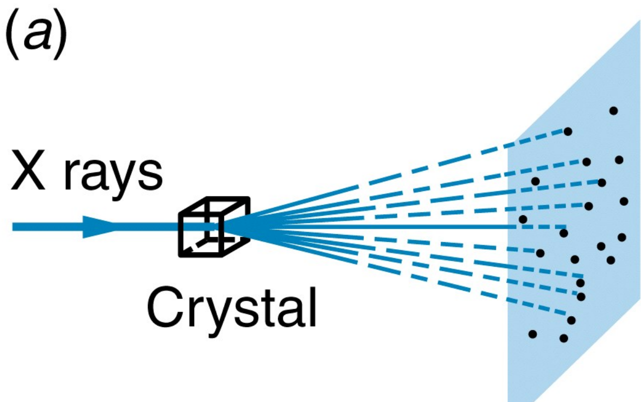
$$\Rightarrow \boxed{n\hbar = mvr}$$

Angular momentum

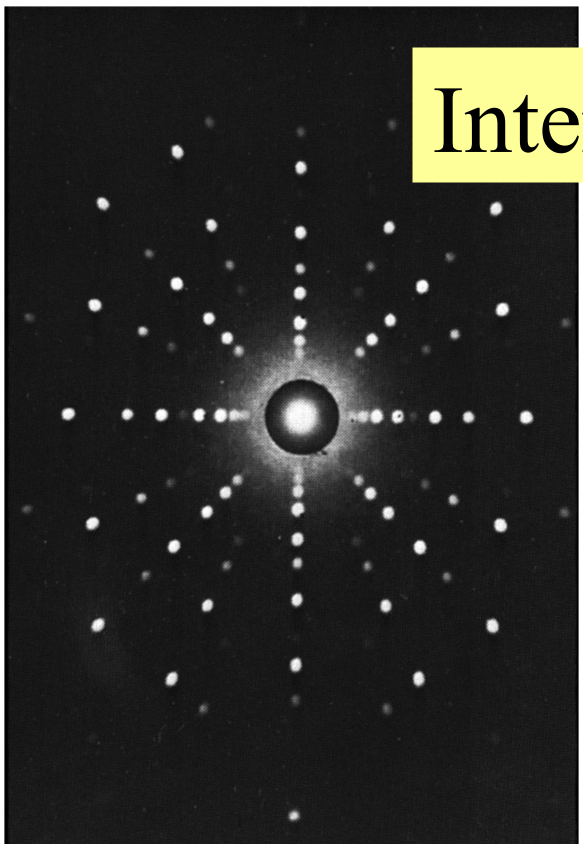
Quantization condition!

This is too intense ! Must verify such "loony tunes" with experiment

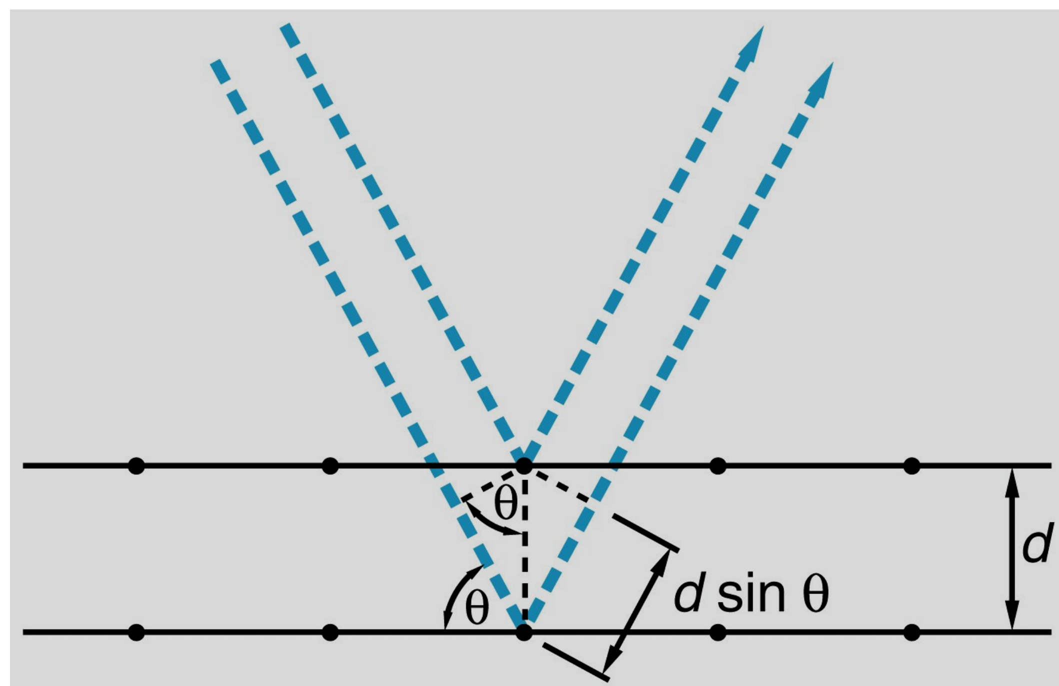
Reminder: Light as a Wave : Bragg Scattering Expt



Range of X-ray wavelengths scatter
Off a crystal sample
X-rays constructively interfere from
Certain planes producing bright spot



