Physics 2D Lecture Slides
Nov 10

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Wave Packet To Describe "Particle" : Localization

To make localized wave packet, add " infinite" # of waves with Well chosen Amplitudes A, Wave# k & angular Freq. w

$$\psi(x,t) = \int_{-\infty}^{\infty} A(k)\, e^{i(kx-wt)} dk$$

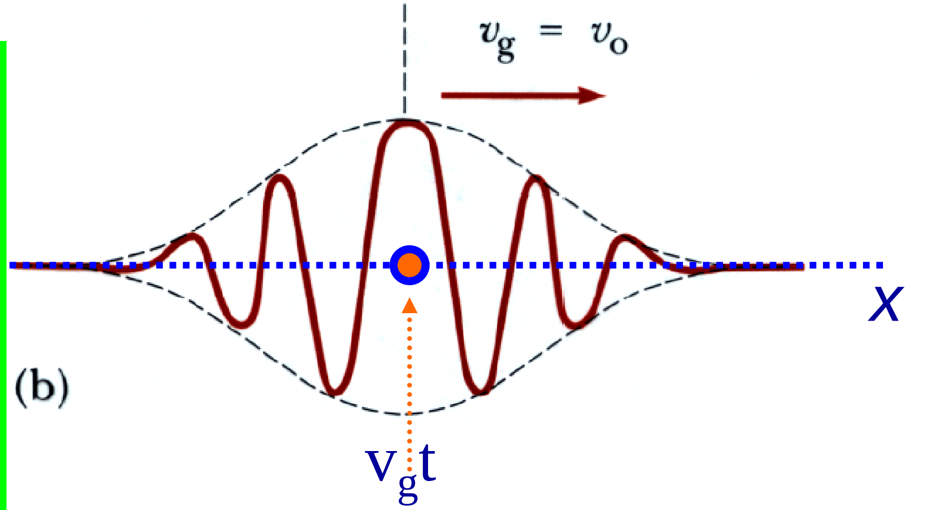
$A(k)$ = Amplitude Fn

$\Rightarrow$ diff waves of diff k

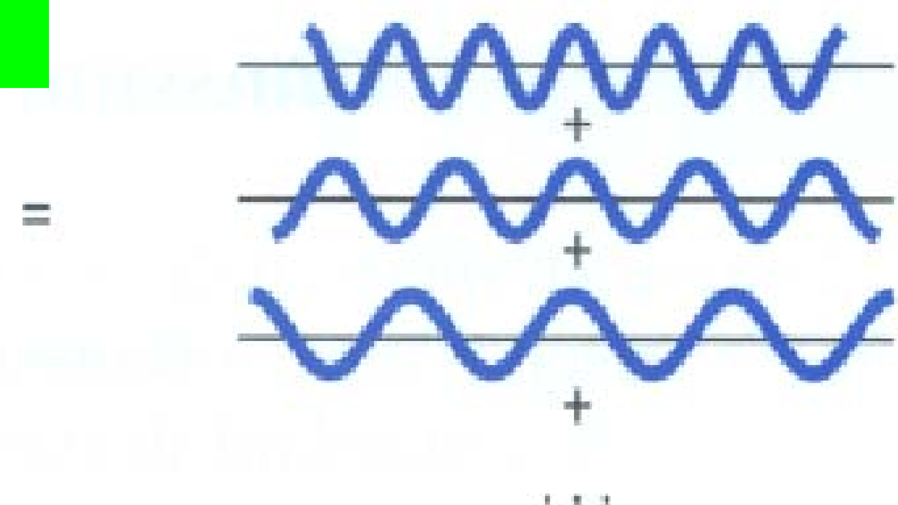
have different amplitudes A(k)

w = w(k), depends on type of wave, media

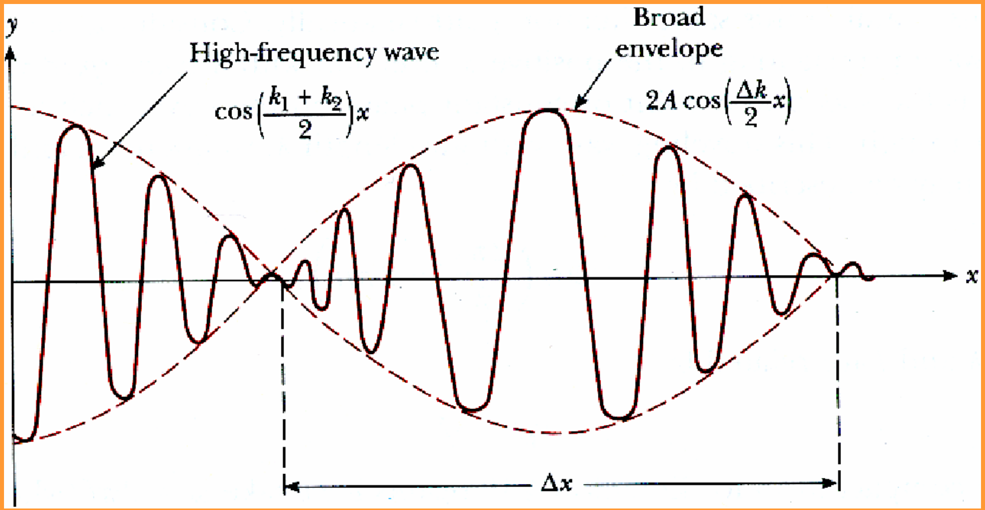
Group Velocity $V_g = \left.\frac{dw}{dk}\right|_{k=k_0}$



localized



Wave Packets & Uncertainty Principle



We added two Sinusoidal waves

$$y = 2A\left[\left(\cos\left(\frac{\Delta k}{2}x - \frac{\Delta w}{2}t\right)\right)\cos(kx - wt)\right]$$

Amplitude Modulation

- Distance ΔX between adjacent minima = $(X_2)_{node}$ - $(X_1)_{node}$
- Define $X_1=0$ then phase diff from $X_1 \rightarrow X_2 = \pi$ (similarly for $t_1 \rightarrow t_2$)

What can we learn from this simple model

Node at y = 0 = $2A\cos\left(\frac{\Delta w}{2}t - \frac{\Delta k}{2}x\right)$, Examine x or t behavior

$\Rightarrow$ in space x: $\boxed{\Delta k.\Delta x = \pi}$ $\Rightarrow$ Need to combine many k to make small Δx pulse

$$\Delta x = \frac{\pi}{\Delta k}, \text{ for small } \Delta x \rightarrow 0 \Rightarrow \Delta k \rightarrow \infty \text{ \& Vice Verca}$$

and In time t : $\boxed{\Delta w.\Delta t = \pi}$ $\Rightarrow$ Need to combine many ω to make small Δt pulse

$$\Delta t = \frac{\pi}{\Delta \omega}, \text{ for small } \Delta t \rightarrow 0 \Rightarrow \Delta \omega \rightarrow \infty \text{ \& Vice Verca}$$

Wave Packets & Uncertainty Principle

in space x: $\boxed{\Delta k . \Delta x = \pi}$ $\Rightarrow$ since $k = \dfrac{2\pi}{\lambda}, \; p = \dfrac{h}{\lambda}$

$$\Rightarrow \quad \boxed{\Delta p . \Delta x = h/2}$$

usually one writes $\boxed{\Delta p . \Delta x \geq \hbar/2}$ approximate relation

In time t : $\boxed{\Delta w . \Delta t = \pi}$ $\Rightarrow$ since $\omega = 2\pi f, E = hf$

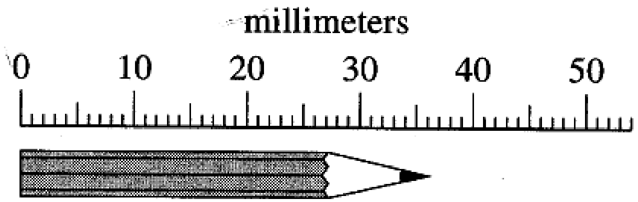
$$\Rightarrow \quad \boxed{\Delta E . \Delta t = h/2}$$

usually one writes $\boxed{\Delta E . \Delta t \geq \hbar/2}$ approximate relation

What do these inequalities mean physically?

