Physics 2D Lecture Slides
Nov 12

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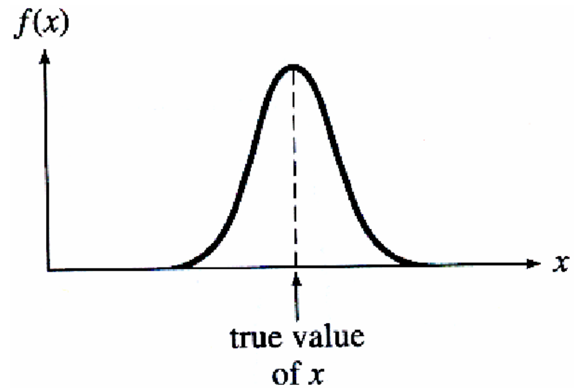
UCSD Physics

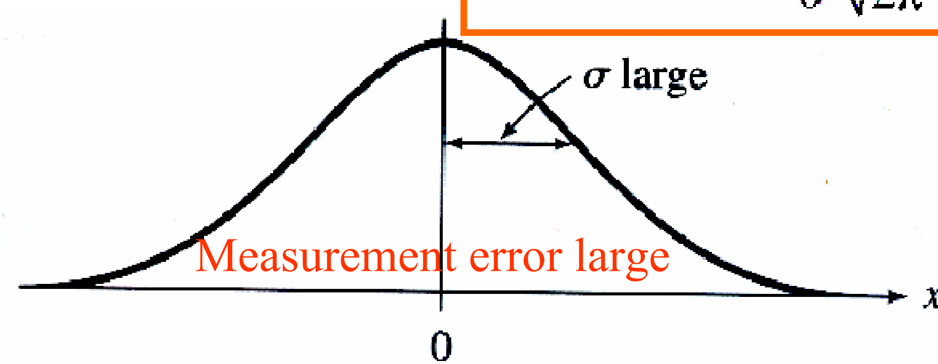
Measurement Error : $x \pm \Delta x$

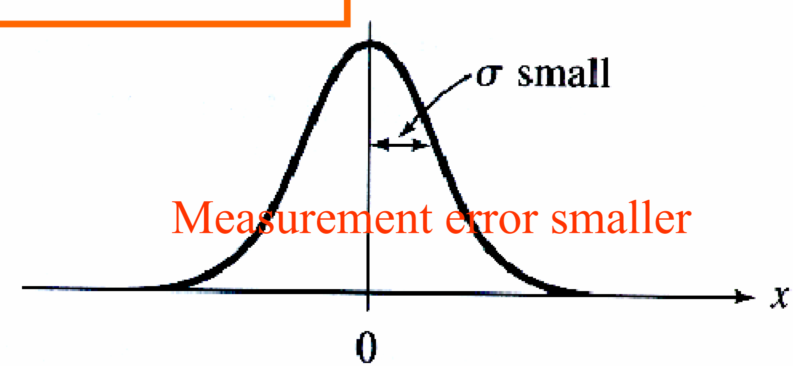
- Measurement errors are unavoidable since the measurement procedure is an experimental one
- True value of an measurable quantity is an abstract concept
- In a set of repeated <u>measurements with random errors</u>, the distribution of measurements resembles a Gaussian distribution characterized by the parameter σ or Δ characterizing the width of the distribution



The Gauss, or Normal, Distribution

$$G_{X,\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{(x - X)^2/2\sigma^2}.$$





Interpreting Measurements with random Error : Δ

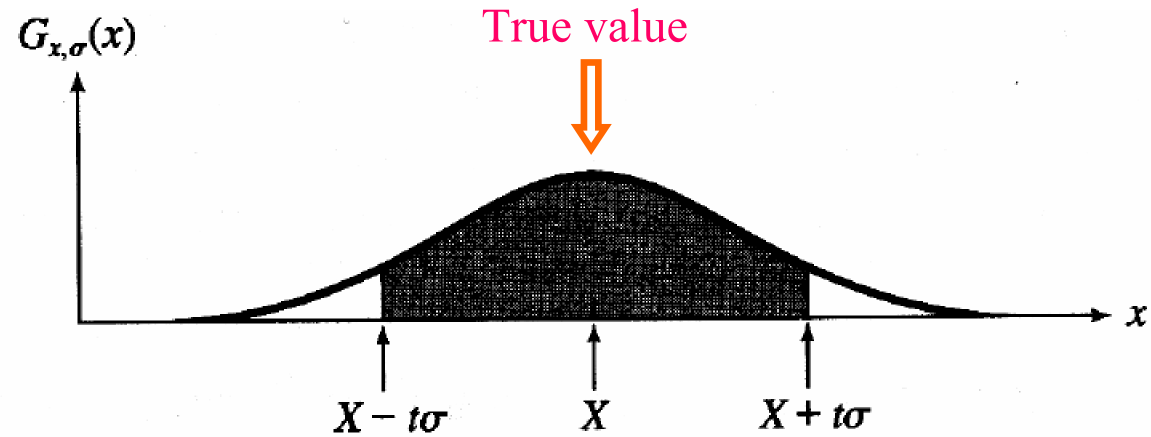
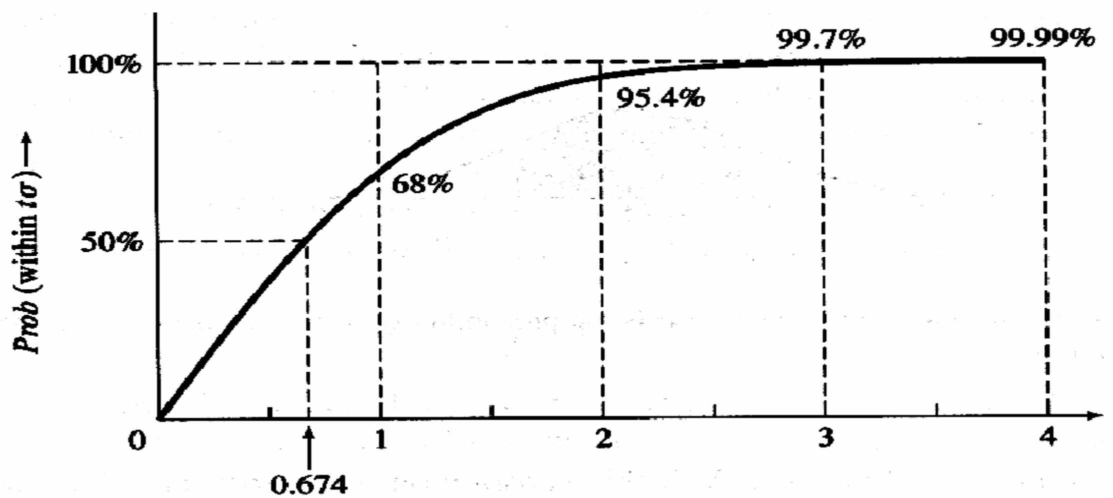


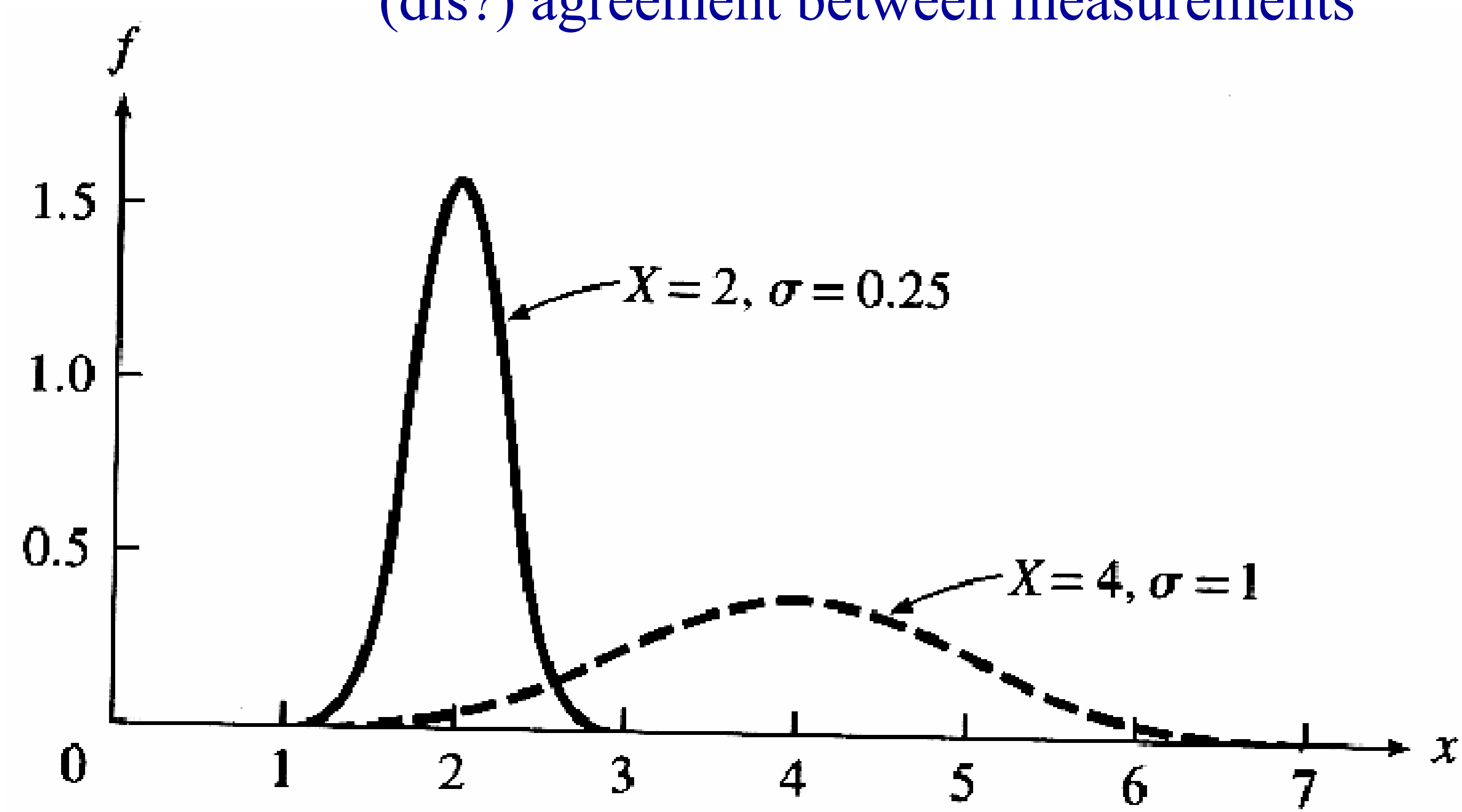
Figure 5.12. The shaded area between $X \pm t\sigma$ is the probability of a measurement within t standard deviations of X.



t	0	0.25	0.5	0.75	1.0	1.25	1.5	1.75	2.0	2.5	3.0	3.5	4.0
Prob (%)	0	20	38	55	68	79	87	92	95.4	98.8	99.7	99.95	99.99

Comparing Measurements With Errors

(dis?) agreement between measurements



Back to Sharma's weight : Mass measured with poor precision

1000 ± 700 kg is consistent with 70 ± 15kg

Measurements with Errors

- If your measuring apparatus has an intrinsic error of Δp
- Then results of measurement of momentum p of an object at rest can easily yield a range of values accommodated by the measurement imprecision :
  - $-\Delta p \le p \le \Delta p$

- Similarly for all measurable quantities !