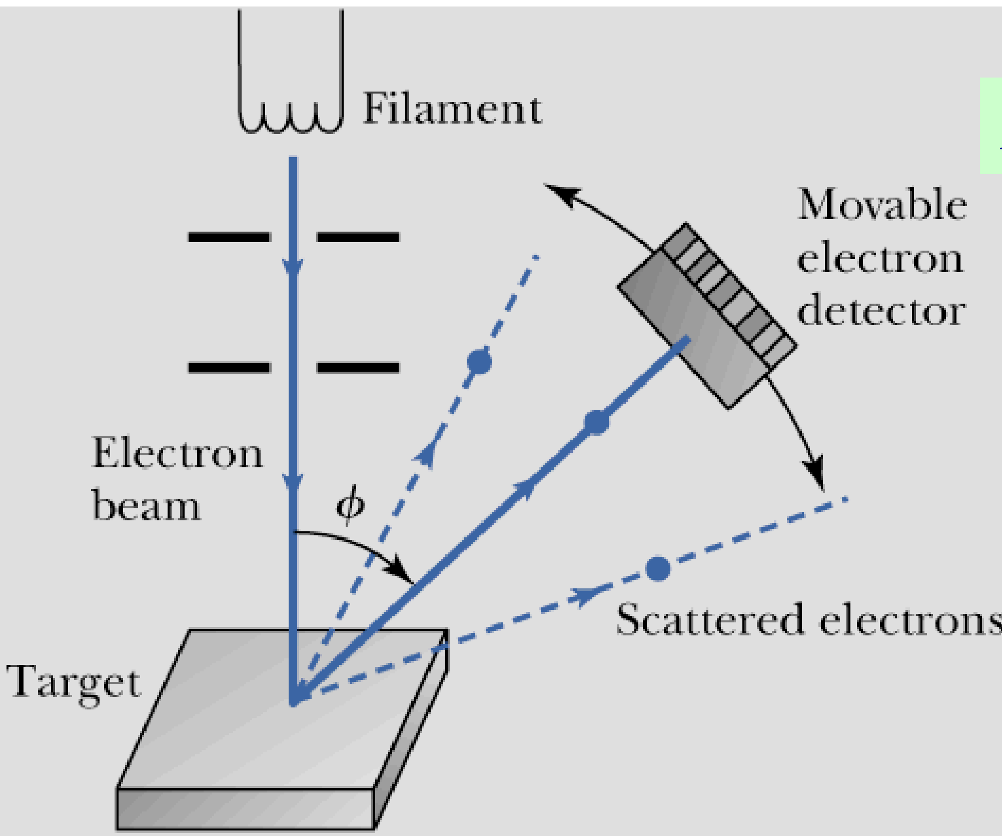
Interference → Path diff=2dsinϑ = nλ



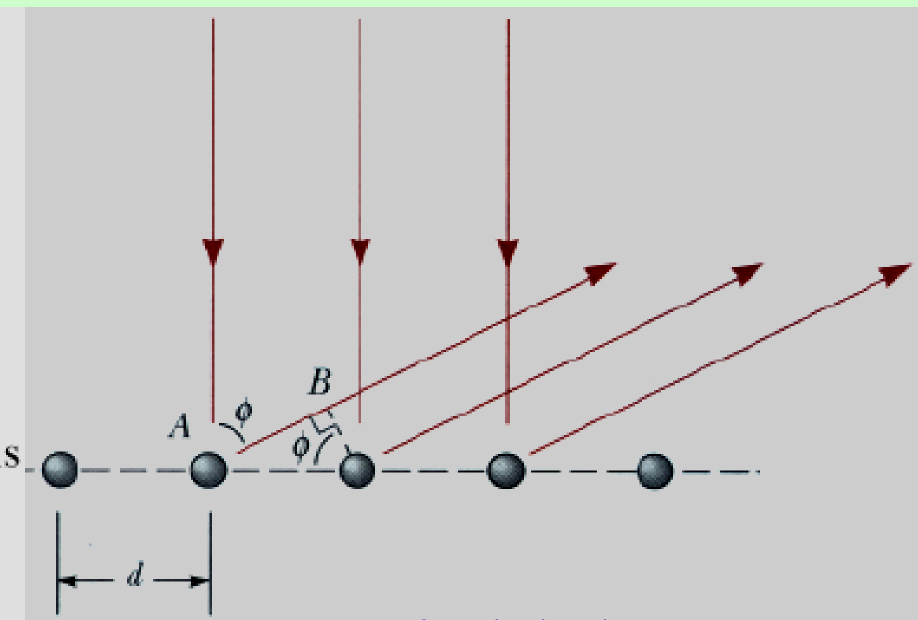
Verification of Matter Waves: Davisson & Germer Expt

If electrons have associated wave like properties → expect interference pattern when incident on a layer of atoms (reflection diffraction grating) with inter-atomic separation d such that