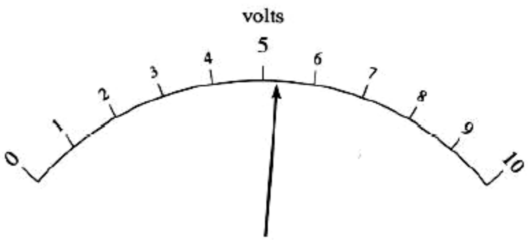
Know the Error of Thy Ways: Measurement Error →Δ

- Measurements are made by observing something : length, time, momentum, energy
- All measurements have some (limited) precision`…no matter the instrument used
- Examples:
  - How long is a desk ? L = (5 ± 0.1) m = L ± ΔL (depends on ruler used)
  - How long was this lecture ? T = (50 ± 1)minutes = T ± ΔT (depends on the accuracy of your watch)
  - How much does Prof. Sharma weigh ? M = (1000 ± 500) kg = m ± Δm
    - Is this an correct measure of my weight ?
      - Correct (because of large error reported) but imprecise
      - My correct weight is covered by the (large) error in observation



Best Estimate Length: 36 mm
Probable Range: 35.5 to 36.5 mm

Length Measure



Best Estimate of Voltage: 5.3 V
Estimated Range: 5.2 to 5.4 mm

Voltage (or time) Measure

Where in the World is Carmen San Diego?

- Carmen San Diego hidden inside a big box of length L
- Suppose you can’t see thru the (blue) box, what is you best estimate

of her location inside box (she could be anywhere inside the box)

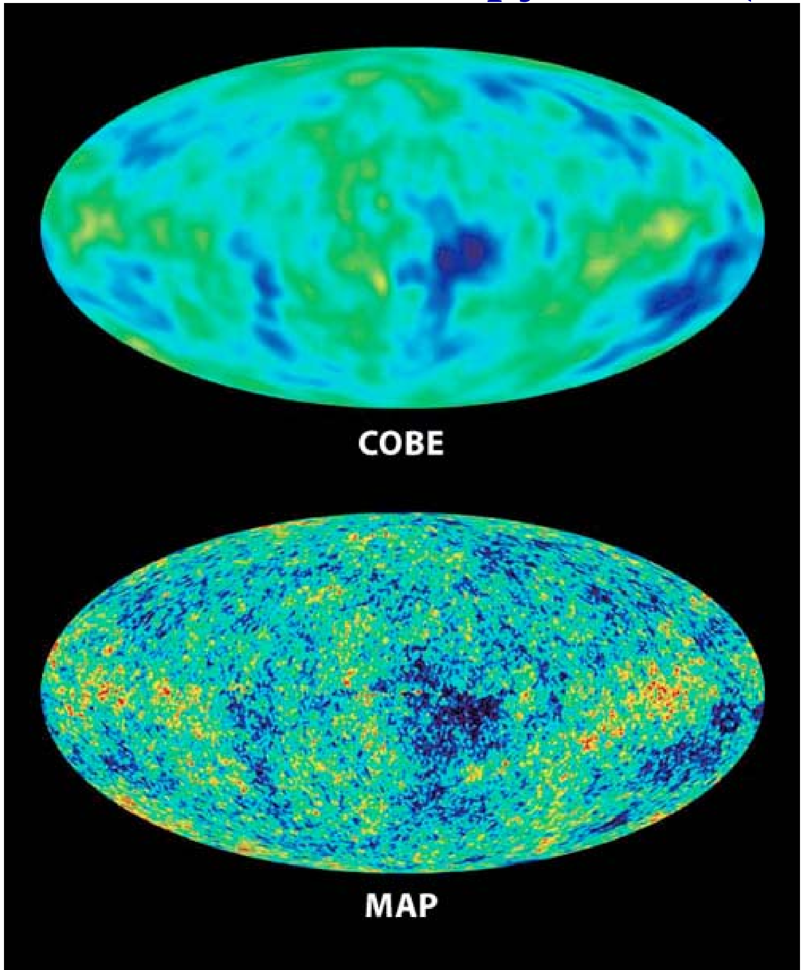
Your best unbiased measure would be $x = L/2 \pm L/2$

There is no perfect measurement, there are always measurement error

Baby Pictures of Our Universe Revealed This Year

- Look at the Intensity, temperature & polarization in cosmic microwave background
- Universe is (13.7 ± .14) Billion years old
- Universe is expanding faster than ever, propelled by a mysterious (unknown) DARK ENERGY
- Measurements give first clear indication of the “dynamite” behind the “big bang”

Microwave Anisotropy Probe (MAP)



Back to Heisenberg's Uncertainty Principle

- $\Delta x . \Delta p \geq h/4\pi \Rightarrow$
  - If the measurement of the position of a particle is made with a precision Δx and a SIMULTANEOUS measurement of its momentum p_x in the X direction , then the product of the two uncertainties (measurement errors) can never be smaller than $\simeq h/4\pi$ irrespective of how precise the measurement tools

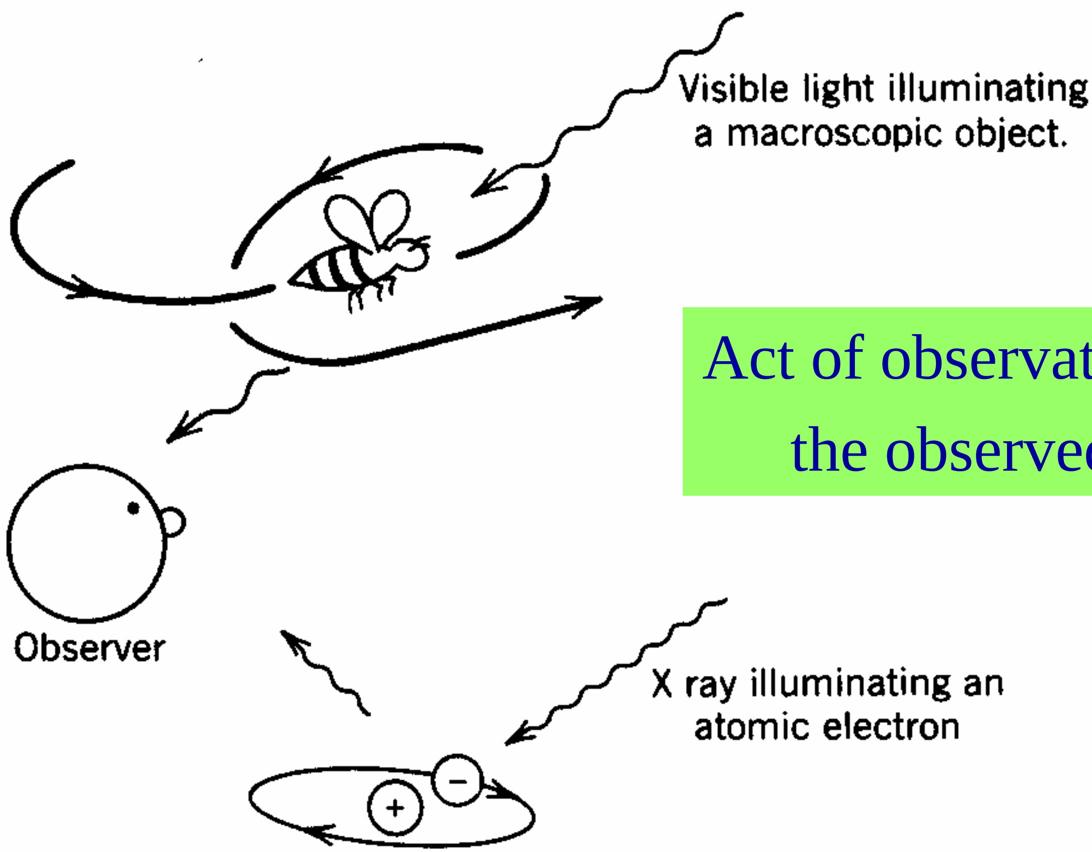
- $\Delta E . \Delta t \geq h/4\pi \Rightarrow$
  - If the measurement of the energy E of a particle is made with a precision ΔE and it took time Δt to make that measurement, then the product of the two uncertainties (measurement errors) can never be smaller than $\simeq h/4\pi$ irrespective of how precise the measurement tools