Wave Packets & Uncertainty Principle

in space x: $\boxed{\Delta k . \Delta x = \pi}$ $\Rightarrow$ since $k = \frac{2\pi}{\lambda}$, $p = \frac{h}{\lambda}$

$\Rightarrow$ $\boxed{\Delta p . \Delta x = h/2}$

usually one writes $\boxed{\Delta p . \Delta x \geq \hbar/2}$ approximate relation

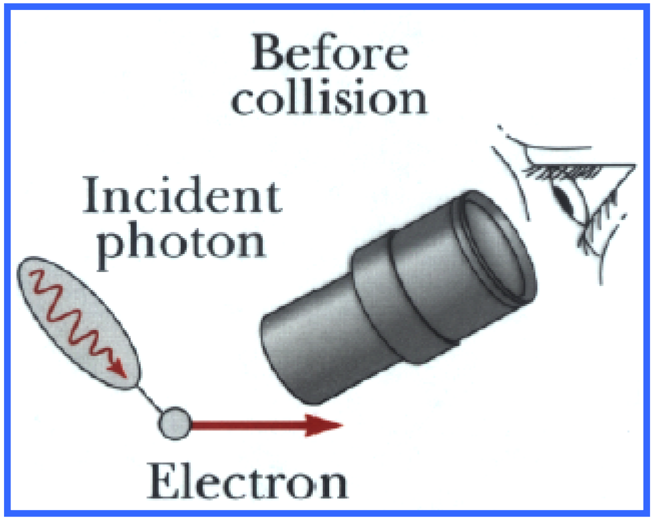
In time t : $\boxed{\Delta w . \Delta t = \pi}$ $\Rightarrow$ since $\omega = 2\pi f$, $E = hf$

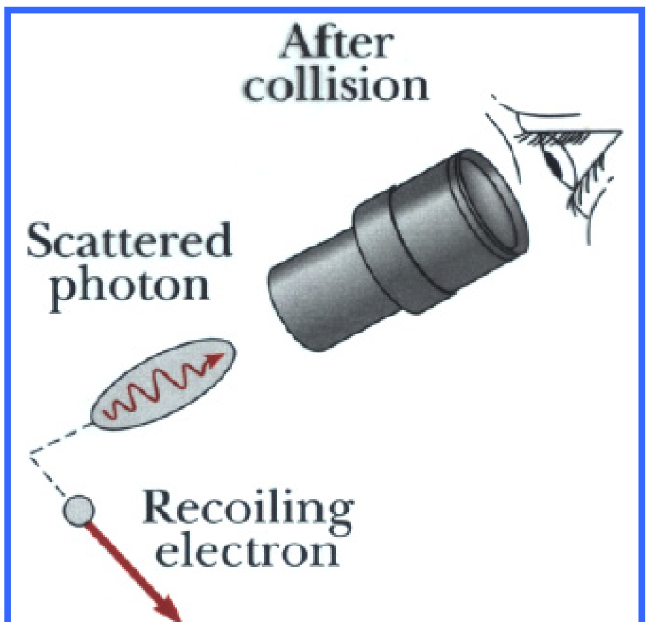
$\Rightarrow$ $\boxed{\Delta E . \Delta t = h/2}$

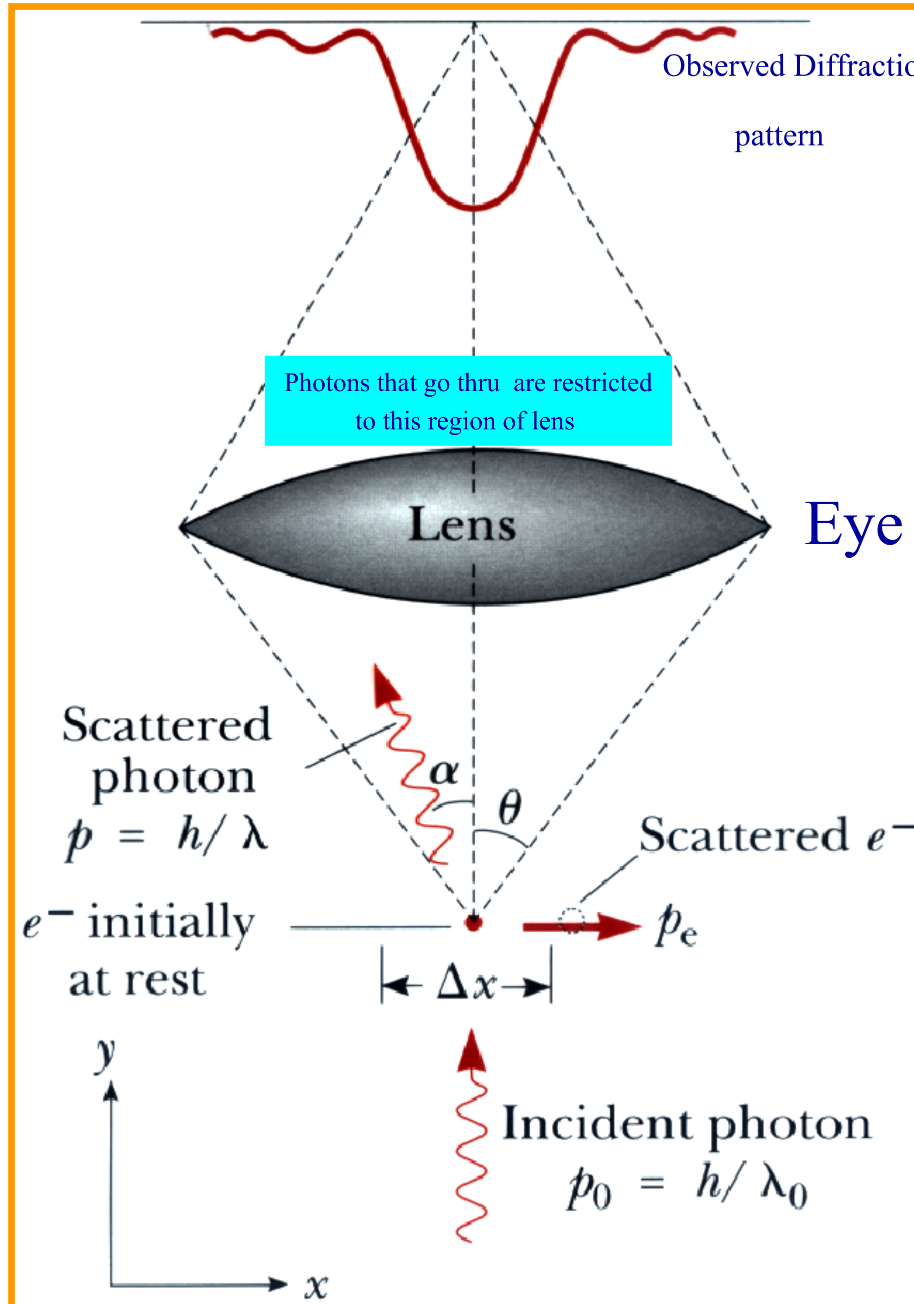
usually one writes $\boxed{\Delta E . \Delta t \geq \hbar/2}$ approximate relation

What do these inequalities mean physically?

Act of Watching: A Thought Experiment







Diffraction By a Circular Aperture (Lens)

See Resnick, Halliday Walker 6th Ed (on S.Reserve), Ch 37, pages 898-900

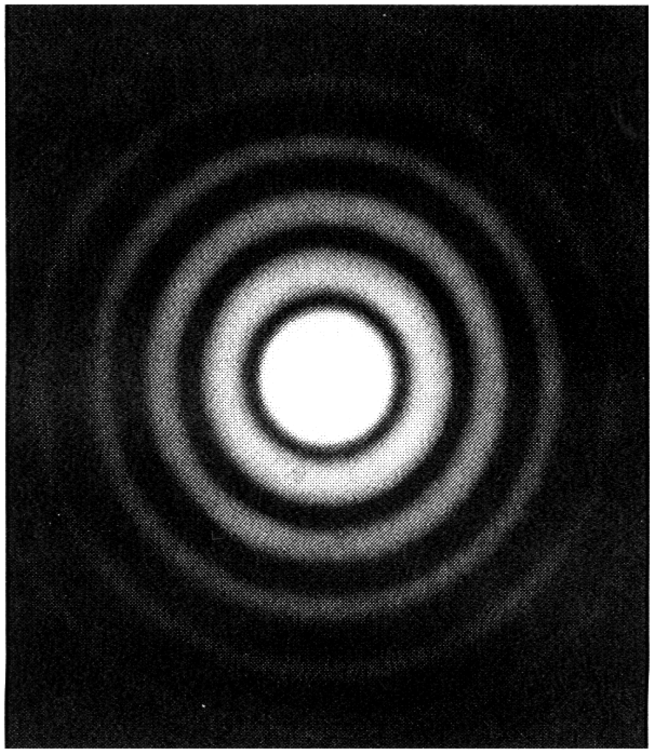


Fig. 37-9 The diffraction pattern of a circular aperture. Note the central maximum and the circular secondary maxima. The figure has been overexposed to bring out these secondary maxima, which are much less intense than the central maximum.

Diffracted image of a point source of light thru a lens (circular aperture of size d)

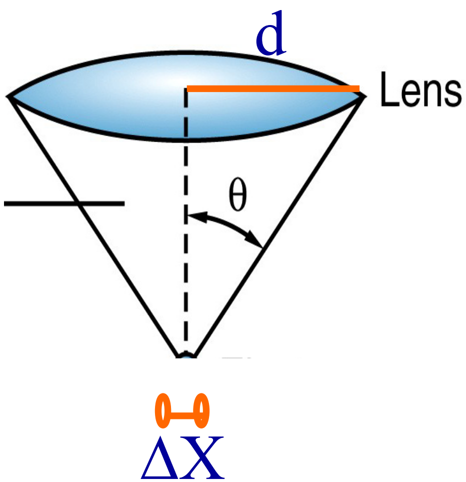
First minimum of diffraction pattern is located by

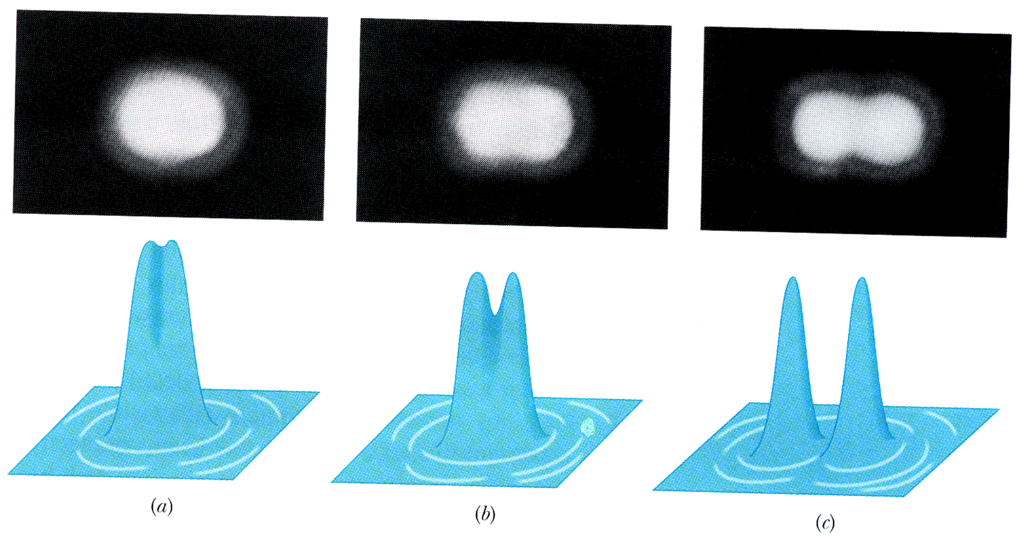
$$\sin\theta = 1.22\frac{\lambda}{d}$$

See previous picture for definitions of ϑ, λ, d

Resolving Power of Light Thru a Lens

Image of 2 separate point sources formed by a converging lens of diameter d, ability to resolve them depends on λ & d because of the Inherent diffraction in image formation



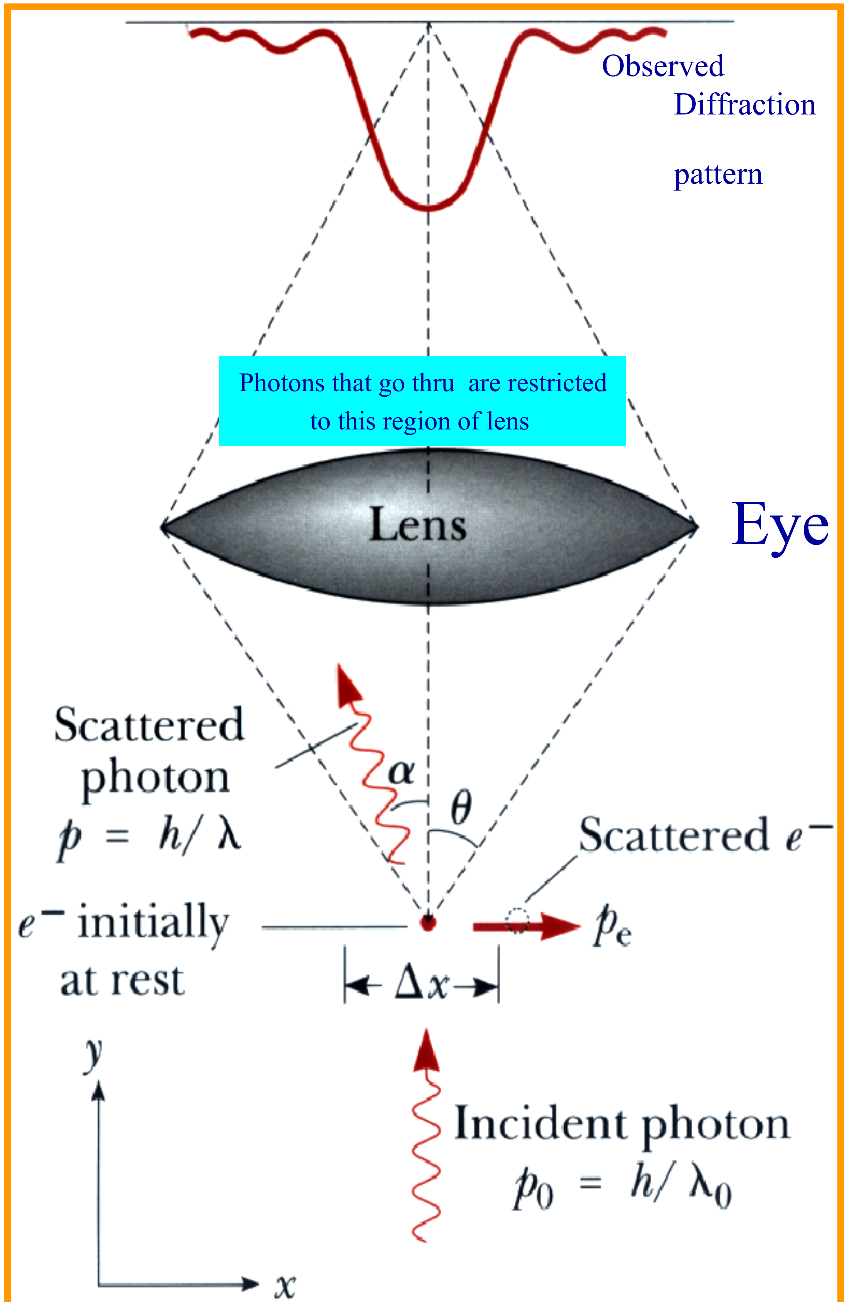


Not resolved   barely resolved   resolved

$$\text{Resolving power } \Delta x \simeq \frac{\lambda}{2\sin\theta}$$

θ Depends on d

Act of Observing an Electron



- Incident light (p,λ) scatters off electron
- To be collected by lens → γ must scatter thru angle α
  - $-\vartheta \leq \alpha \leq \vartheta$
- Due to Compton scatter, electron picks up momentum
  - P_X , P_Y