path diff AB= $d\sin\vartheta = n\lambda$

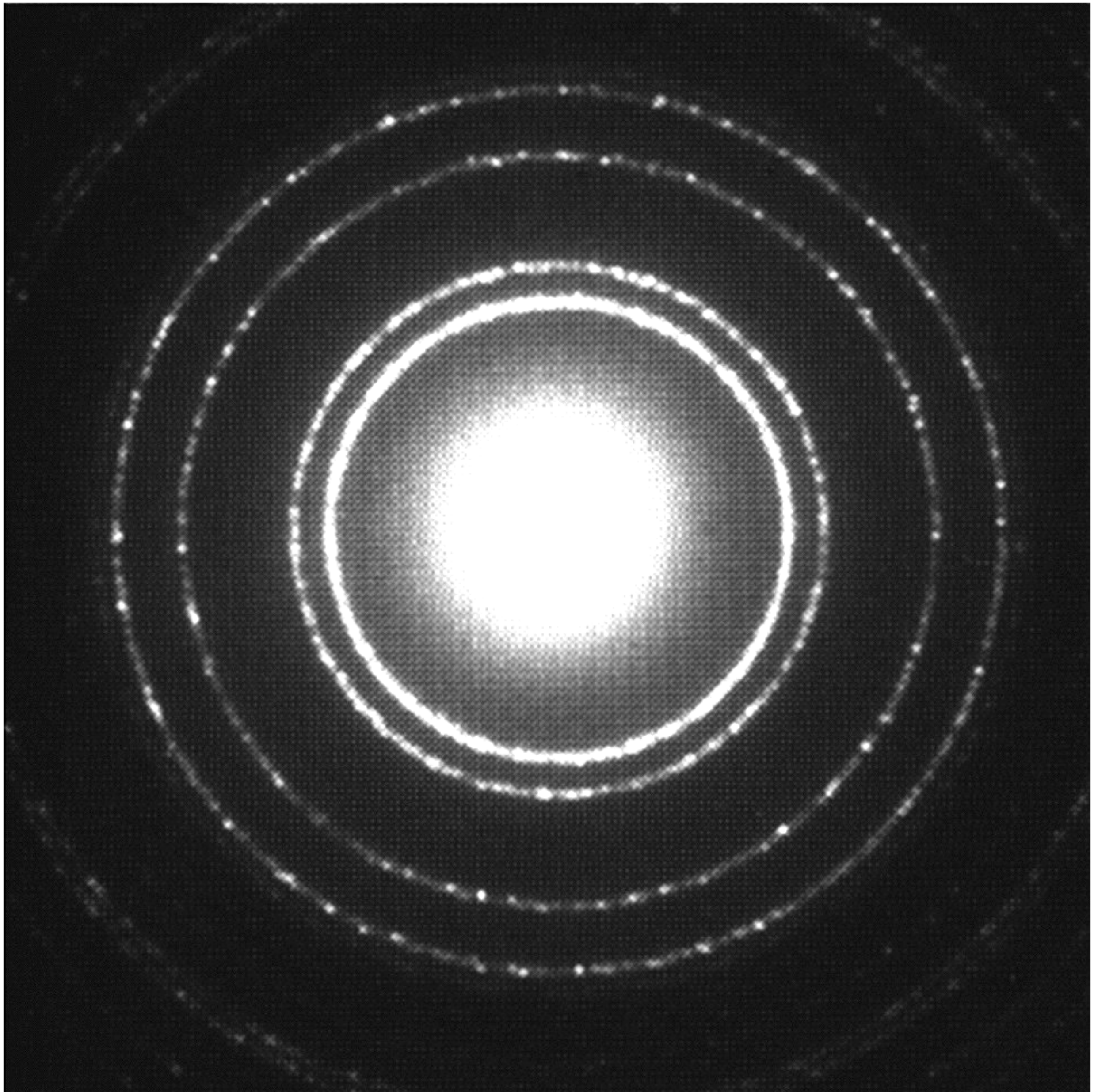


Atomic lattice as diffraction grating



Layer of Nickel atoms

Electrons Diffract in Crystal, just like X-rays

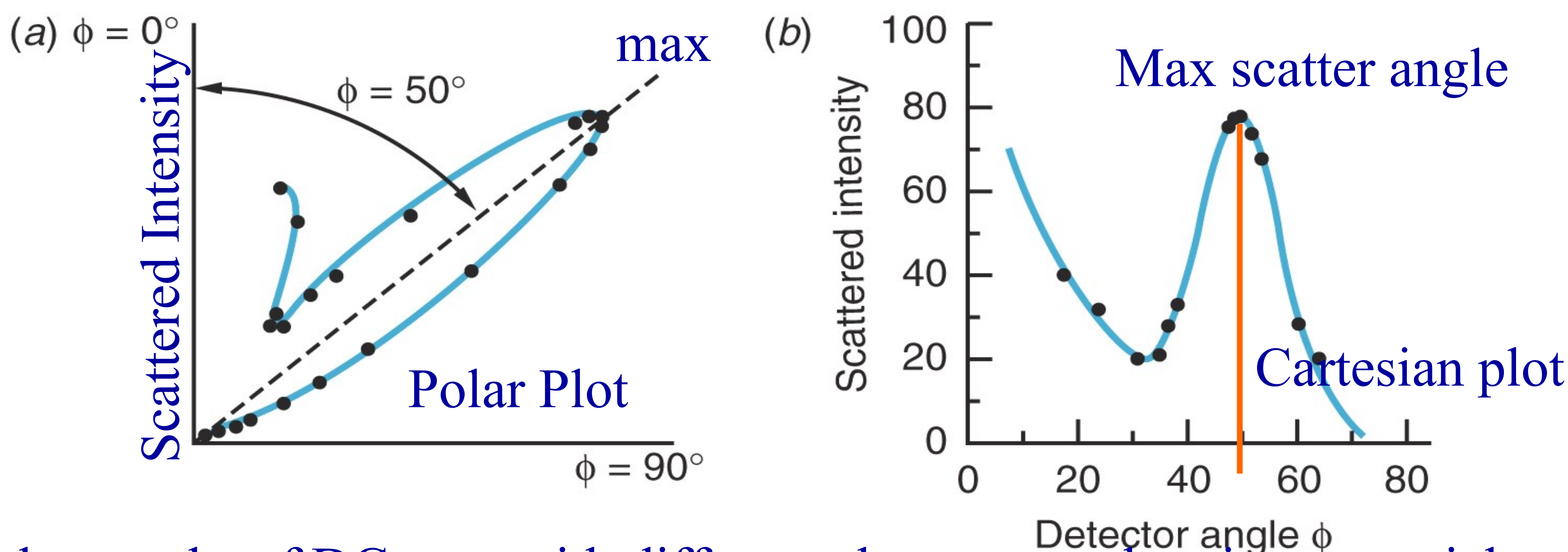


Diffraction pattern produced by 600eV electrons incident on a Al foil target

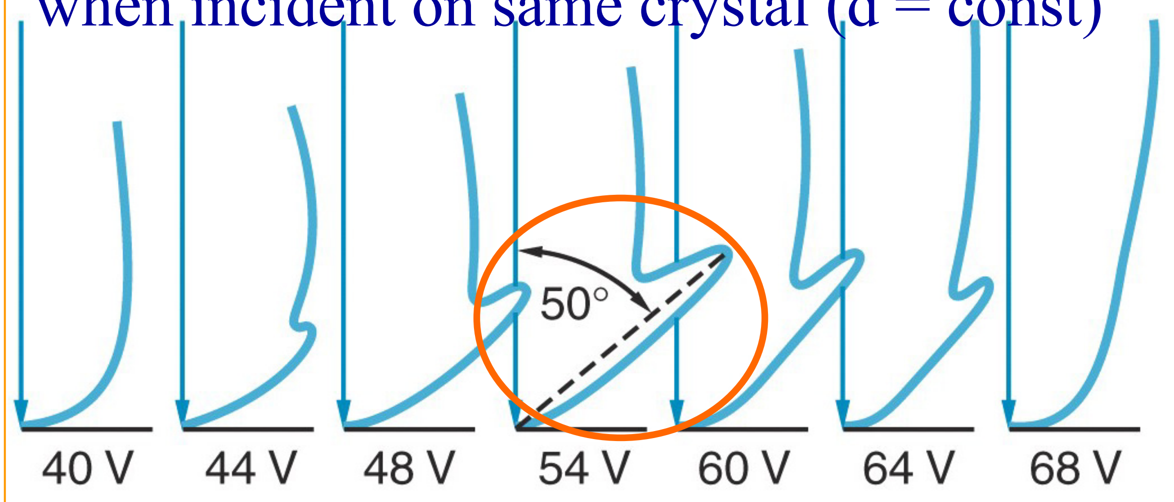
Notice the waxing and waning of scattered electron Intensity.

What to expect if electron had no wave like attribute

Davisson-Germer Experiment: 54 eV electron Beam



Polar graphs of DG expt with different electron accelerating potential when incident on same crystal (d = const)

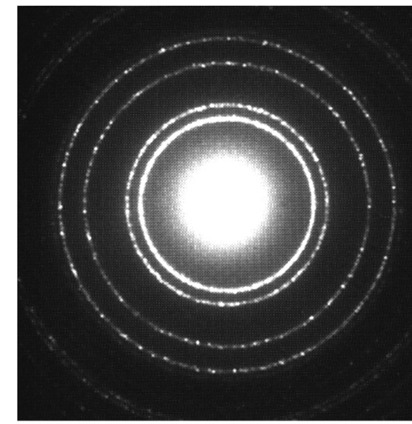