These rules arise from the way we constructed the Wave packets describing Matter "pilot" waves

Perhaps these rules Are bogus, can we verify this with some physical picture ??

The Act of Observation (Compton Scattering)

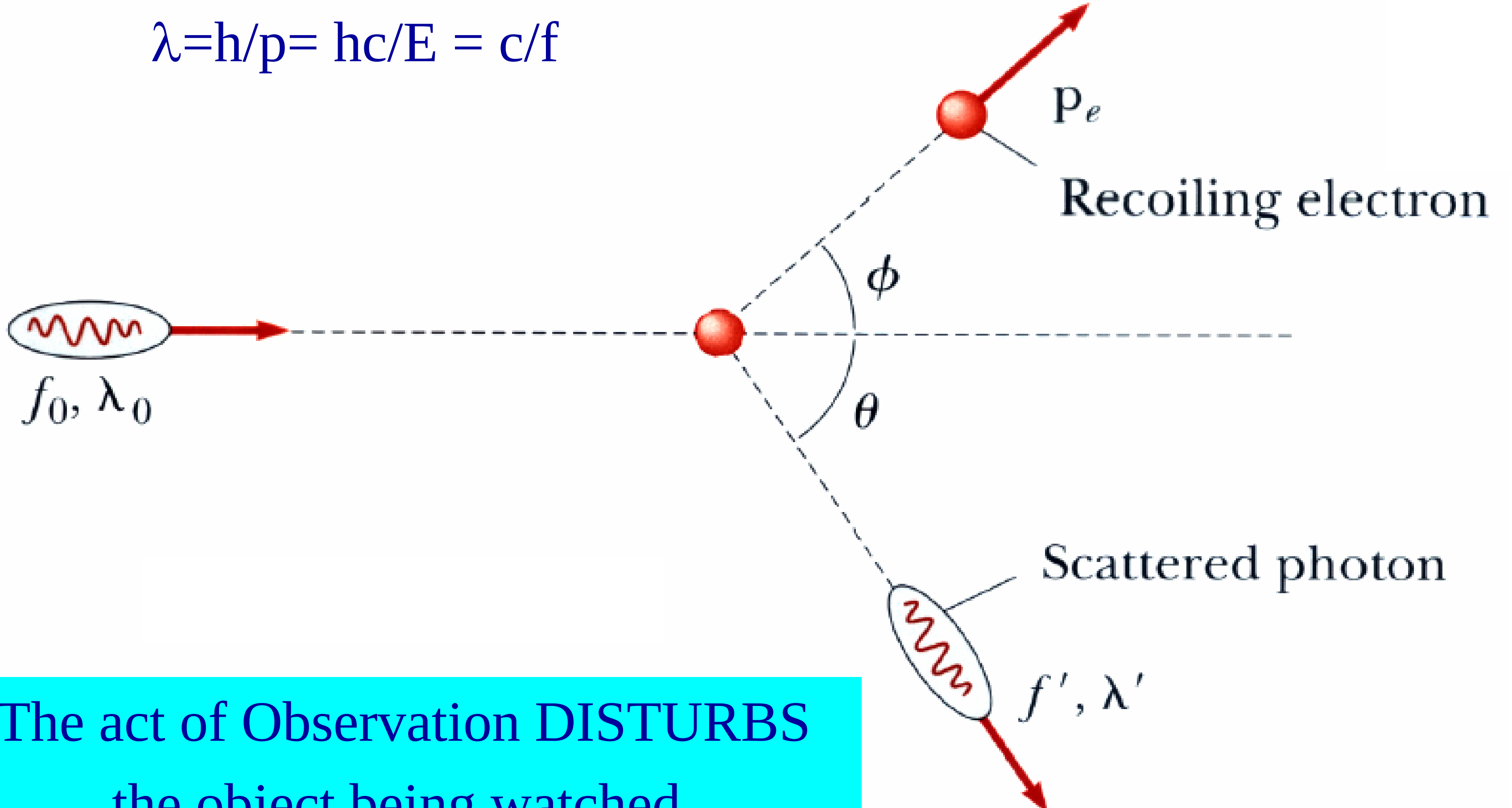
Observations of particle motion by means of scattered illumination. When the incident wavelength is reduced to accommodate the size of the particle, the momentum transferred by the photon becomes large enough to disturb the observed motion.



Act of observation disturbs the observed system

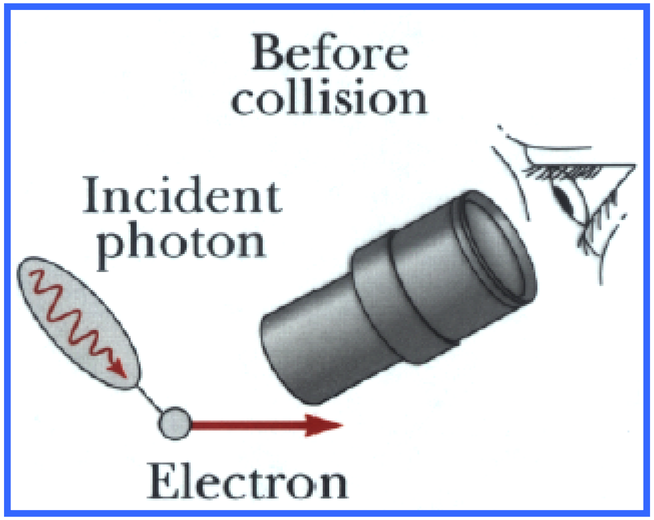
Compton Scattering: Shining light to observe electron