$$-\frac{h}{\lambda}\sin\theta \leq P_x \leq \frac{h}{\lambda}\sin\theta$$

electron momentum uncertainty is

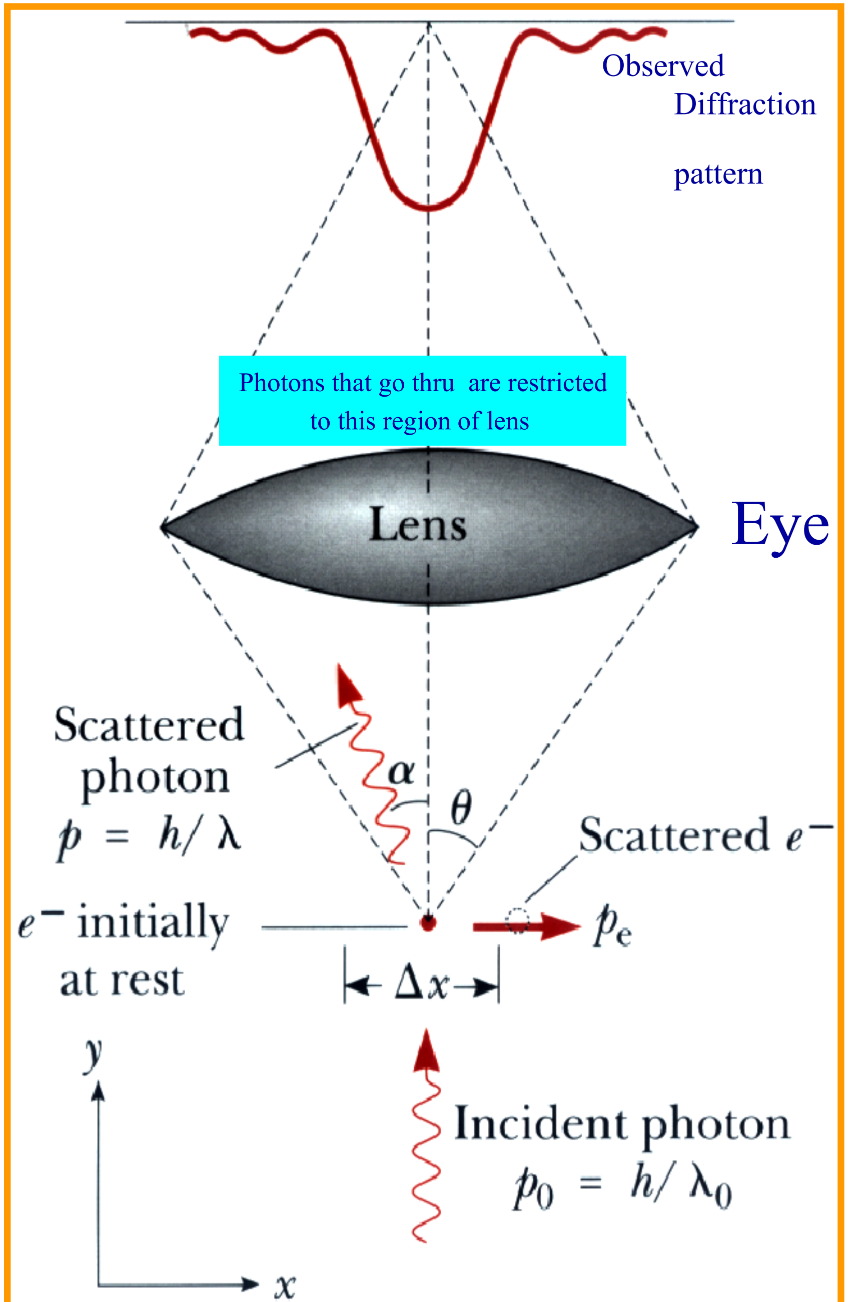
$$\Delta p \cong \frac{2h}{\lambda}\sin\theta$$

- After passing thru lens, photon "diffracts", lands somewhere on screen, image (**of electron**) is fuzzy
- How fuzzy ? Optics says shortest distance between two resolvable points is :

$$\Delta x = \frac{\lambda}{2\sin\theta}$$

- Larger the lens radius, larger the $\vartheta \Rightarrow$ better resolution

Putting it all together: act of Observing an electron



Putting them together

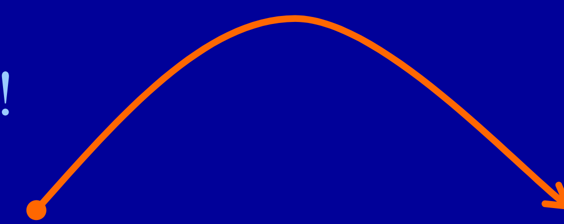
$$\Rightarrow \Delta p \,.\, \Delta x \simeq \left(\frac{2h\sin\theta}{\lambda}\right)\left(\frac{\lambda}{2\sin\theta}\right) = h$$

$$\Rightarrow \Delta p.\Delta x \geq \hbar/2$$

- Can not EXACTLY measure Location and momentum of particle at the same time
- Can measure both P_X and Y component exactly but not P_x and X

Pseudo-Philosophical Aftermath of Uncertainty Principle

- Newtonian Physics & Deterministic physics topples over
  - Newton's laws told you all you needed to know about trajectory of a particle
    - Apply a force, watch the particle go !
      - Know every thing ! X, v, p , F, a
      - Can predict exact trajectory of particle if you had perfect device

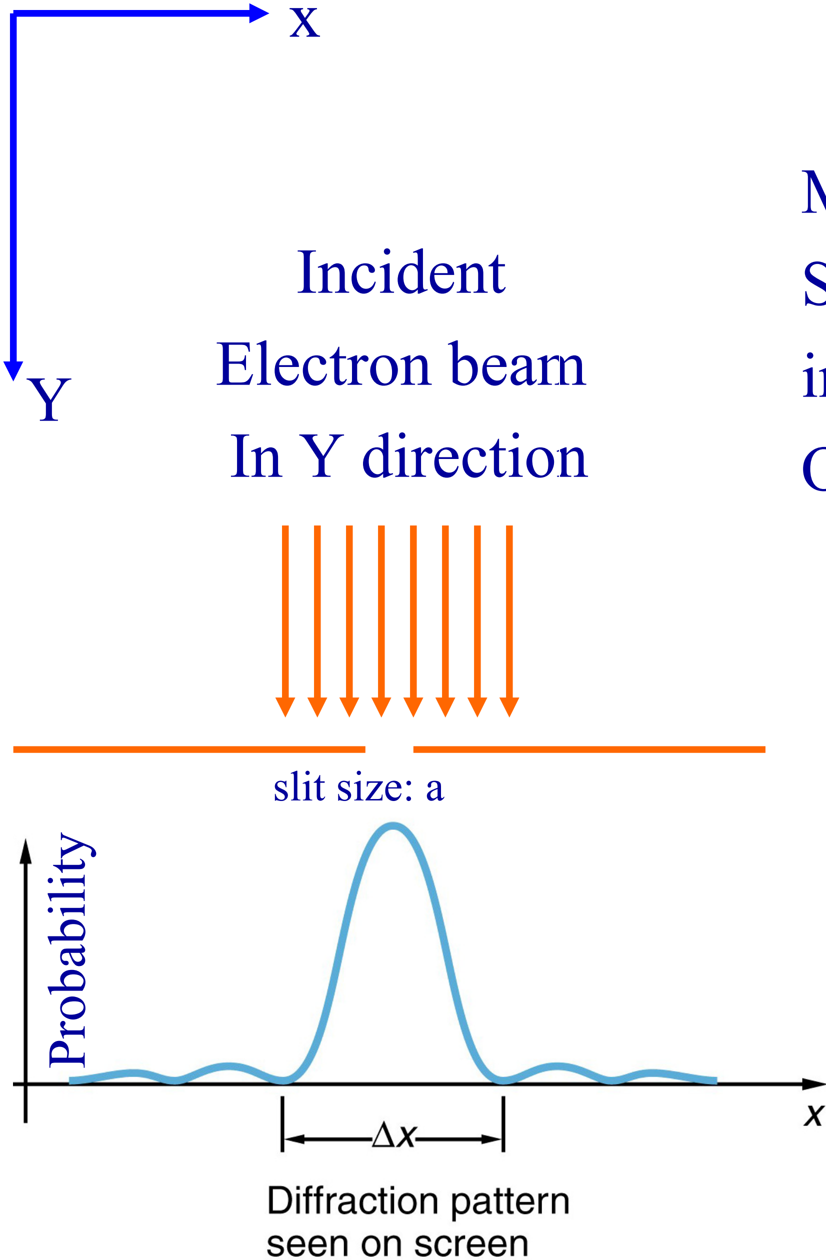


- No so in the subatomic world !
  - Of small momenta, forces, energies
  - Cant predict anything exactly
    - Can only predict probabilities
      - There is so much chance that the particle landed here or there
      - Cant be sure !....cognizant of the errors of thy observations

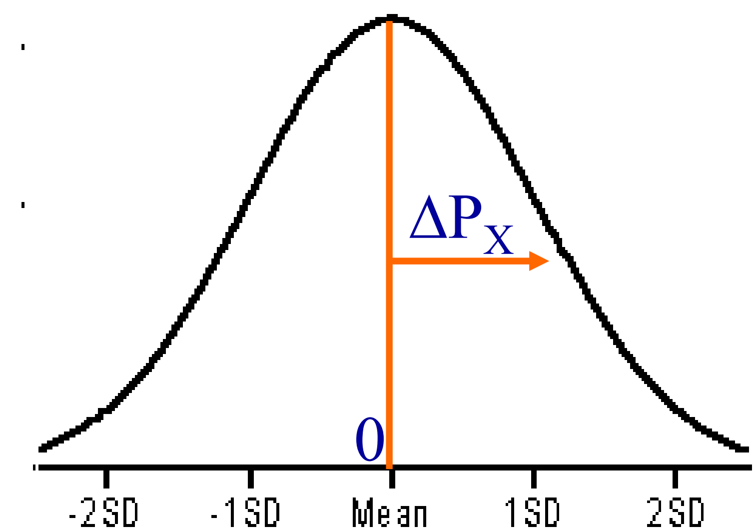
Philosophers went nuts !...what has happened to nature

Philosophers just talk, don't do real life experiments!