Peak at $\Phi = 50^o$ when $V_{acc} = 54$ V

Analyzing Davisson-Germer Expt with de Broglie idea

de Broglie λ for electron accelerated thru V_{acc} =54V

- $\frac{1}{2}mv^2 = K = \frac{p^2}{2m} = eV \Rightarrow v = \sqrt{\frac{2eV}{m}}$; $p = mv = m\sqrt{\frac{2eV}{m}}$



If you believe de Broglie

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{m\sqrt{\frac{2eV}{m}}} = \boxed{\frac{h}{\sqrt{2meV}} = \lambda^{predict}}$$

For $V_{acc} = 54$ Volts $\Rightarrow \lambda = 1.67\times10^{-10}m$ (de Broglie)

Exptal data from Davisson-Germer Observation:

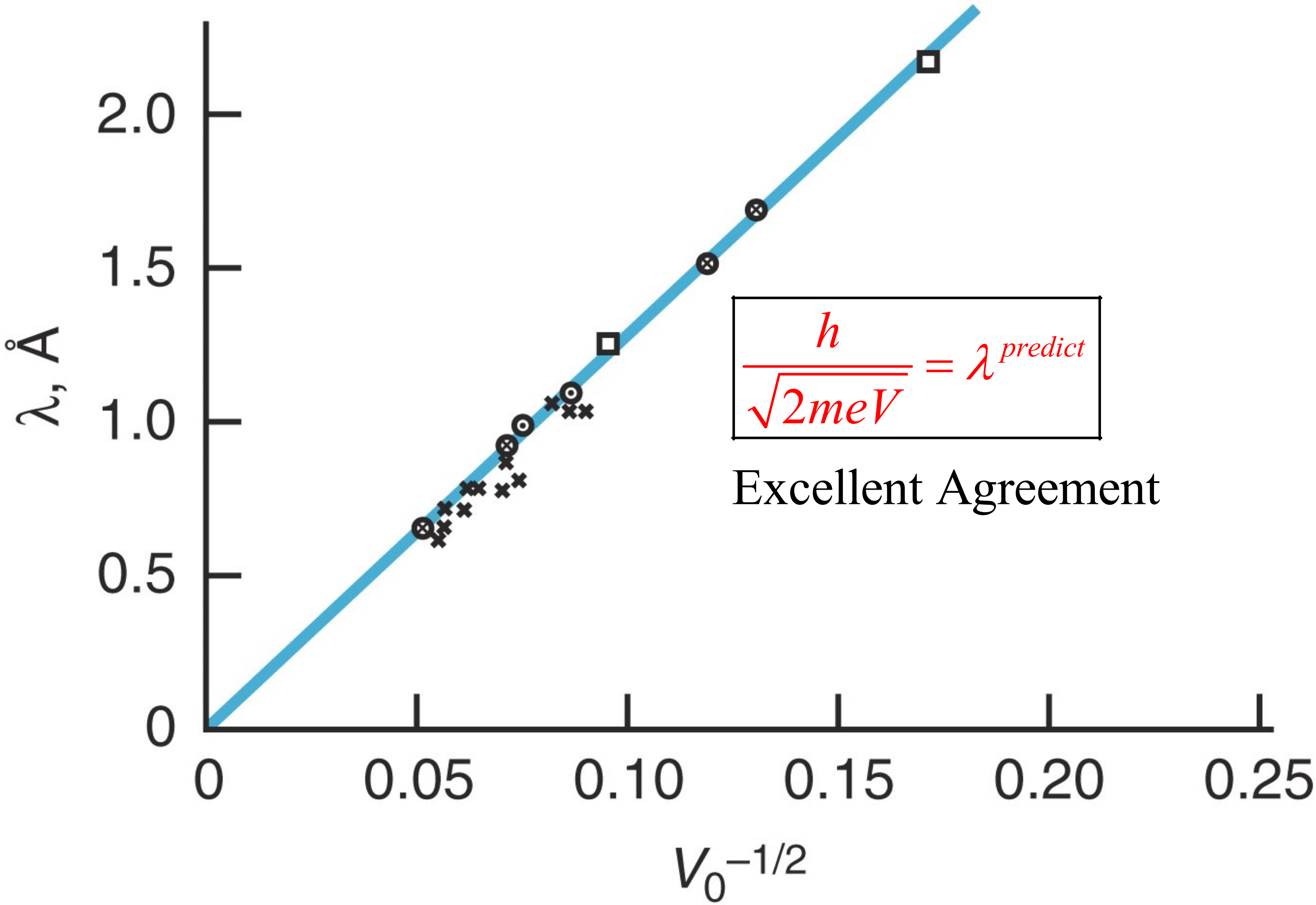
d_{nickel} = 2.15 Å = $2.15\times10^{-10}m$ (from Bragg Scattering)