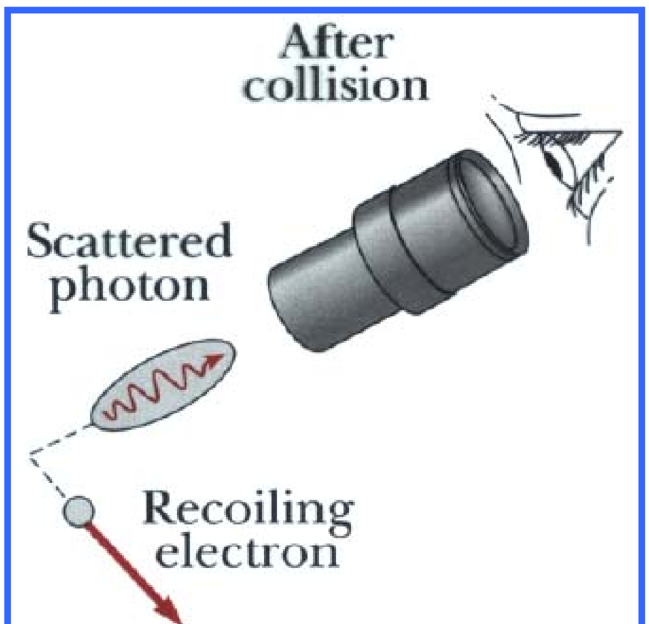
$\lambda = h/p = hc/E = c/f$

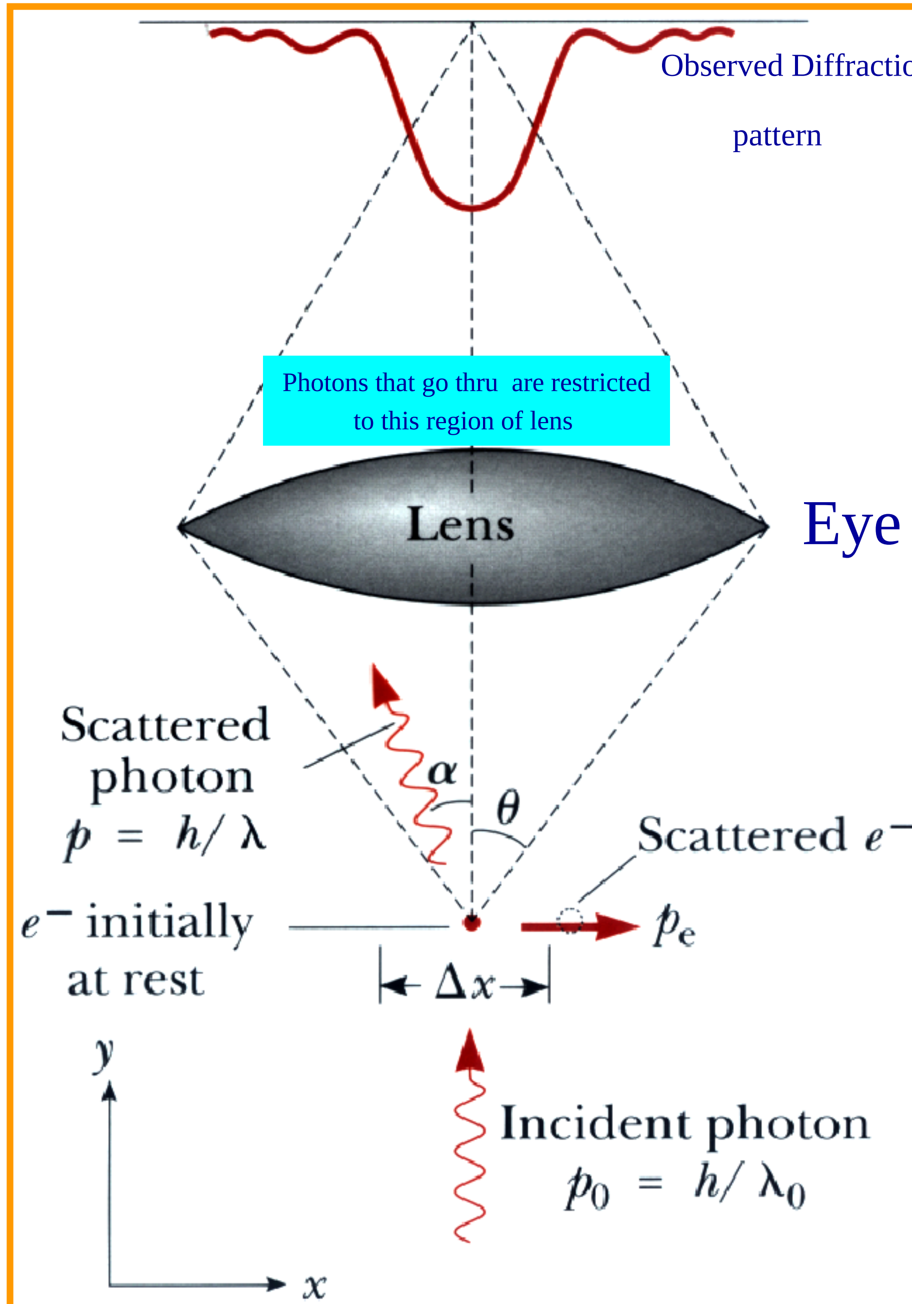


The act of Observation DISTURBS
the object being watched,
here the electron moves away from
where it was originally

Act of Watching: A Thought Experiment







Diffraction By a Circular Aperture (Lens)

See Resnick, Halliday Walker 6th Ed (on S.Reserve), Ch 37, pages 898-900

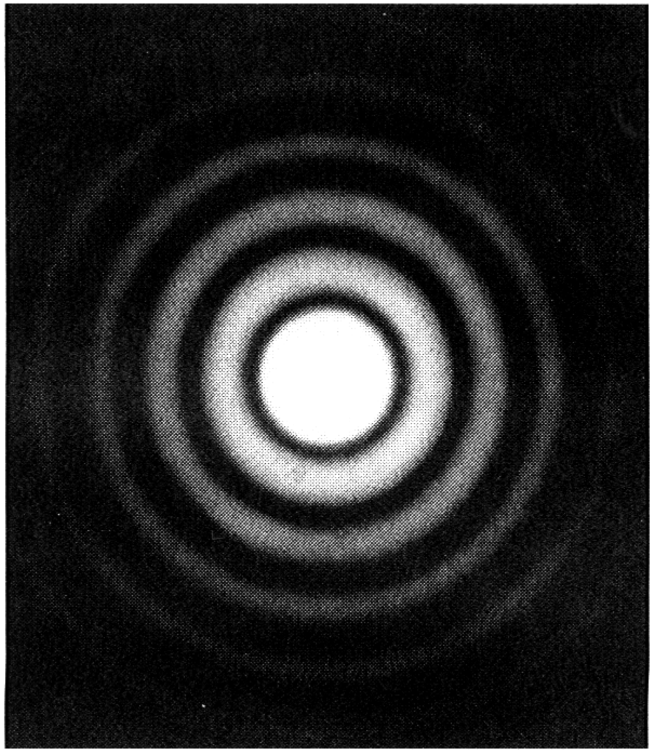


Fig. 37-9 The diffraction pattern of a circular aperture. Note the central maximum and the circular secondary maxima. The figure has been overexposed to bring out these secondary maxima, which are much less intense than the central maximum.

Diffracted image of a point source of light thru a lens (circular aperture of size d)

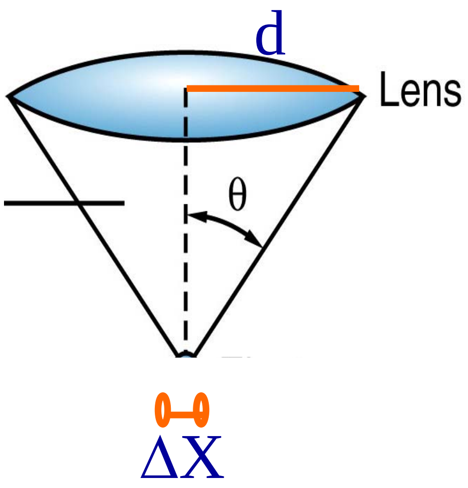
First minimum of diffraction pattern is located by