Matter Diffraction & Uncertainty Principle



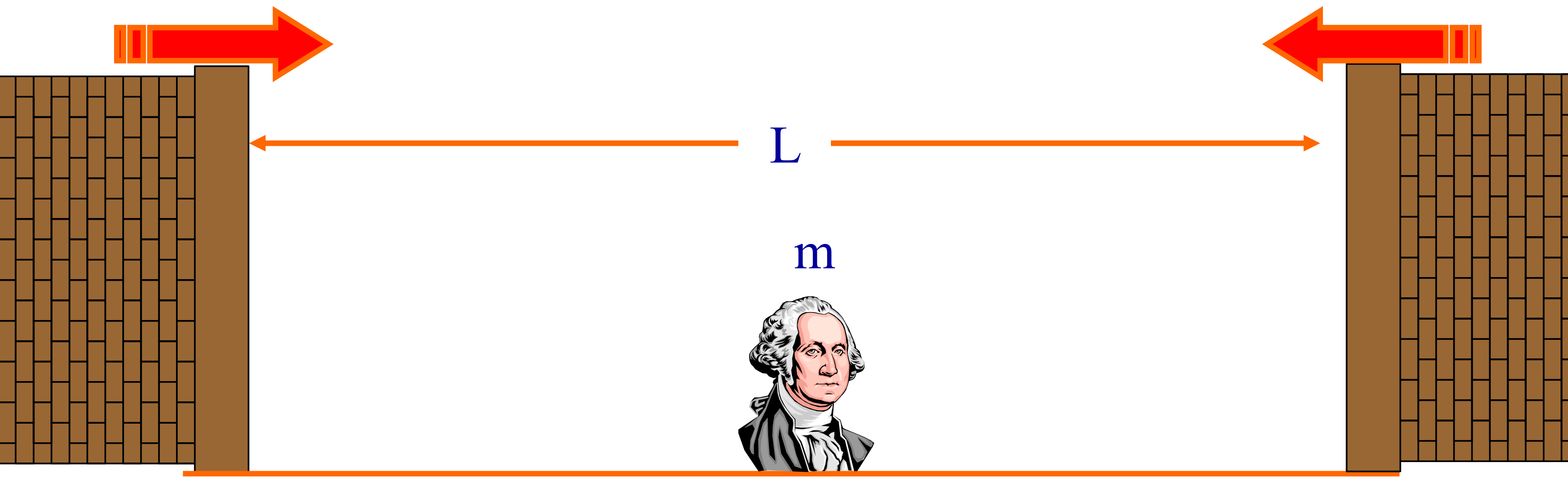
Momentum measurement beyond Slit show particle not moving exactly in Y direction, develops a X component Of motion $\Delta P_X = h/(2\pi a)$

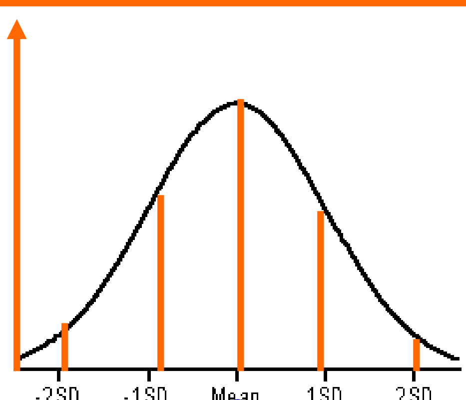


X component P_X of momentum

Particle at Rest Between Two Walls

- Object of mass M at rest between two walls originally at infinity
- What happens to our perception of George as the walls are brought in ?





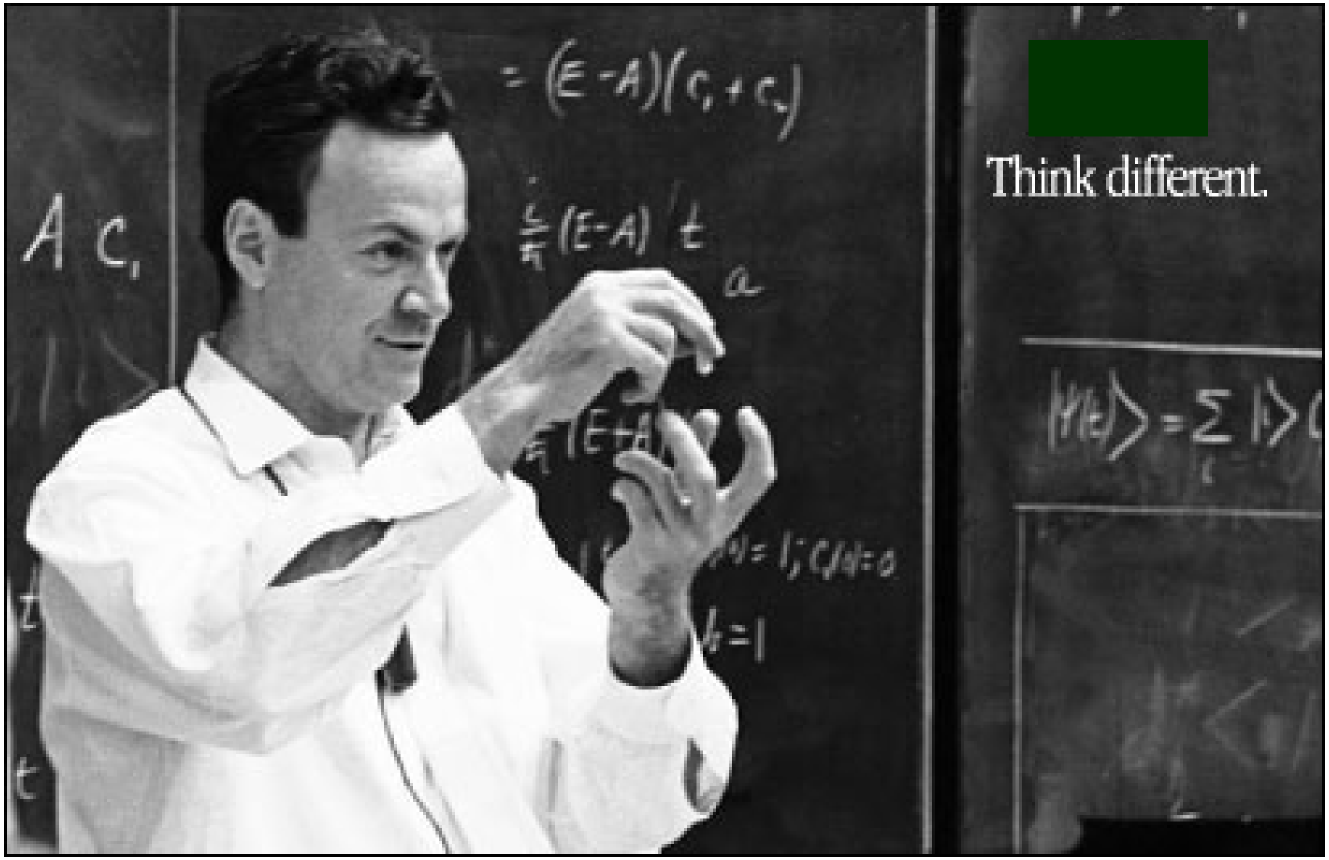
On average, measure <p> = 0

but there are quite large fluctuations!

Width of Distribution $= \Delta P$

$$\Delta P = \sqrt{(P^2)_{ave} - (P_{ave})^2}; \quad \Delta P \sim \frac{\hbar}{L}$$

Quantum Behavior : Richard Feynman

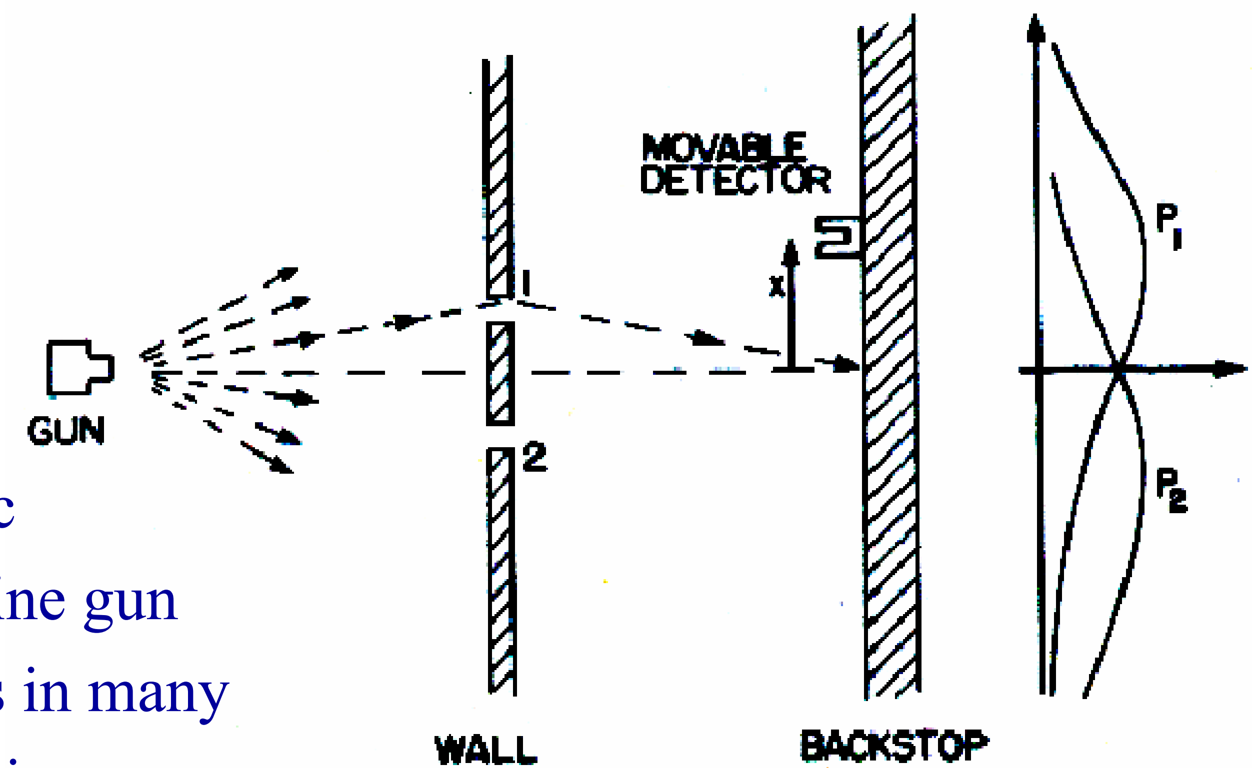


See Chapters 1 & 2 of Feynman Lectures in Physics   Vol III

Or   Six Easy Pieces by Richard Feynman : Addison Wesley Publishers

An Experiment with Indestructible Bullets

Probability P_{12} when Both holes open



Erratic Machine gun sprays in many directions

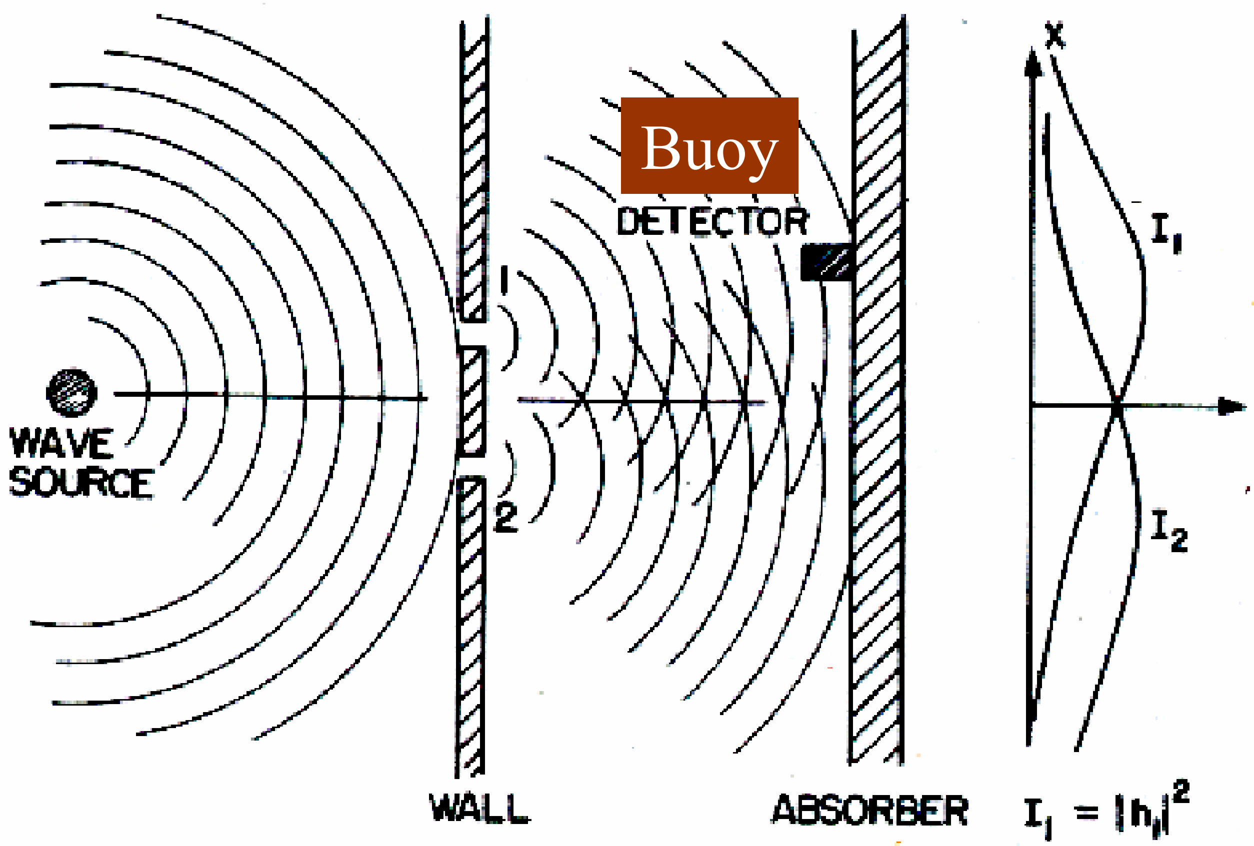
Made of Armor plate

$$P_{12} = P_1 + P_2$$

An Experiment With Water Waves

Measure Intensity of Waves
(by measuring amplitude of displacement)