$\theta_{diff}^{max} = 50^o$ (observation from scattering intensity plot)

$\boxed{\text{Diffraction Rule : d } \sin\phi = n\lambda}$

$\Rightarrow$ For Principal Maxima (n=1); λ^{meas} = (2.15 Å)(sin 50°) $\boxed{\begin{array}{l}\lambda^{predict} = 1.67\,\dot{A} \\ \lambda^{observ} = 1.65\,\dot{A}\end{array}}$ *Excellent* agreement

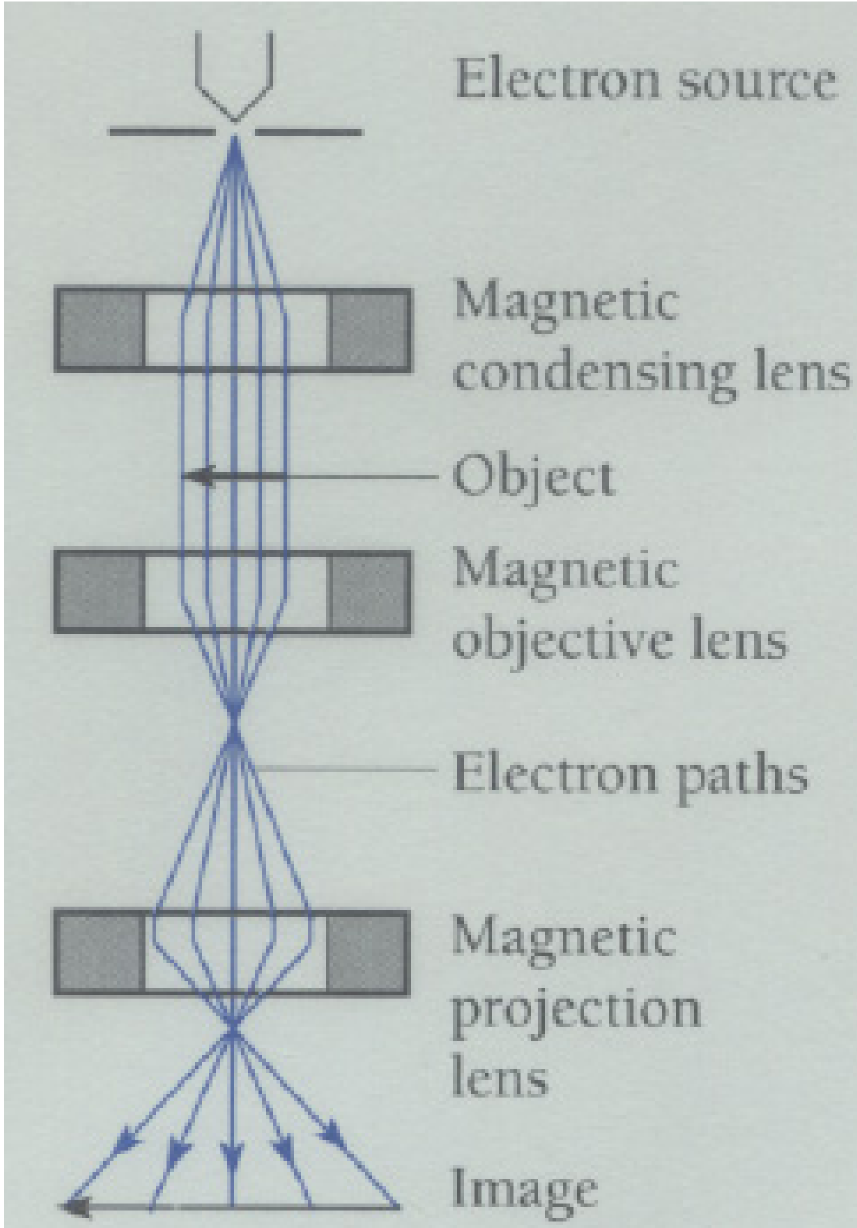
Davisson Germer Experiment: Matter Waves !