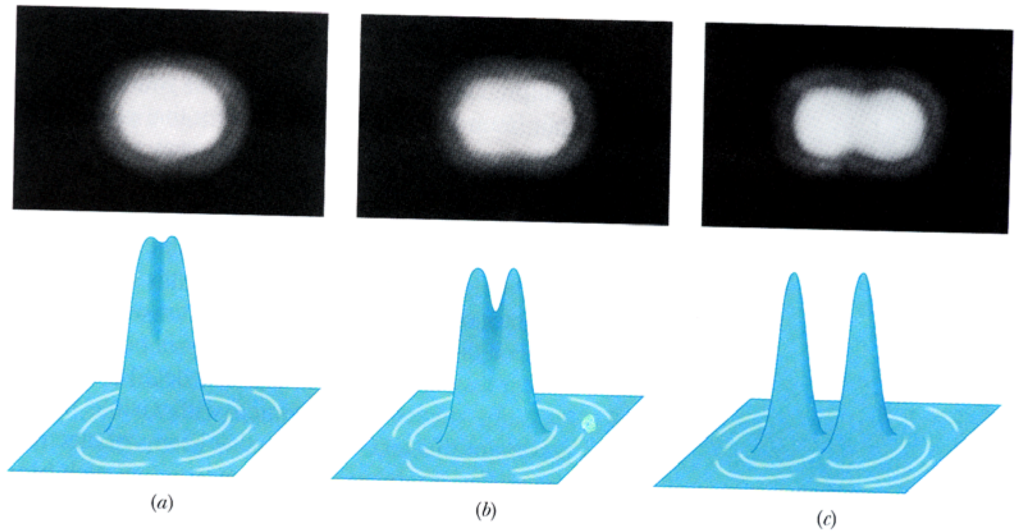
$$\sin\theta = 1.22\frac{\lambda}{d}$$

See previous picture for definitions of ϑ, λ, d

Resolving Power of Light Thru a Lens

Image of 2 separate point sources formed by a converging lens of diameter d, ability to resolve them depends on λ & d because of the Inherent diffraction in image formation



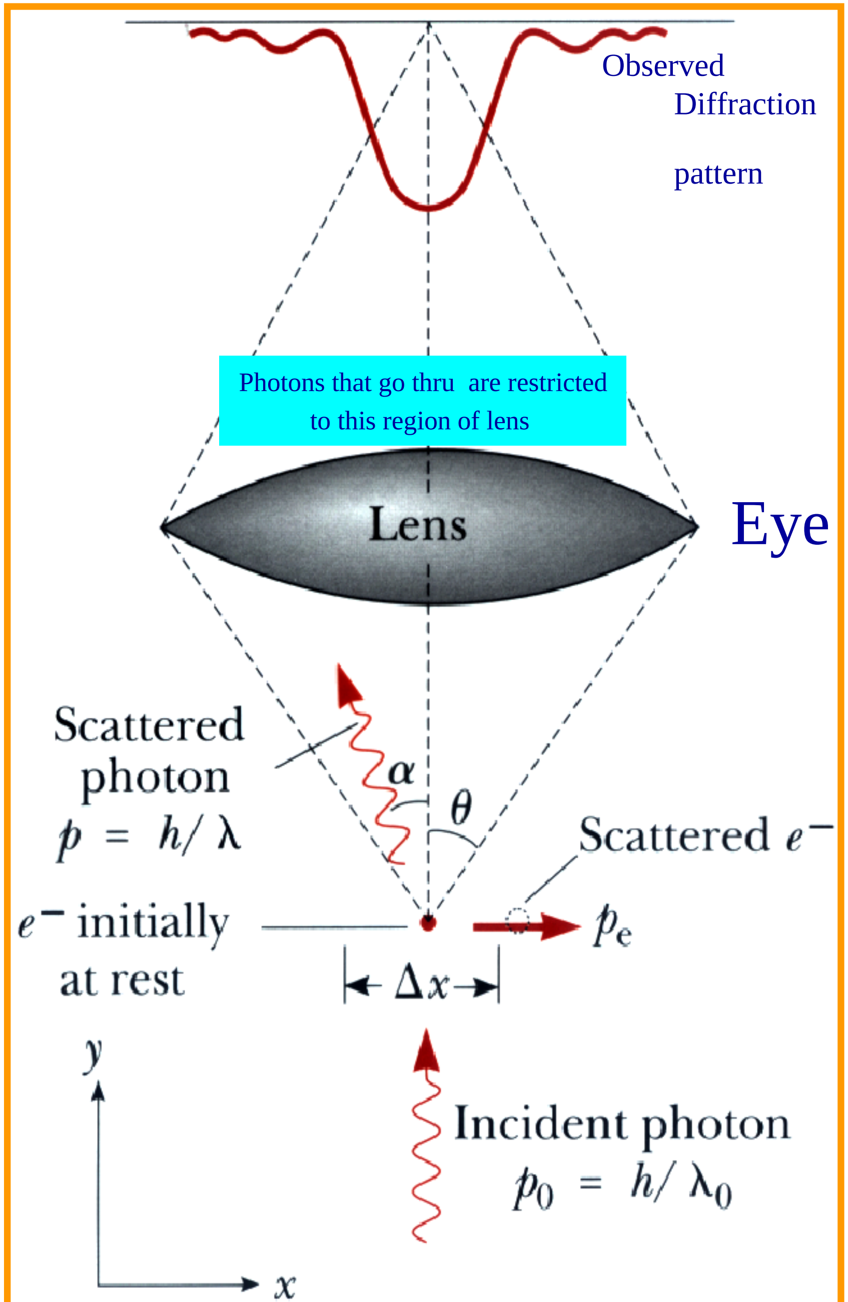


Not resolved barely resolved resolved

$$\text{Resolving power } \Delta x \simeq \frac{\lambda}{2\sin\theta}$$

ϑ Depends on d

Putting it all together: act of Observing an electron



- Incident light (p,λ) scatters off electron
- To be collected by lens → γ must scatter thru angle α
  - $-\vartheta \leq \alpha \leq \vartheta$
- Due to Compton scatter, electron picks up momentum
  - P_X , P_Y

$$-\frac{h}{\lambda}\sin\theta \leq P_x \leq \frac{h}{\lambda}\sin\theta$$

electron momentum uncertainty is

$$\Delta p \cong \frac{\sim 2h}{\lambda}\sin\theta$$

- After passing thru lens, photon diffracts, lands somewhere on screen, image (of electron) is fuzzy
- How fuzzy ? Optics says shortest distance between two resolvable points is :

$$\Delta x = \frac{\lambda}{2\sin\theta}$$

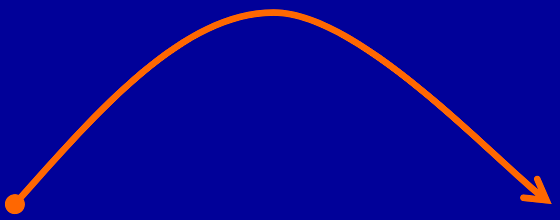
- Larger the lens radius, larger the $\vartheta \Rightarrow$ better resolution

$$\Rightarrow \Delta p.\Delta x \simeq \left(\frac{2h\sin\theta}{\lambda}\right)\left(\frac{\lambda}{2\sin\theta}\right) = h$$

$$\Rightarrow \Delta p.\Delta x \geq \hbar/2$$

Pseudo-Philosophical Aftermath of Uncertainty Principle

- Newtonian Physics & Deterministic physics topples over
  - Newton’s laws told you all you needed to know about trajectory of a particle
    - Apply a force, watch the particle go !
      - Know every thing ! X, v, p , F, a
      - Can predict exact trajectory of particle if you had perfect device



- No so in the subatomic world !
  - Of small momenta, forces, energies
  - Cant predict anything exactly
    - Can only predict probabilities
      - There is so much chance that the particle landed here or there
      - Cant be sure !....cognizant of the errors of thy observations

Philosophers went nuts !...what has happened to nature

Philosophers just talk, don’t do real life experiments!

Can Electrons Exist Within the Nucleus?