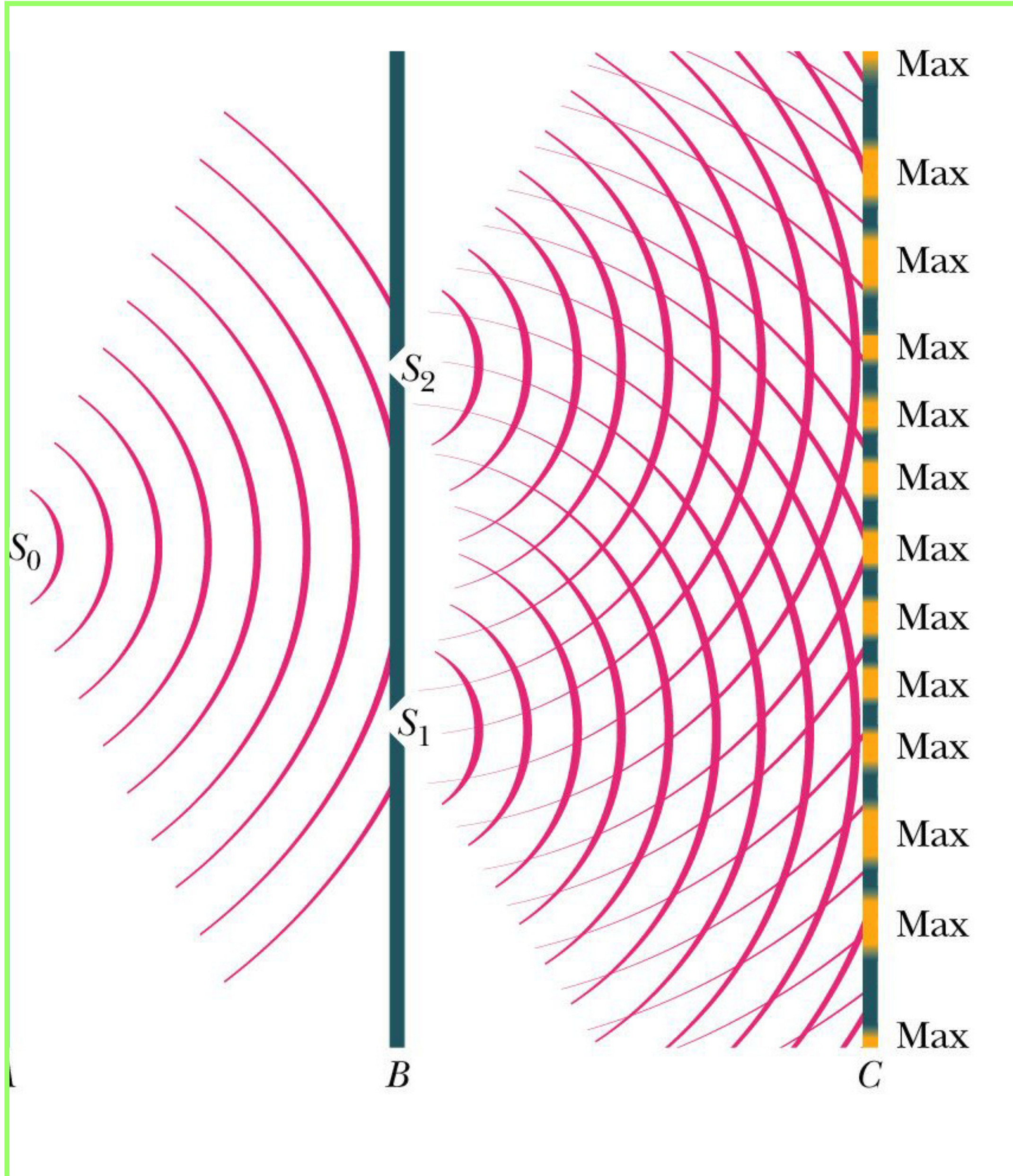
Intensity I_{12} when
Both holes open



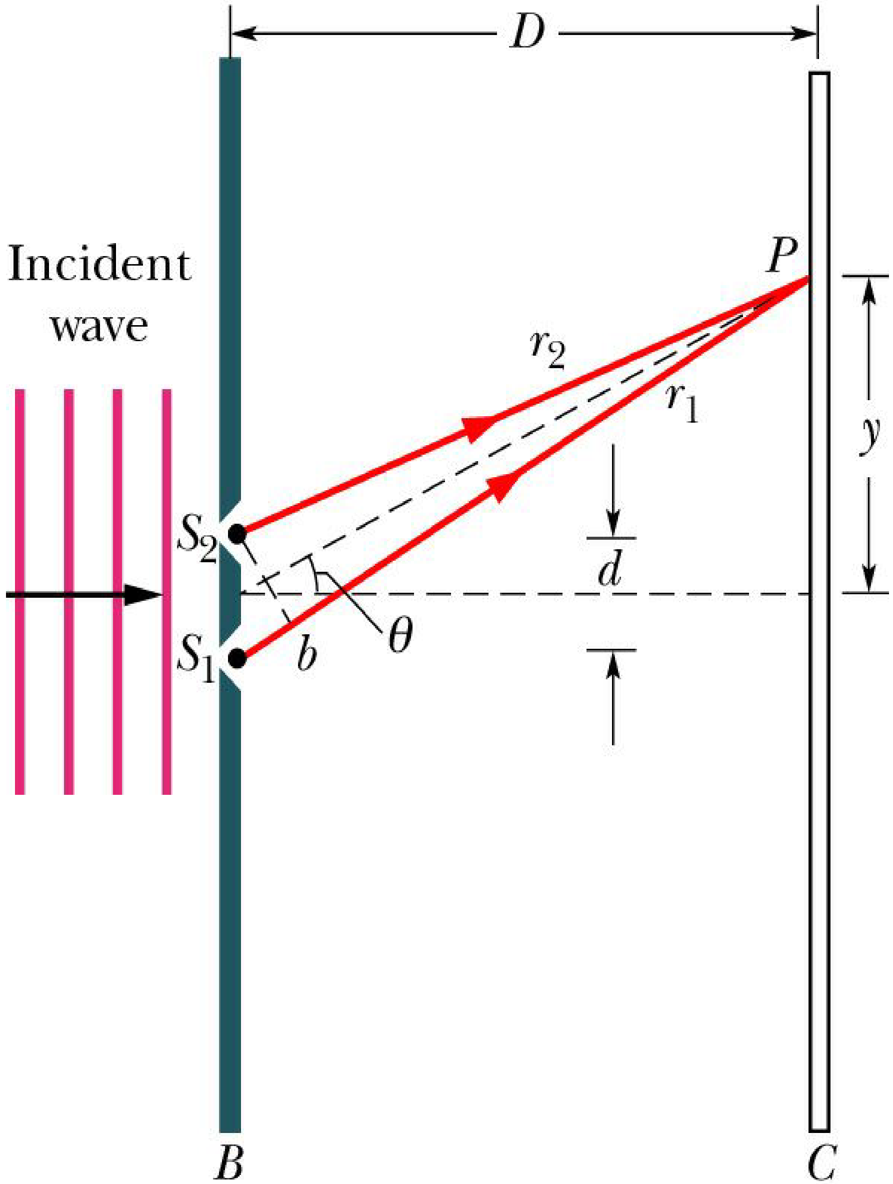
$I_1 = |h_1|^2$
$I_2 = |h_2|^2$

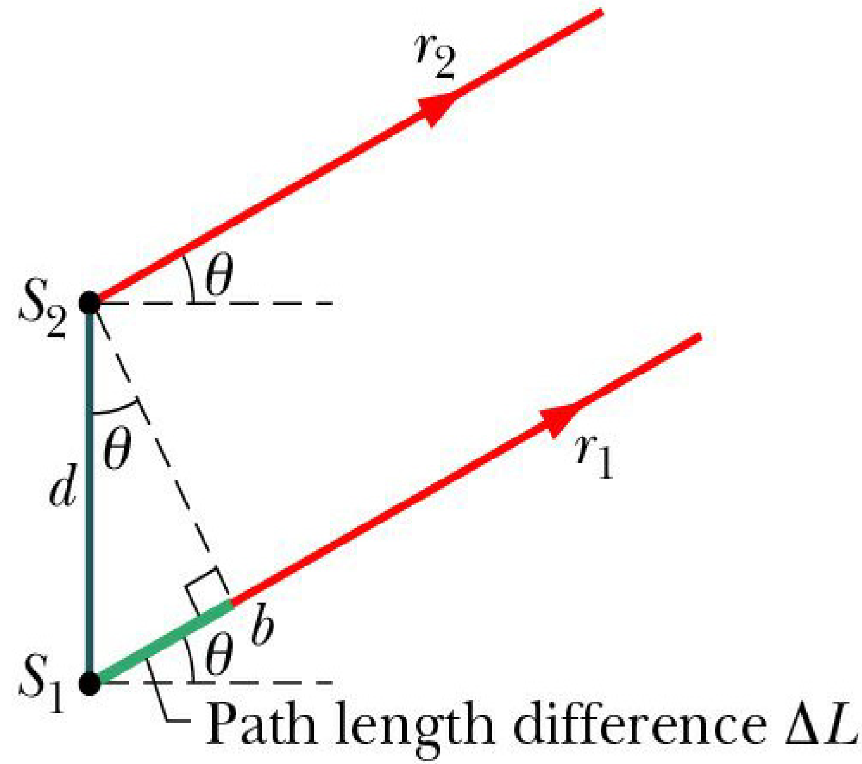
$$I_{12} = |h_1 + h_2|^2 = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