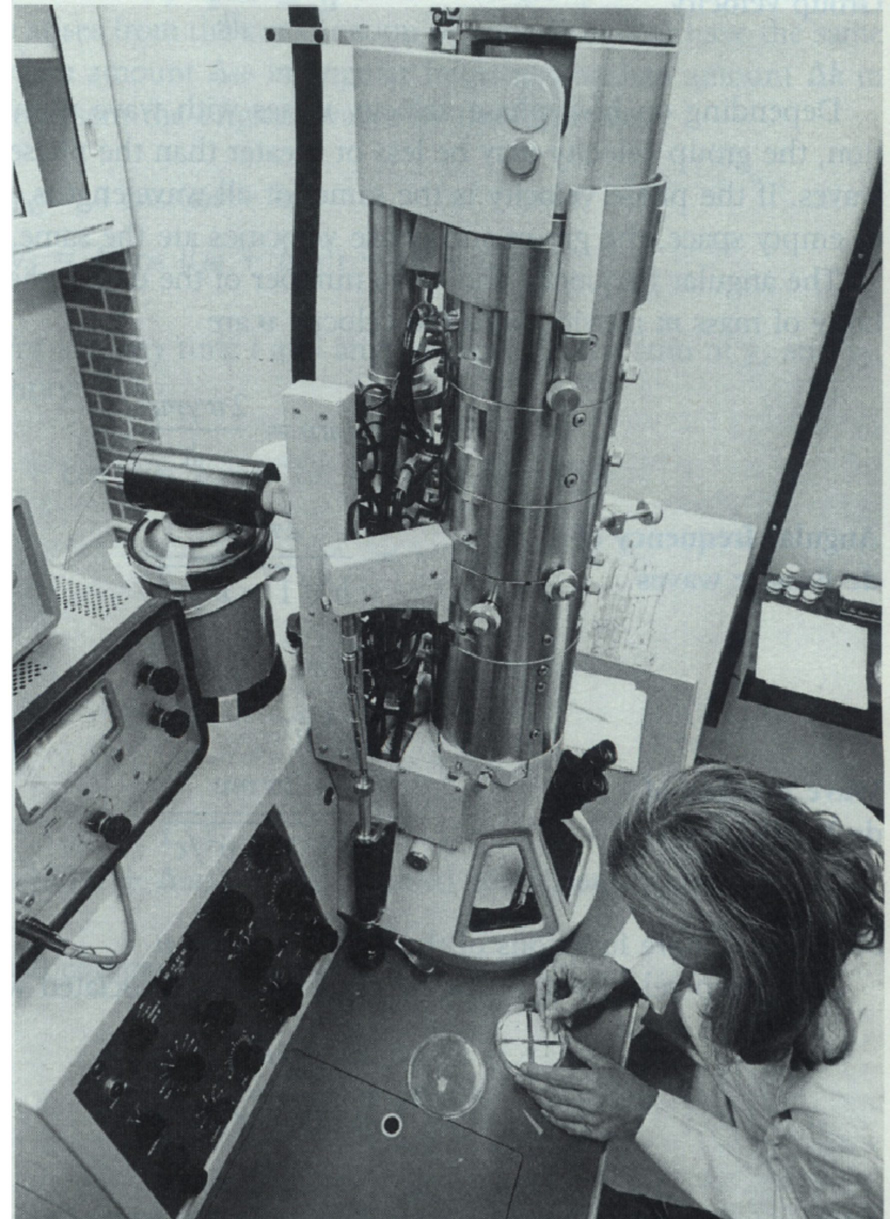


$$\boxed{\frac{h}{\sqrt{2meV}} = \lambda^{predict}}$$

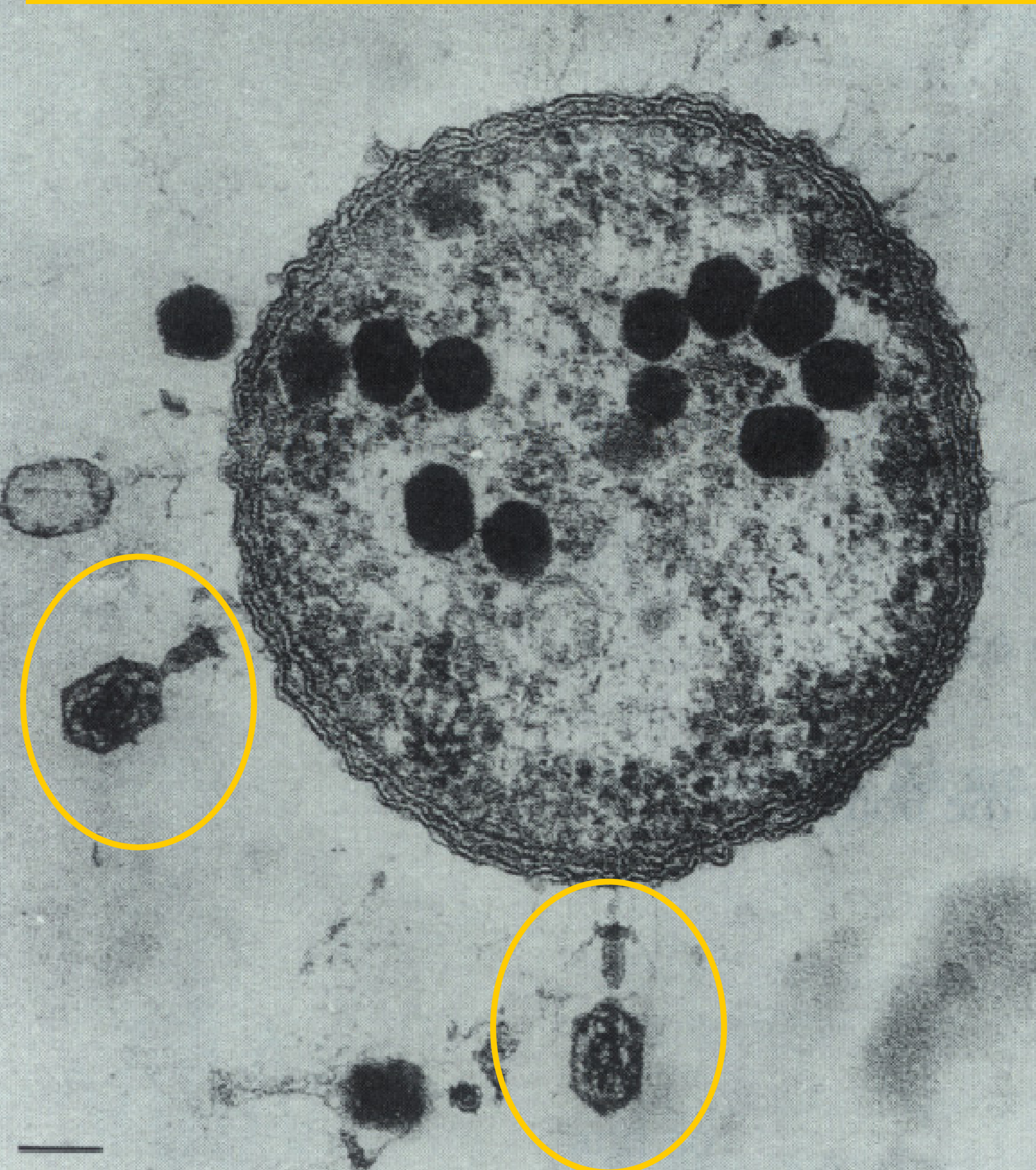