- Example of "where in the world is Carmen San Diego" !
- Size of Nucleus : d = 1.0 x 10^{-14} m
- Electron somewhere within nucleus …don't know exactly where
- Take $\Delta x = d/2 \Rightarrow$ error in knowledge of its momentum $\Delta p \geq h/(4\pi \Delta x)$..now do the numbers using the Heisenberg Uncertainty principle

$$\Delta p_x \geq \frac{\hbar}{2\Delta x} = \frac{6.58\times10^{-16}eV.s}{1.0\times10^{-14}m}\frac{3.0\times10^{8}m/s}{c}$$

$$\geq 2.0\times10^{7}\frac{eV}{c}$$

so electron momentum can be -20 MeV/c $\leq p_x \leq 20$ MeV/c

Looks large, lets go relativistic in calculation (cant hurt)

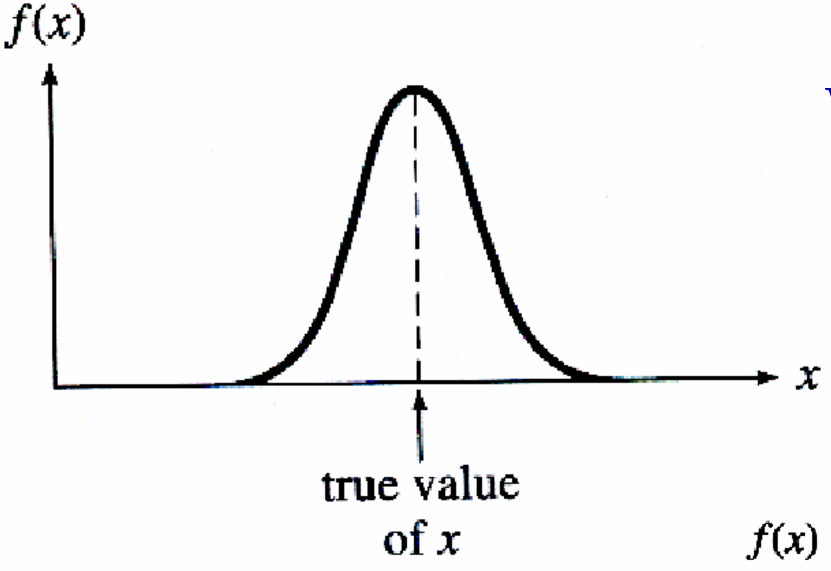
$\mathrm{E}^2 = (pc)^2 + (m_e c^2)^2$, substitute #s $\Rightarrow \mathrm{E}^2 > 400(MeV)^2$

$\Rightarrow E \geq 20MeV$, Kinetic energy KE = E - $m_e c^2 \geq 19.2$ MeV

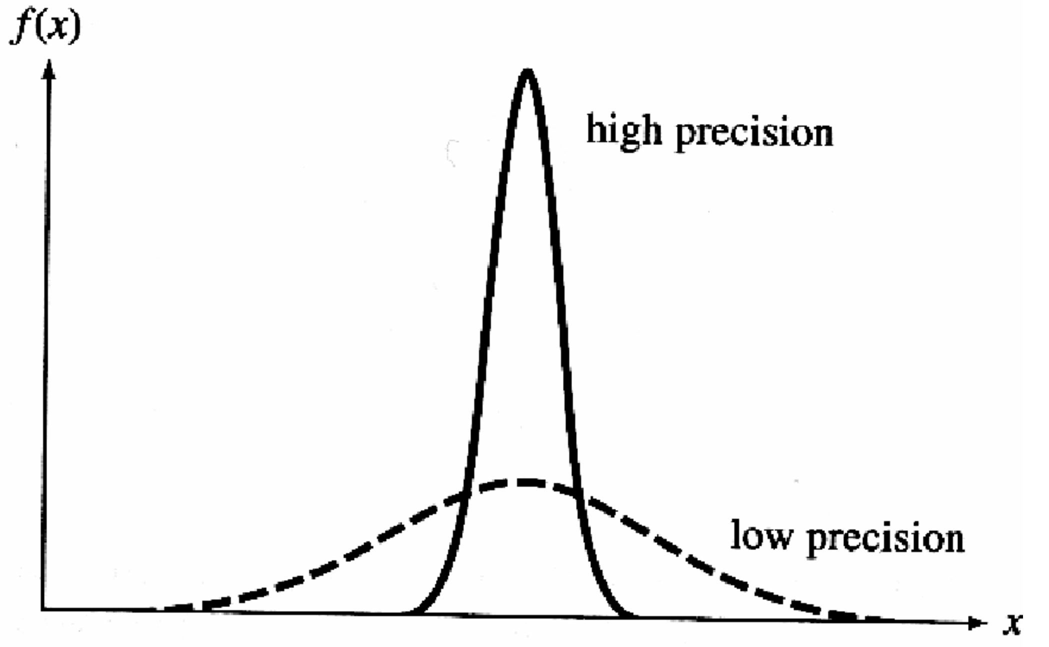
E>> 13.6 eV, even larger than typical energy in radioactivity

$\Rightarrow$ Electron can not exist inside Nucleus

Measurement Error : Δ



What do I weigh : x = 1000 +- 700 kg



Measurement Error : Δ

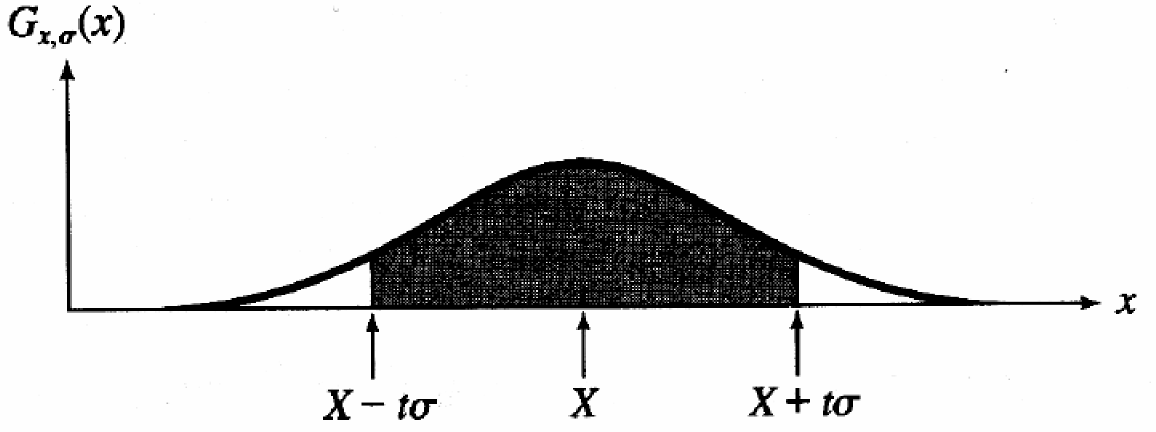
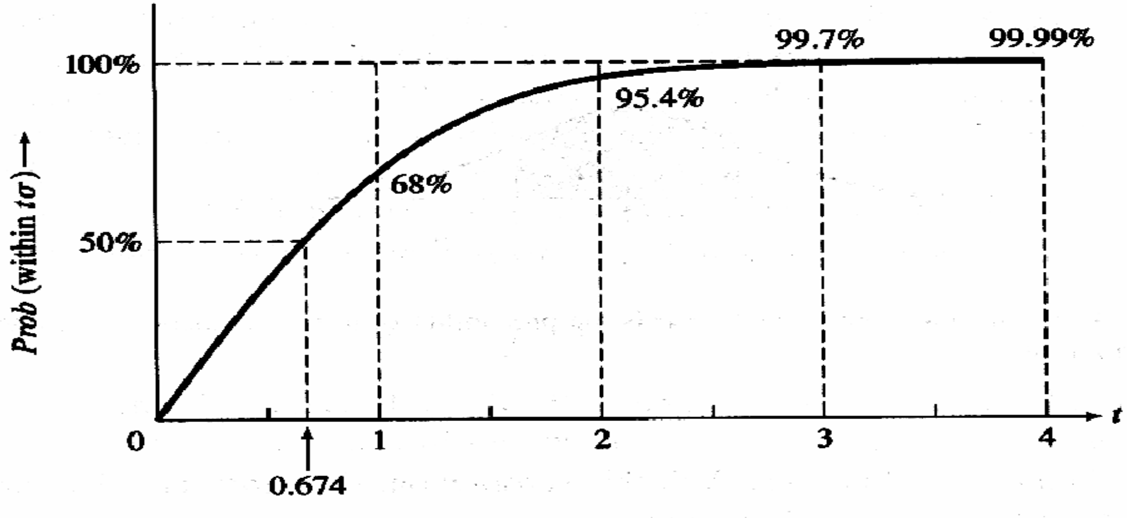