Interference and Diffraction: Ch 36 & 37, RHW



Interference Phenomenon in Waves

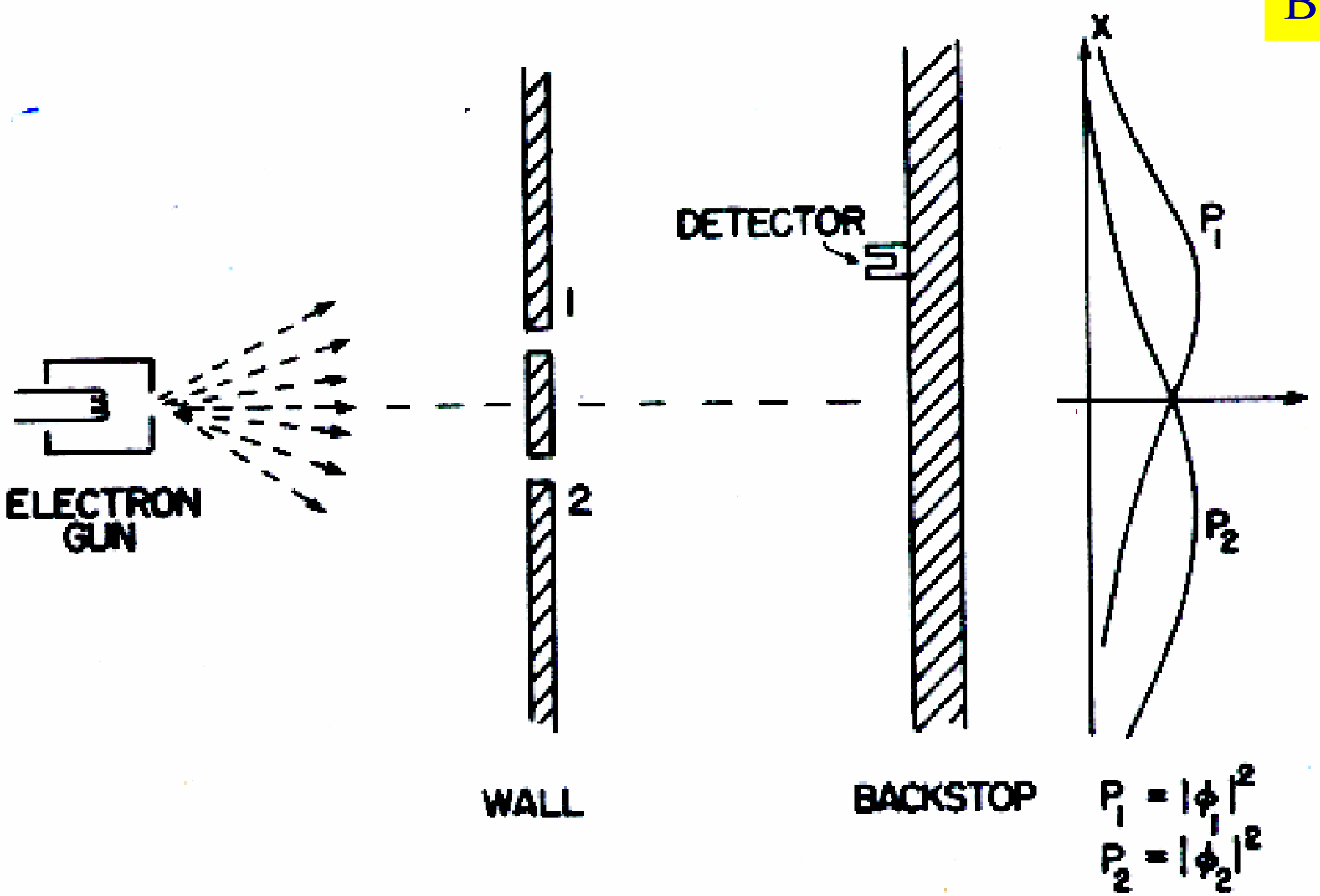




$$n\lambda = d\sin\theta$$

An Experiment With Electrons

Probability P_{12} when Both holes open

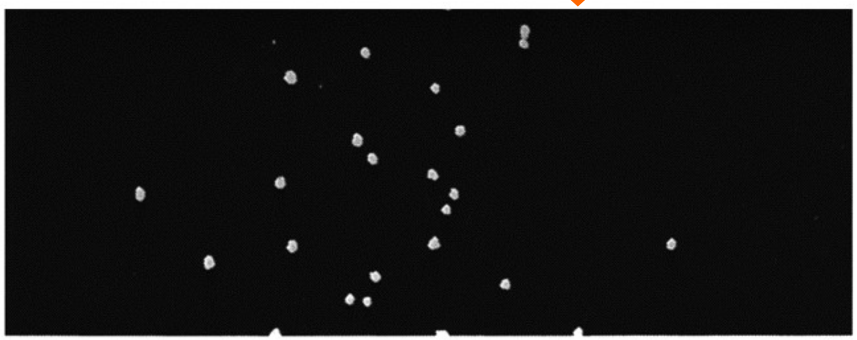


$$P_{12} \neq P_1 + P_2$$

Interference in Electrons Thru 2 slits

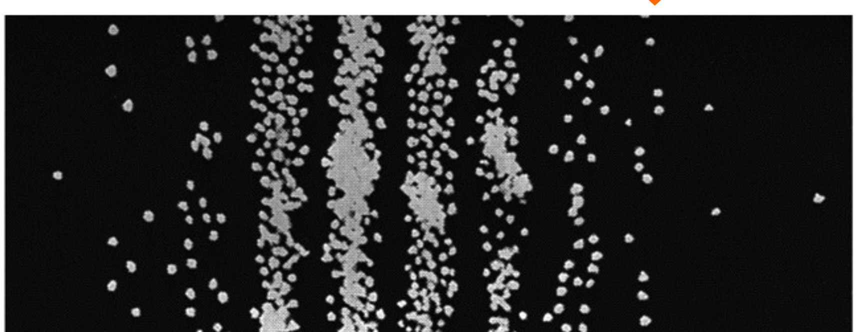
Growth of 2-slit Interference pattern thru different exposure periods
Photographic plate (screen) struck by:

28 electrons



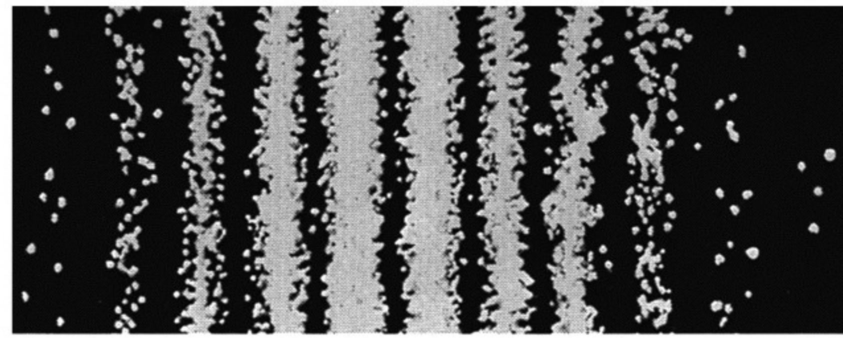
(a)

1000 electrons



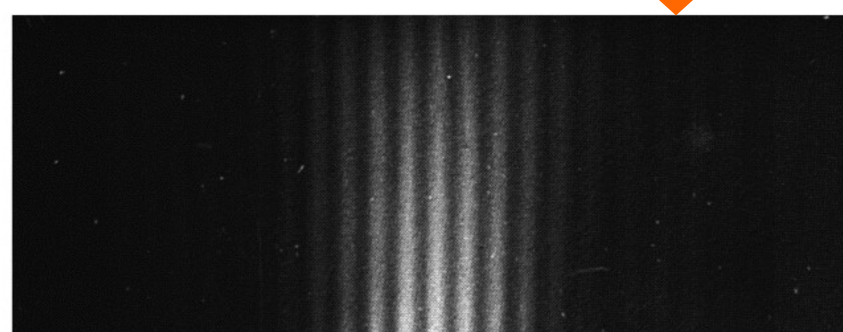
(b)

10,000 electron



(c)

10^6 electrons

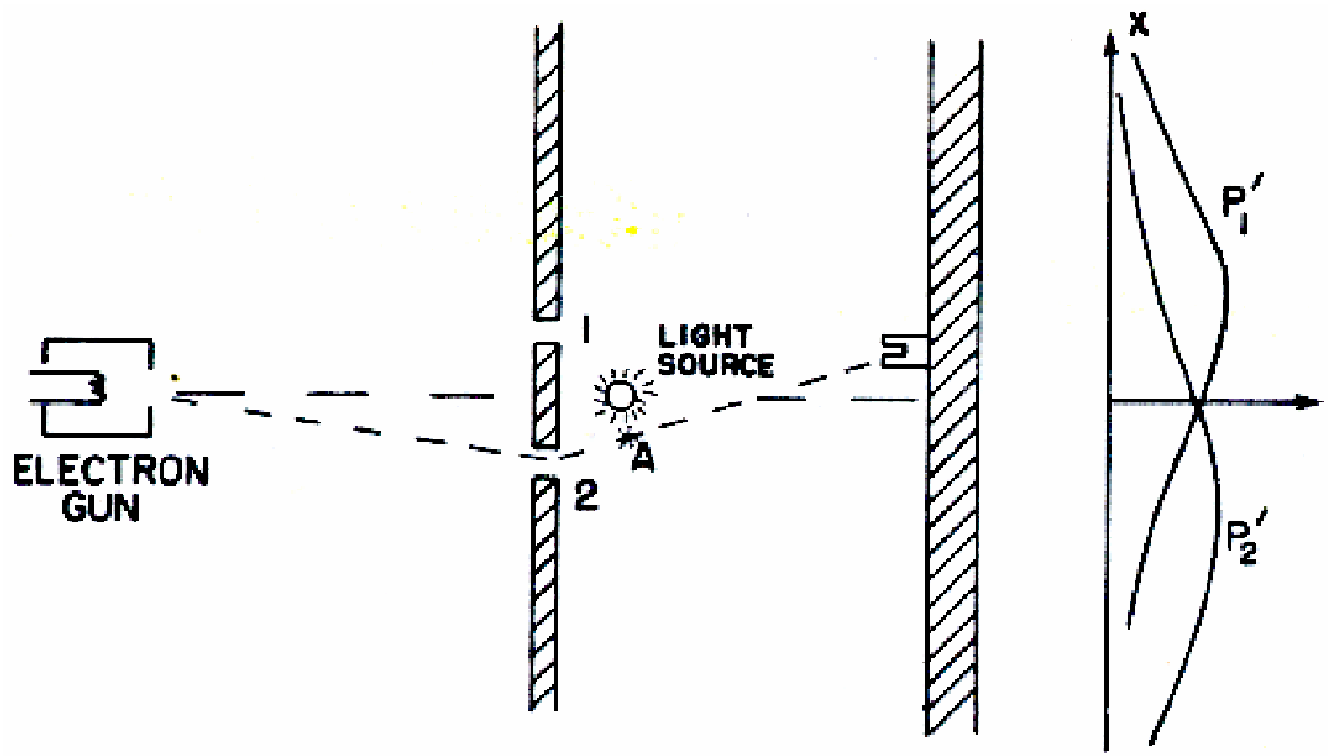


(d)

White dots simulate presence of electron
No white dots at the place of destructive Interference (minima)

Watching The Electrons With Intense Light

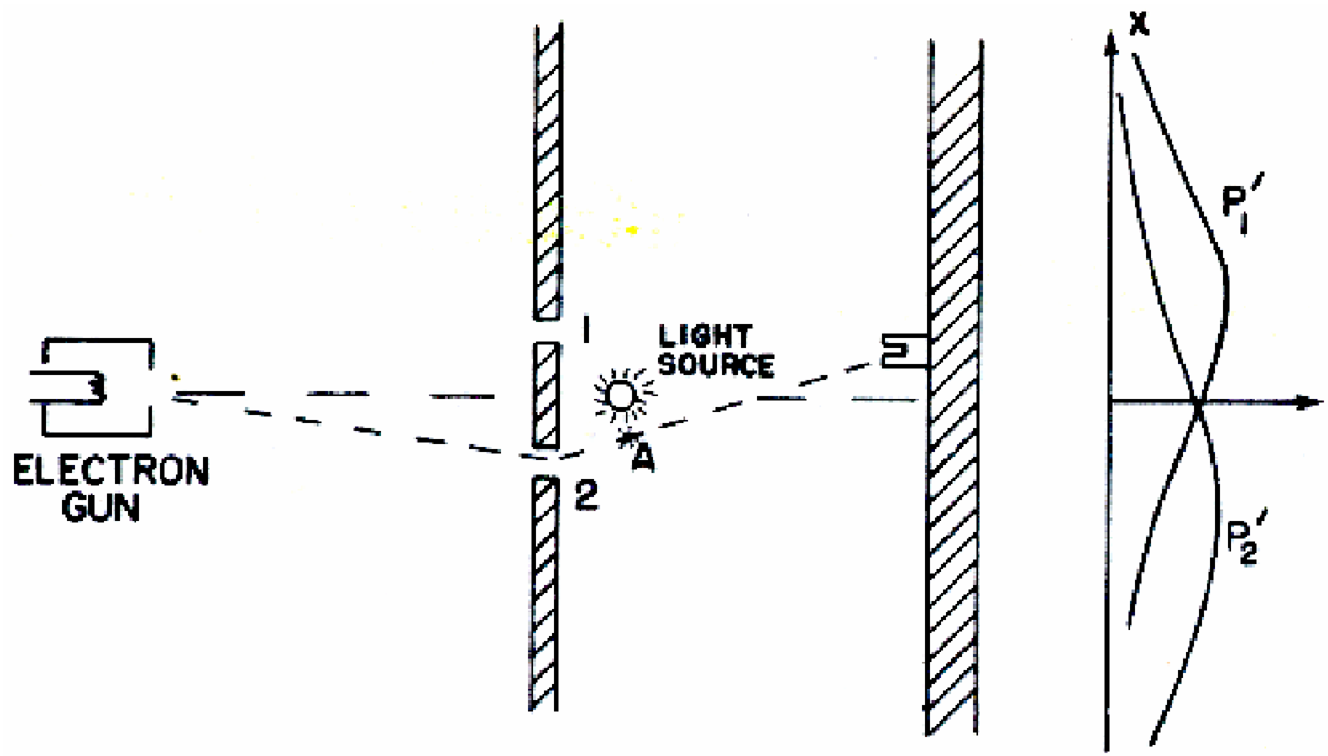
Probability P_{12} when both holes open and I see which hole the electron came thru



$$P'_{12} = P'_1 + P'_2$$

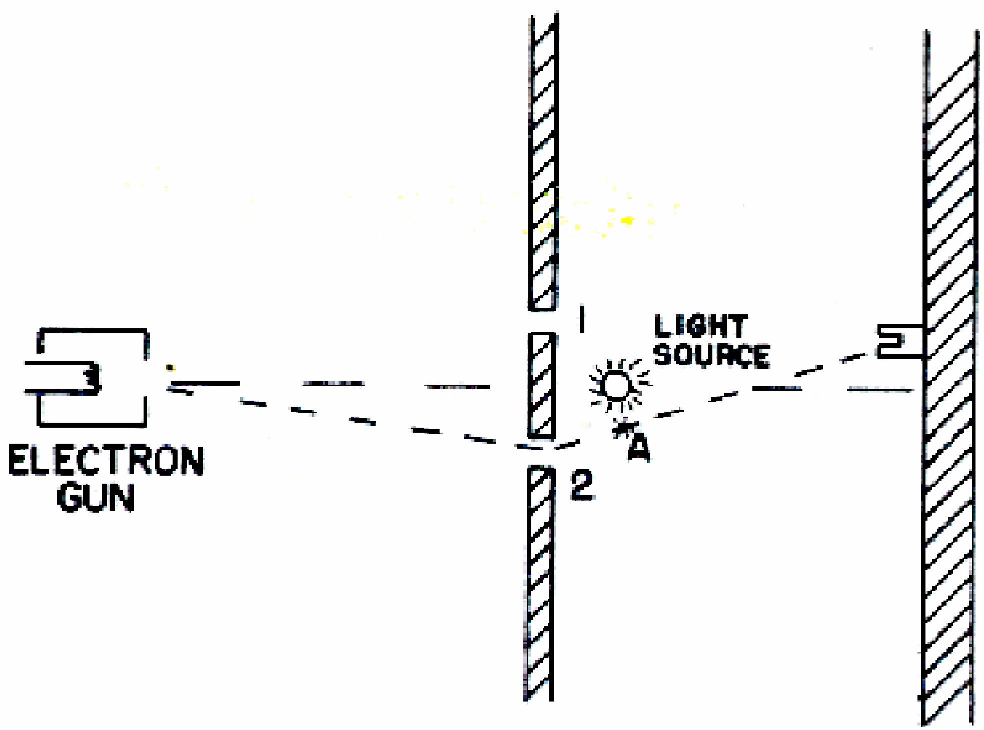
Watching The Electrons With Dim Light

Probability P_{12} when both holes open and I see which hole the electron came thru