Excellent Agreement

Practical Application : Electron Microscope





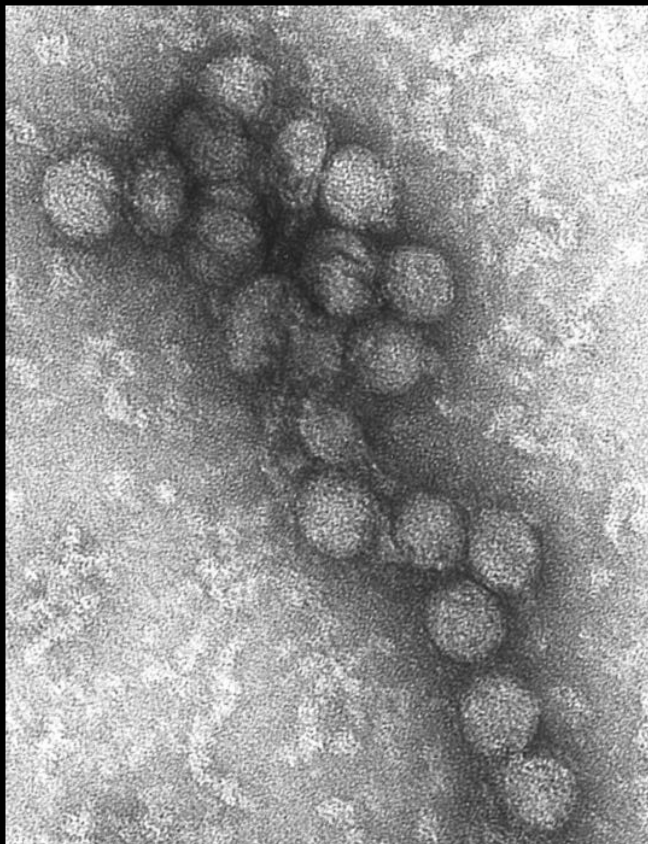
Electron Microscope : Excellent Resolving Power