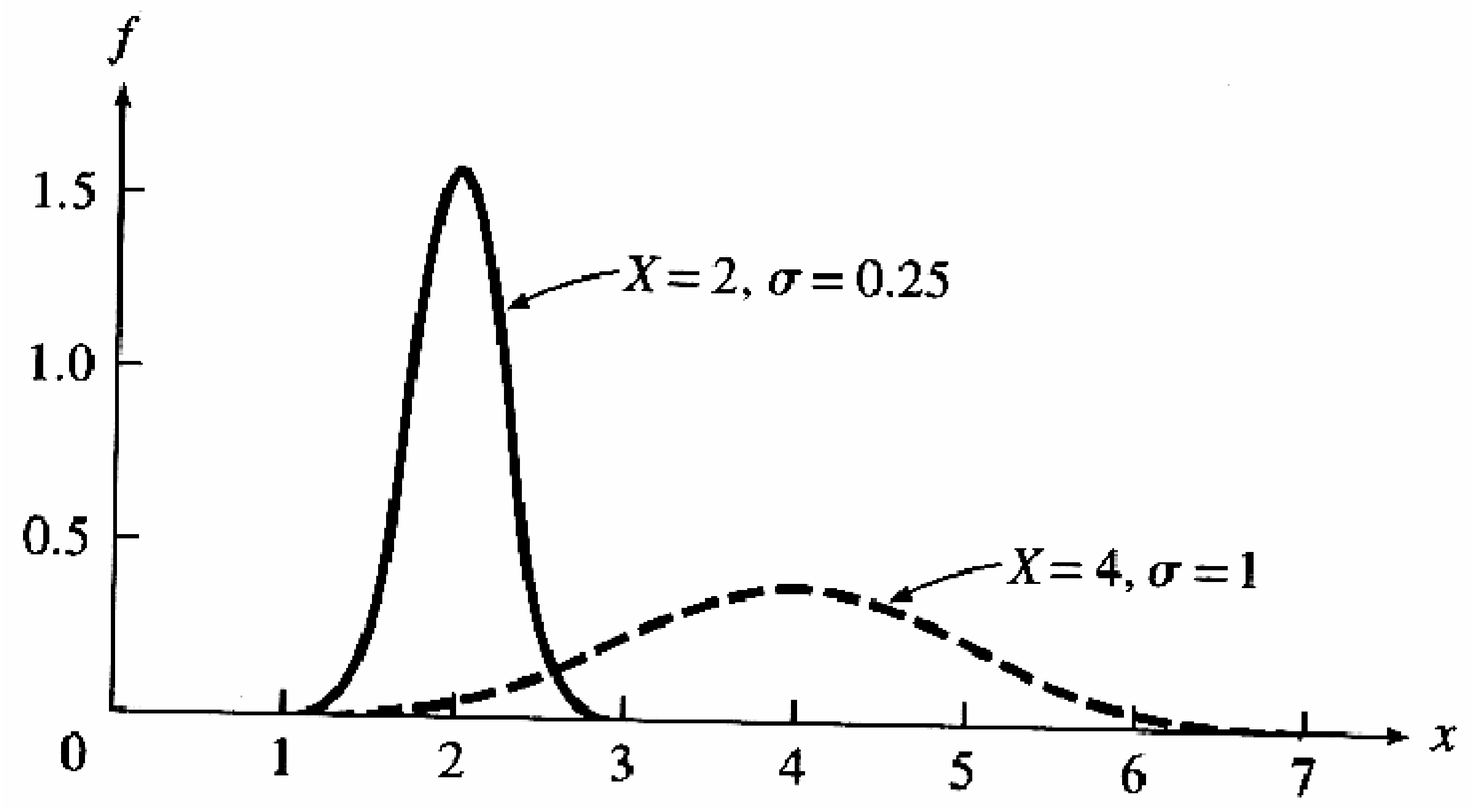
Figure 5.12. The shaded area between $X \pm t\sigma$ is the probability of a measurement within t standard deviations of X.



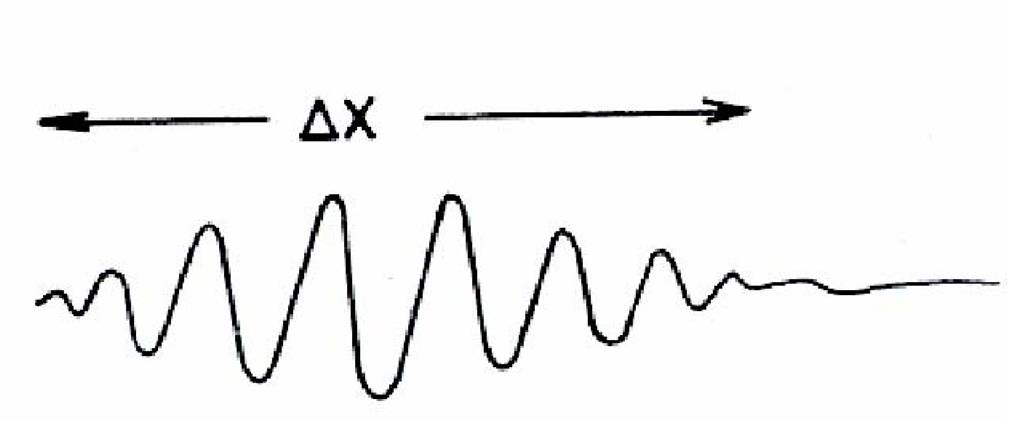
t	0	0.25	0.5	0.75	1.0	1.25	1.5	1.75	2.0	2.5	3.0	3.5	4.0
Prob (%)	0	20	38	55	68	79	87	92	95.4	98.8	99.7	99.95	99.99

Measurement Error : Δ

(dis?) Agreement between Measurements



Wave Packets & Matter Waves



What is the Wave Length of this wave packet?