Watching The Electrons With Dim Light

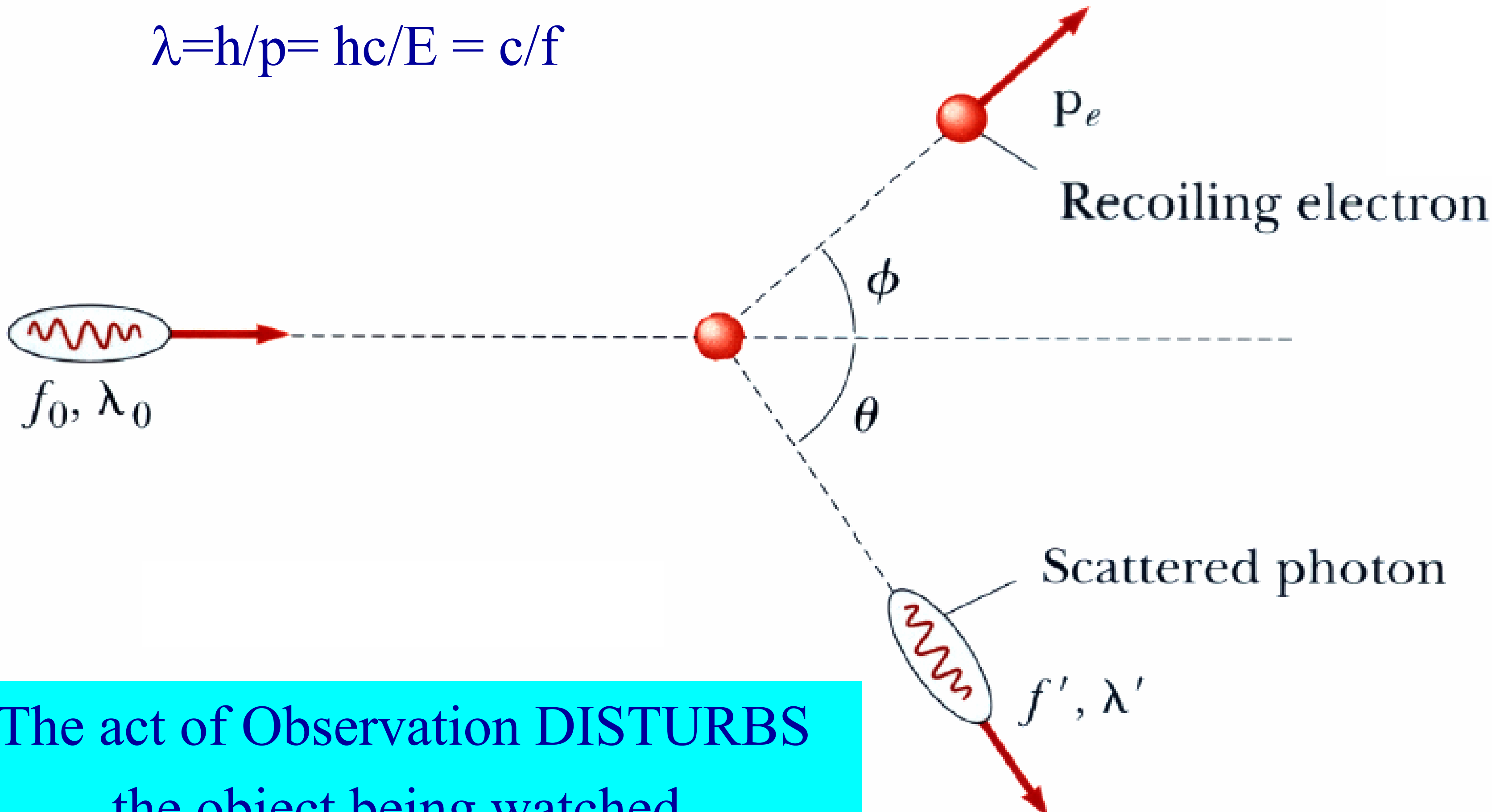
Probability P_{12} when both holes open and I Don't see which hole the electron came thru



$$P_{12} = |\phi_1 + \phi_2|^2$$

Compton Scattering: Shining light to observe electron

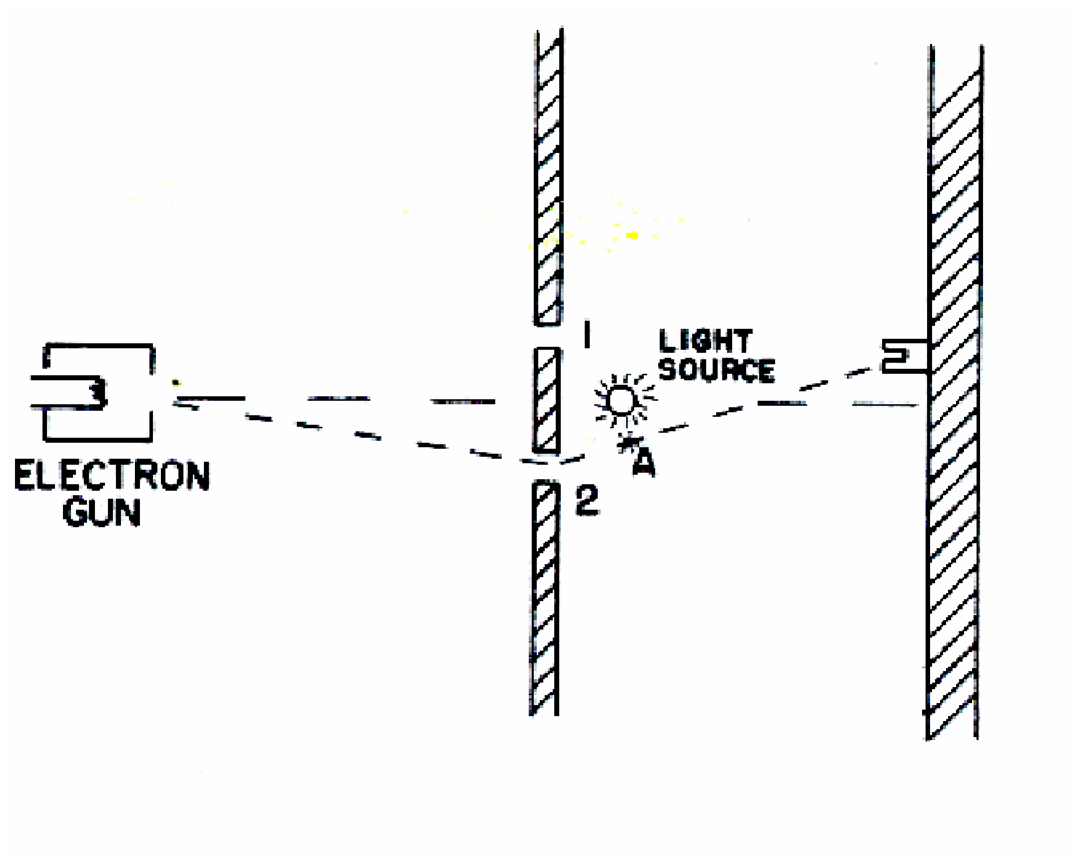
$\lambda = h/p = hc/E = c/f$



The act of Observation DISTURBS the object being watched, here the electron moves away from where it was originally

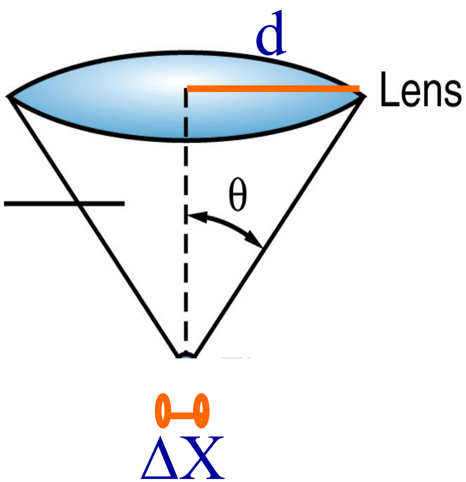
Watching Electrons With Light of $\lambda >>$ slitsize but High Intensity

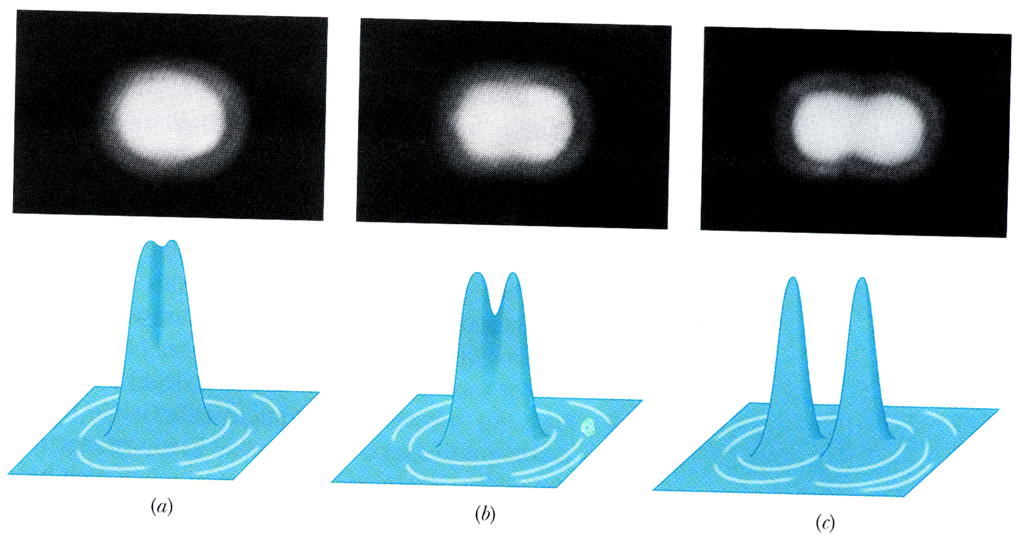
Probability P_{12} when both holes open but cant tell from flash which hole the electron came thru



Why Fuzy Flash? → Resolving Power of Light

Image of 2 separate point sources formed by a converging lens of diameter d, ability to resolve them depends on λ & d because of the Inherent diffraction in image formation