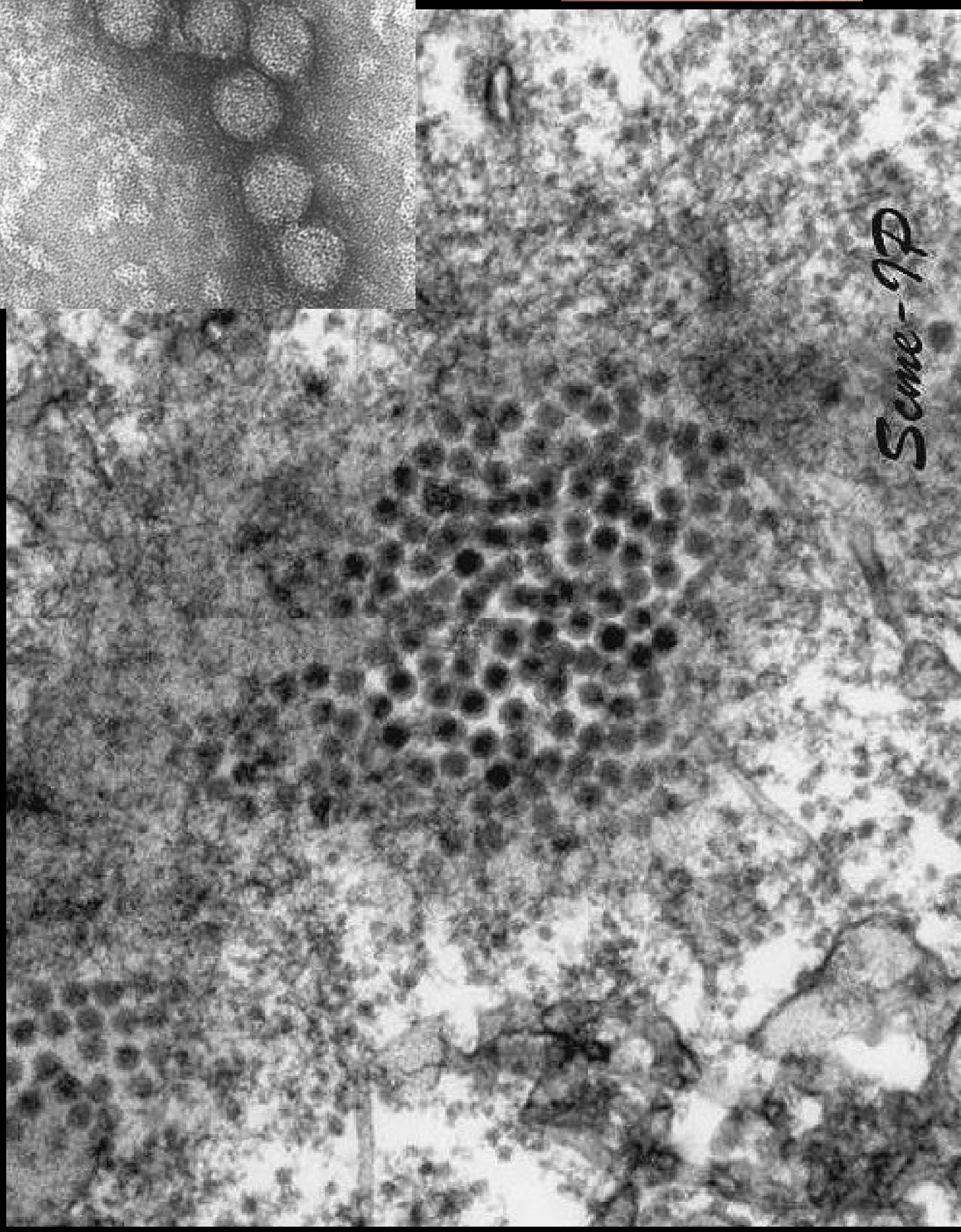


Electron Micrograph
Showing Bacteriophage
Viruses in E. Coli bacterium

The bacterium is $\cong 1\mu$ size

West Nile Virus extracted from a crow brain

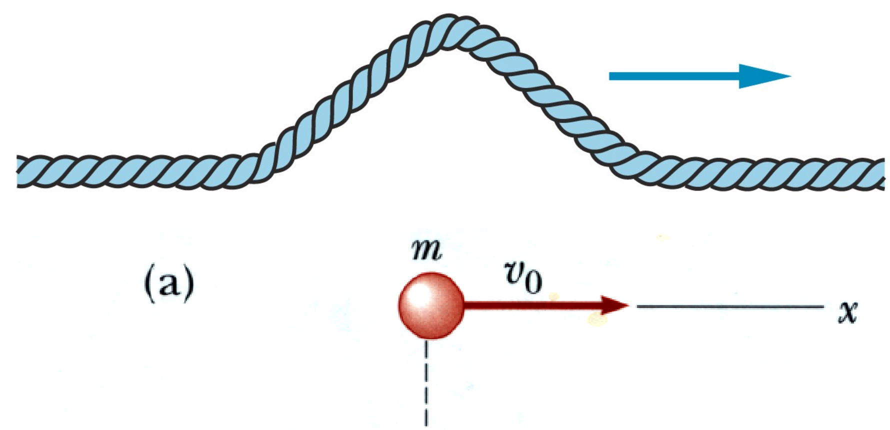




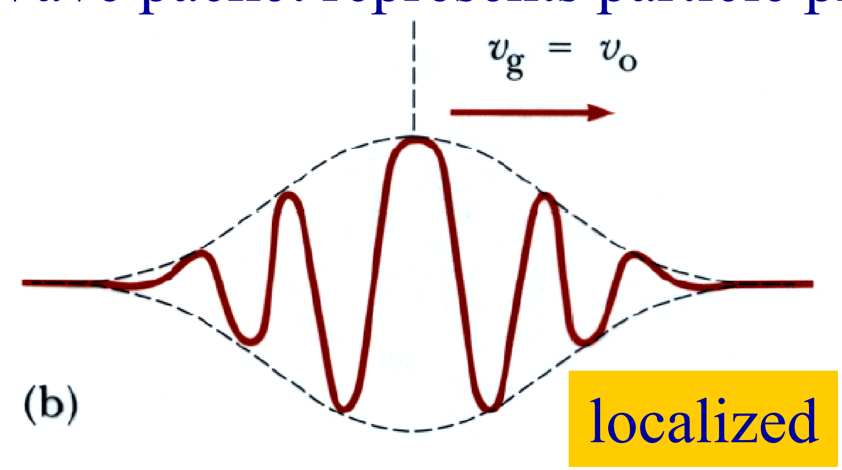
Just **WHAT** is Waving in Matter Waves ?

- For waves in an ocean, it's the water that "waves"
- For sound waves, it's the molecules in medium
- For light it's the **E** & **B** vectors
- What's waving for matter waves ?
  - **It's the PROBABLILITY OF FINDING THE PARTICLE that waves !**
  - **Particle can be represented by a wave packet in**
    - **Space**
    - **Time**
    - **Made by superposition of many sinusoidal waves of different λ**
    - **It's a "pulse" of probability**

Imagine Wave pulse moving along a string: its localized in time and space (unlike a pure harmonic wave)



Wave packet represents particle prob



Making Wave packets with Sinusoidal Waves: Model

Ex: Phenomenon of "Beating" in Sound:

Add two waves of slightly different λ, f

$$\Rightarrow \text{ Wave with : } f = \left(\frac{f_1 + f_2}{2}\right), \text{ Amplitude } A \propto \left(\frac{f_1 - f_2}{2}\right)$$

Start with two waves

$$y_1 = ACos(k_1 x - w_1 t),\ \ y_2 = ACos(k_2 x - w_2 t) : k = \frac{2\pi}{\lambda}, w = 2\pi f$$