$$\lambda-\Delta\lambda < \lambda < \lambda+\Delta\lambda$$

De Broglie wavelength $\lambda = h/p$

→Momentum Uncertainty: $p-\Delta p < p < p+\Delta p$

Similarly for frequency ω or f

$$\omega-\Delta\omega < \omega < \omega+\Delta\omega$$

Planck's condition $E = hf = h\omega/2$

→ $E-\Delta E < E < E + \Delta E$

Wave Packets & Uncertainty Principle

in space x: $\boxed{\Delta k.\Delta x = \pi}$ $\Rightarrow$ since $\mathrm{k} = \dfrac{2\pi}{\lambda}, \mathrm{p} = \dfrac{\mathrm{h}}{\lambda}$

$$\Rightarrow \quad \boxed{\Delta p.\Delta x = h/2}$$

usually one writes $\boxed{\Delta p.\Delta x \geq \hbar/2}$ approximate relation

In time t : $\boxed{\Delta w.\Delta t = \pi}$ $\Rightarrow$ since $\omega = 2\pi f, E = hf$

$$\Rightarrow \quad \boxed{\Delta E.\Delta t = h/2}$$

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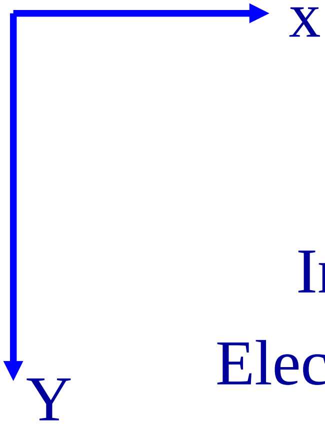
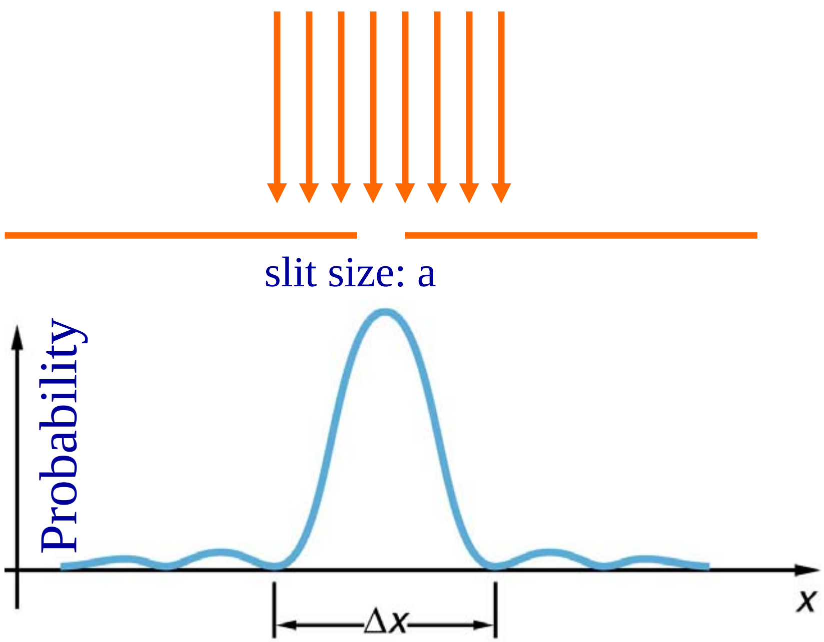
What do these inequalities mean physically?

Matter Diffraction & Uncertainty Principle

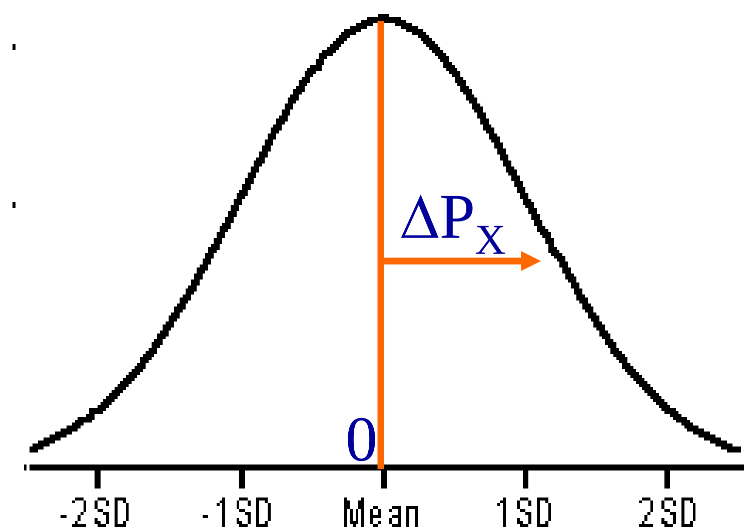


Incident
Electron beam
In Y direction

Momentum measurement beyond
Slit show particle not moving exactly
in Y direction, develops a X component
Of motion $\Delta P_X = h/(2\pi\ a)$



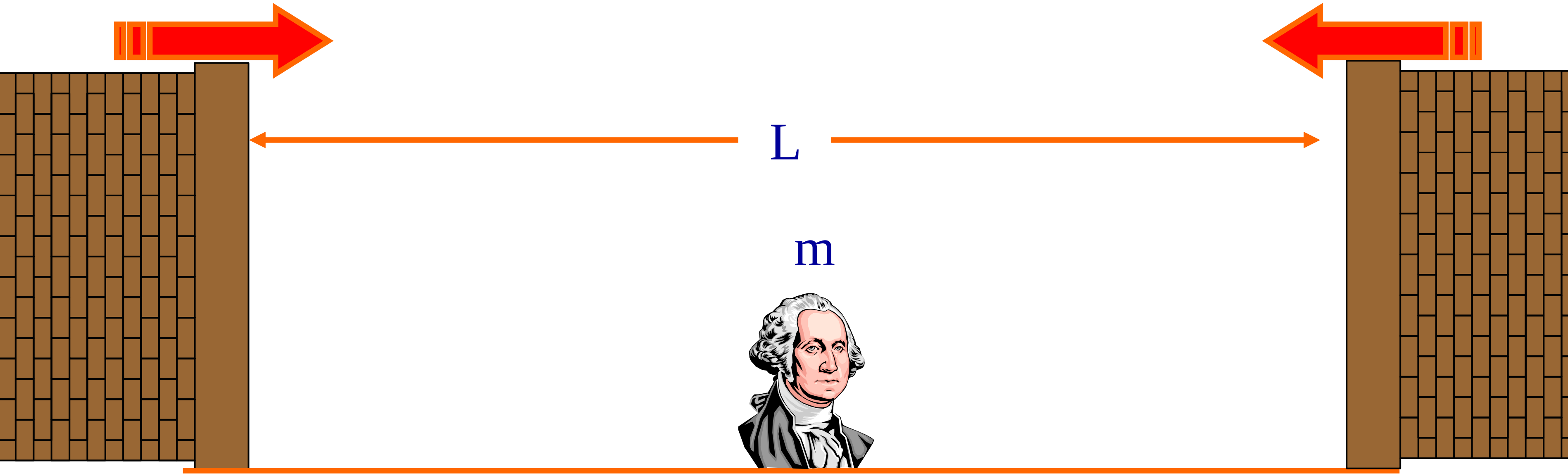
Diffraction pattern
seen on screen

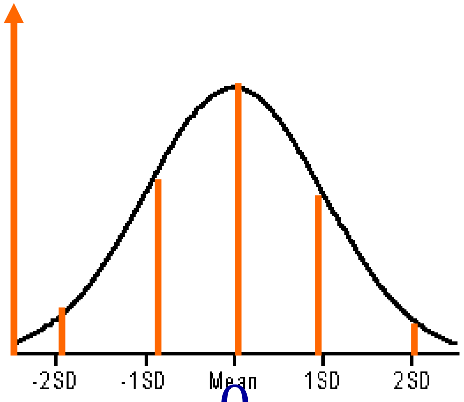


X component P_X of momentum

Particle at Rest Between Two Walls

- Object of mass M at rest between two walls originally at infinity
- What happens to our perception of George as the walls are brought in ?





George's Momentum p

On average, measure <p> = 0

but there are quite large fluctuations!

Width of Distribution $= \Delta P$

$$\Delta P = \sqrt{(P^2)_{ave} - (P_{ave})^2}; \quad \Delta P \sim \frac{\hbar}{L}$$