Not resolved barely resolved resolved

$$\text{Resolving power } \Delta x \simeq \frac{\lambda}{2\sin\theta}$$

Summary of Experiments So Far

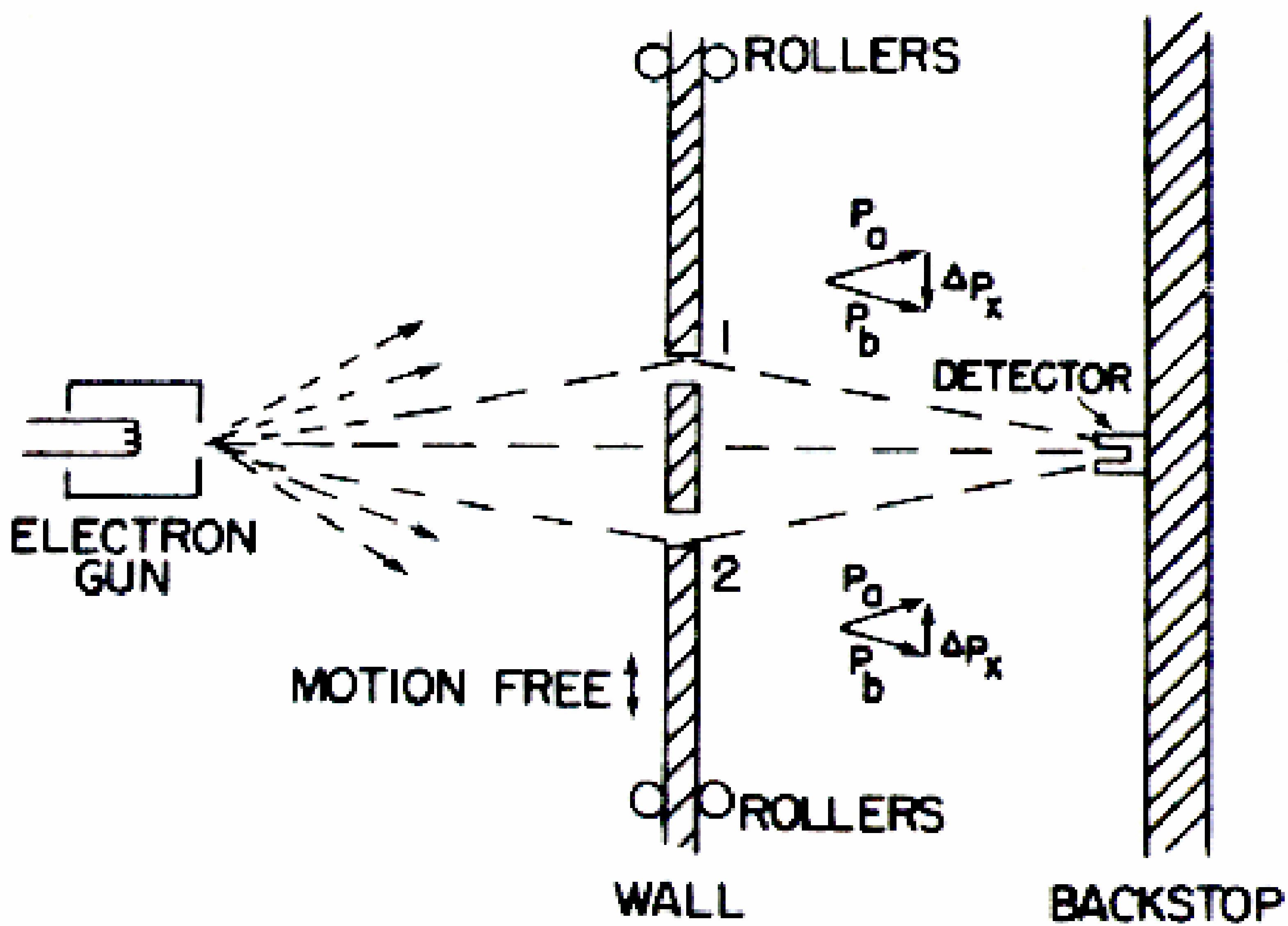
1. Probability of an event is given by the square of amplitude of a complex # Ψ: Probability Amplitude
2. When an event occurs in several alternate ways, probability amplitude for the event is sum of probability amplitudes for each way considered seperately. There is interference:
   - $\Psi = \Psi_1 + \Psi_2$

     $P_{12} = |\Psi_1 + \Psi_2|^2$
3. If an experiment is done which is capable of determining whether one or other alternative is actually taken, probability for event is just sum of each alternative
   - Interference pattern is LOST !

Is There No Way to Beat Uncertainty Principle?

- How about NOT watching the electrons!
- Lets be a bit crafty
- Since this is a Thought experiment→ ideal conditions
  - Mount the wall on rollers, put a lot of grease → frictionless
  - Wall will move when electron hits it
  - Watch recoil of the wall containing the slits when the electron hits it
  - By watching whether wall moved up or down I can tell
    - Electron went thru hole # 1
    - Electron went thru hole #2
- Will my ingenious plot succeed?

Measuring The Recoil of The Wall: Not Watching Electron !

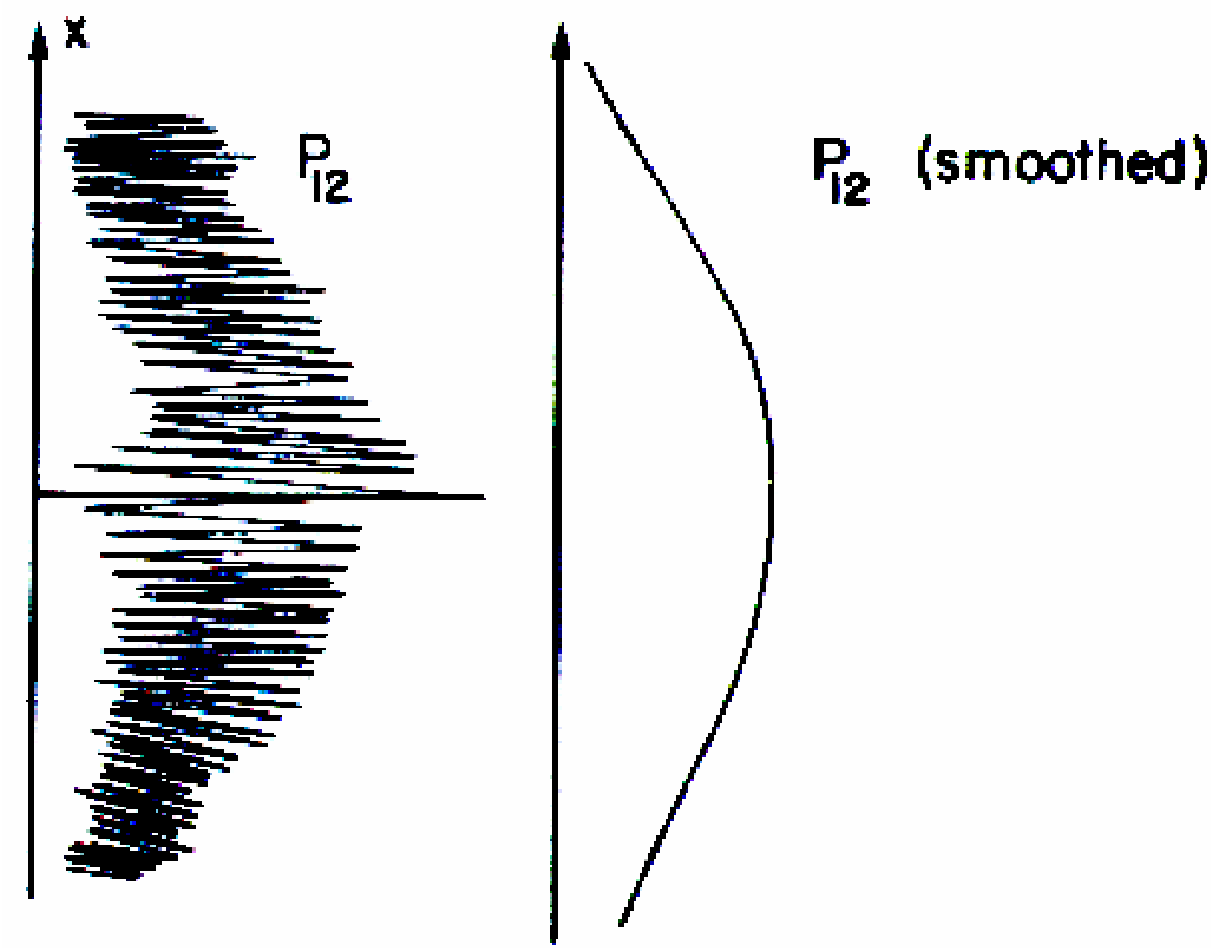


?

Losing Out To Uncertainty Principle

- To measure the RECOIL of the wall $\Rightarrow$
  - must know the initial momentum of the wall before electron hit it
  - Final momentum after electron hits the wall
  - Calculate vector sum → recoil
- Uncertainty principle :
  - To do this $\Rightarrow$ $\Delta P = 0$ → $\Delta X = \infty$ [can not know the position of wall exactly]
  - If don't know the wall location, then down know where the holes are
  - Holes will be in different place for every electron that goes thru
  - → The center of interference pattern will have different (random) location for each electron
  - Such random shift is just enough to Smear out the pattern so that no interference is observed !
- Uncertainty Principle Protects Quantum Mechanics !

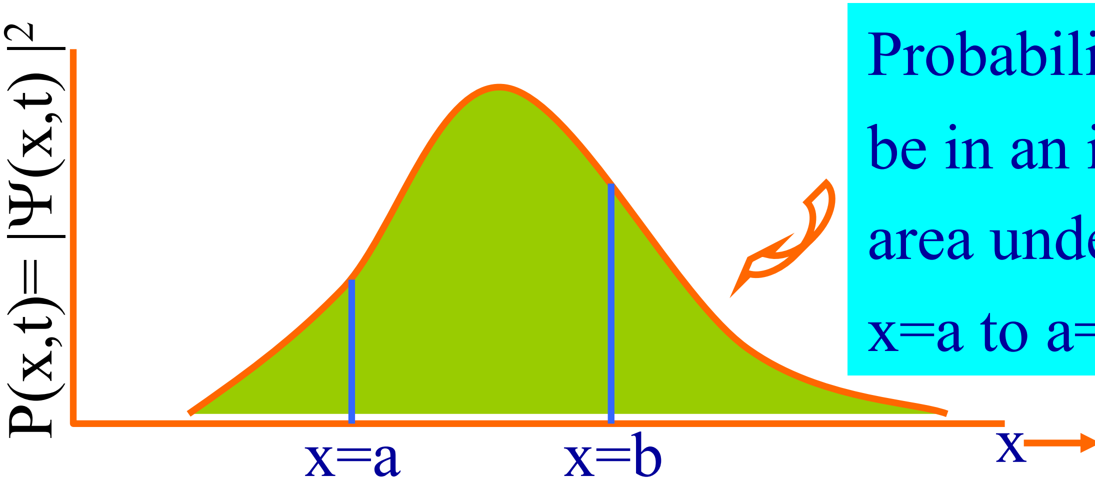
The Bullet Vs The Electron: Each Behaves the Same Way



Quantum Mechanics of Subatomic Particles

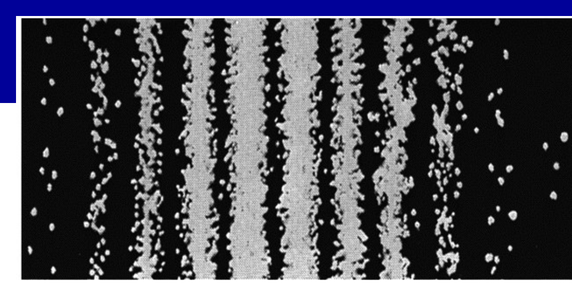
- Act of Observation destroys the system (No watching!)
- If can't watch then All conversations can only be in terms of Probability P
- Every particle under the influence of a force is described by a Complex wave function $\Psi(x,y,z,t)$
- Ψ is the ultimate DNA of particle: contains all info about the particle under the force (in a potential e.g Hydrogen)
- Probability of per unit volume of finding the particle at some point (x,y,z) and time t is given by
  - $P(x,y,z,t) = \Psi(x,y,z,t) \, . \, \Psi^*(x,y,z,t) = | \Psi(x,y,z,t) |^2$
- When there are more than one path to reach a final location then the probability of the event is
  - $\Psi = \Psi_1 + \Psi_2$
  - $P = | \Psi^* \Psi| = |\Psi_1|^2 + |\Psi_2|^2 + 2 |\Psi_1 |\Psi_2| \cos\phi$

Wave Function of "Stuff" & Probability Density



Probability of a particle to be in an interval $a \leq x \leq b$ is area under the curve from x=a to a=b

- Although not possible to specify with certainty the location of particle, its possible to assign probability P(x)dx of finding particle between x and x+dx
- $P(x)\,dx = |\Psi(x,t)|^2\,dx$
- E.g intensity distribution in light diffraction pattern is a measure of the probability that a photon will strike a given point within the pattern



Ψ: The Wave function Of A Particle

- The particle must be some where

$$\int_{-\infty}^{+\infty} |\psi(x,t)|^2 \, dx = 1$$

- Any Ψ satisfying this condition is NORMALIZED
- Prob of finding particle in finite interval

$$P(a \le x \le b) = \int_a^b \psi^*(x,t) \; \psi(x,t) dx$$

- Fundamental aim of Quantum Mechanics
  - Given the wavefunction at some instant (say t=0) find Ψ at some subsequent time t
  - Ψ(x,t=0) → Ψ(x,t) …evolution
  - Think of a probabilistic view of particle's "newtonian trajectory"
    - We are replacing Newton's 2nd law for subatomic systems

The Wave Function is a mathematical function that describes a physical object → Wave function must have some rigorous properties :

- Ψ must be finite
- Ψ must be continuous fn of x,t
- Ψ must be single-valued
- Ψ must be smooth fn → $\frac{d\psi}{dx}$ must be continuous

WHY ?